

OPTIMISING ASH WOOD CHAIRS

Stig-Inge Gustafsson,
IKP/Wood Technology Institute of Technology,
S 58183 Linköping, Sweden

Abstract

In Sweden, forest research has been emphasised on mainly two species of wood, i.e. pine and spruce. However, we have also a number of hardwoods which could be utilised for furniture manufacturing, cabinets etc. Nowadays, these hardwoods are a slumbering resource in our country. Most of our broad leafed species are found as small stands inside our soft wood forests and hence not utilised in the most profitable way. For example, much of our birch wood is ground to paper fibres even if it would be perfect for high valued veneer. Instead, most of our birch veneer is imported from Finland. In order to increase the interest for Swedish hardwoods we therefore have started research in this field and have now designed a chair made of ash wood, *Fraxinus excelsior*. Most chairs are made up of structural elements called indetermined frames which makes it a rather tedious task to analyse the internal forces in the frame. However, by using the Finite Element Method, FEM, it has been possible to reduce this drawback. This paper shows how a chair could be analysed, and designed, by use of methods common in other disciplines than furniture manufacturing. We also present results, in the form of stress-strain diagrams, from tests made on Swedish ash.

INTRODUCTION

Furniture design is almost always based on experience from traditions in handcraft manufacturing. As far as we know, no carpentry in Sweden or elsewhere, use static analyses for finding the internal forces inside the wooden members of a e. g. a chair. However, some academic research groups have shown interest in this topic and the first to mention must be the work of C. A. Eckelman starting already in 1966, see Reference [1]. In the paper he shows that a chair could be analysed as a structure for taking up loads. By use of strain gauges he also presented some values for the maximum moments at different parts of the chair. Eckelman also continued his efforts and we have found seven additional papers published the latest three years 1992-1995. He has also been the author of some text books and among them, Reference [2], seems to be the one most comprehensive. In this book he analyses, not only chairs, but also book shelves, sofas etc. as structural devices.

In Poland the research seems to have been concentrated around the Poznan University. Several papers have been published and some of them also dealt with Finite Element Methods, FEM, and chairs. The most interesting paper for us, however, seems to only have been published in Polish, Reference [3].

Another research team in USA has also published papers in this field, see Reference [4]. They have examined analytical and experimental results for chairs, sofas and book shelves. Further, they have studied the joints between different chair members in detail.

A number of other researchers have dealt with different types of furniture, e.g. cabinets, and here only three of them are mentioned, References [5], [6] and [7]. We have also contributed ourselves to this scientific field with, up to now, three papers, viz. References [8], [9] and [10].

MATERIALS TESTING

When structures are to be analysed by use of solid mechanics there is a need for knowledge about the properties of the wood used in that structure. In the literature it is easy to find values for e.g. the Young's modulus, the strength in tension and compression and so forth. We have used a table found in Reference [11], page 163, where such values for European ash are presented. A closer look, however, reveals that the modulus of elasticity is calculated from tests in bending and not from tensile or compression tests. This is understandable because the properties for wood differs so much between tension and compression. We, therefore, need more experience of how, for instance, ash wood behaves when it is exposed to loads in different directions. Below, some tests are shown from our own laboratory in order to improve the situation. In Figure 1 a tensile test for an ash rod specimen is presented.

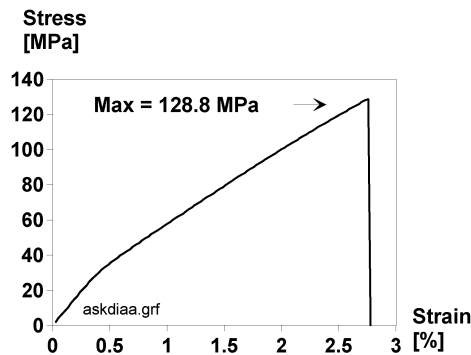


Figure 1: Tensile test for European ash, *Fraxinus excelsior*

The rods we used have been manufactured in accordance with details found in Reference [12] page 324. The tensile test in Figure 1 shows two different moduli of elasticity. The highest modulus, see the slope of the curve, is found for a stress below approximately 35 MPa, equalling about 7,300 MPa, while the lower one is found above this point, i.e. 3,900 MPa. Interesting is also to note the behaviour of the rupture. The stress increased in a linear way up to the point where a total rupture occurred. Usually, there is a significant part of the stress - strain line which is not linear. Instead the Young's modulus gets lower and lower and finally the slope is negative. Interesting to note is also the fact

that our specimens did not show the high strength that is reported in literature. In Reference [12] page 295, Young's modulus for ash is 16,100 MPa, which is substantially higher than we have found. From Figure 1 it is also obvious that the tensile strength for ash found in Reference [11], i.e. 161 MPa, is significantly higher than the one we experienced, 129 MPa. To a small part this could be explained by the relatively high moisture content in our specimens which was 8.5 %.

We also elaborated some compression tests and one of them is shown in Figure 2.

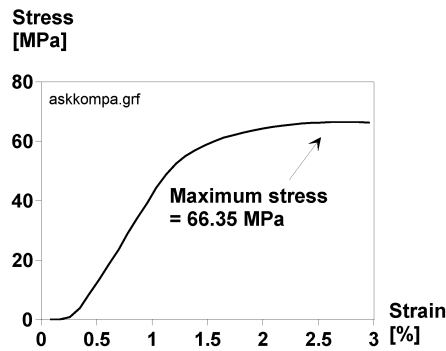


Figure 2: Compression test for European ash, *Fraxinus excelsior*

The maximum stress was 66.4 MPa which is somewhat higher than found in Reference [11], 51 MPa. Young's modulus for compression was calculated to approximately 5,600 MPa, i.e. a bit lower than the one found for tension.

Further, we made some bending tests and one of those is presented in Figure 3.

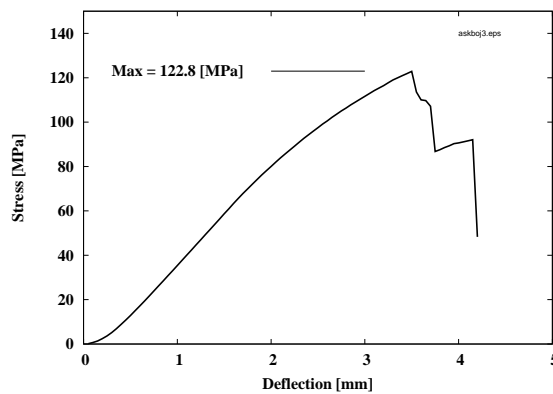


Figure 3: Bending test for European ash, *Fraxinus excelsior*

The maximum stress, the Modulus of Rupture or MOR, for our specimen

was found to be 122.8 MPa which is almost the same as the one found in literature, Reference [11], which is 118 MPa. The Modulus of Elasticity, MOE, was calculated to approximately 9,970 MPa which is somewhat lower than the value found in Reference [11], 13,100 MPa. The highest values from our bending tests was 139.6 and 10,609 MPa for the MOR and MOE respectively.

The tests above show that information in literature not always could be used without consideration. The largest discrepancies were found for the tensile tests where our tests showed a significantly lower strength than presented in e.g. Reference [11]. The behaviour in rupture differs also a lot from "classic" theory and for example ash wood seems to reach the point of rupture in a fairly linear slope and then break immediately. This is different from our birch tests found in Reference [9].

CASE STUDY

Common chairs include a stretcher between the front and back rails. This is very wise because this structural member will reduce the moment in the back rail immediately under the seat, in a significant way. Usually, this stretcher is absolutely horizontal, but we have shown that this is not an optimal location. Instead, the stretcher should be lower at the back rail and higher at the front chair leg. In Reference [8] we presented the solution shown in Figure 4.

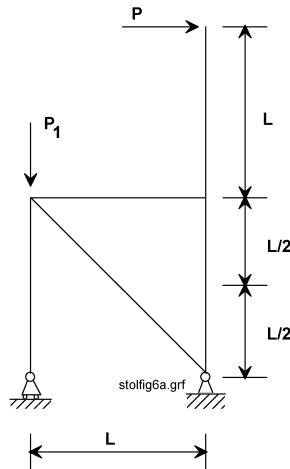


Figure 4: Chair with diagonal stretcher, [8]

We have also shown that the moments in the stretcher are very small but that it was subject to axial forces that could not be neglected. The stretcher we designed deflected in the direction perpendicular to the frame plane, due to stability problems, see Reference [10], and therefore we concluded that it should have a cross section in the form of, almost, a square or a circle, and not a pronounced rectangle. The moment in the right part of the seat was

about half the value of that in the back rail above the seat. The seat rail could therefore be thinner than it originally was, which also should make the joint easier to construct. In Reference [9] the seat and the back rails had identical cross sections. The dowel joint used in the construction made it necessary to make a relatively large hole in the back rail which therefore lost most of its strength and, subsequently, the chair could not endure the expected loads.

Consider for a start the seat rail. It must endure the load of a sitting person and it could be assumed that the load is evenly distributed along the rail. Assume also that the person sitting on the chair weights about 90 kg. In our case the rail is 0.4 m in length, L , and thus the load, q , will become:

$$\frac{90 \times 9.81}{0.4 \times 2} = 1,104 \text{ N/m}$$

Note that we have two seat rails, one on each side of the chair. The moment in the middle of the rail is calculated as:

$$\frac{q \times L^2}{8} = 22 \text{ Nm.}$$

Assume, further, a rail with a cross section of 0.01 times 0.02 m and use the expression:

$$\sigma = \frac{M \times z}{I} \quad \text{where} \quad I = \frac{b \times h^3}{12}$$

and we end up with a stress, σ , equalling 33 MPa. This is substantially lower than the allowed stress for ash which due to Reference [11], page 164, is about 118 MPa which is the MOR in bending. Our own tests showed the same magnitude. We know from previous work, see Reference [9] that the moment in the right end of the rail is about 60 Nm so this later moment will be decisive for the seat rail. In indeterminate frames we must use the deformations of the structure, in order to calculate the internal forces. In a determined frame we only need static equations. In our case, see Figure 4, the frame is indeterminate because the members are firmly connected to each other. When indeterminate frames are analysed we also must make assumptions of the cross sections for each member before it is possible to solve the problem. For a start, assume therefore a back rail with a cross section of 0.015 \times 0.03 m. We have used 0.01 \times 0.03 m in Reference [9] but showed that it was difficult to solve the dowel joints. The seat rail is, with the above discussion in mind, assumed to have a cross section of 0.005 \times 0.02 m and the stretcher 0.01 \times 0.01 m. In Reference [9], 0.005 \times 0.02 m was chosen with resulting stability problems so now a more quadratic cross section is used.

In Figure 5, a simplified frame is presented, where all determinate parts have been excluded.

We will solve this problem with the method of deformations, or displacements, which is one of the finite element methods available, see Reference [13] page 261 and the following, for all details. The first thing to do is, however, to split the problem into two, i. e. one particular and one complementary problem. This is so because of the load, q , between the joints. From so called elementary cases, Reference [13] page 372, we find that this load could be replaced by two moments, one in each end of the seat rail. The moments are $\frac{q \times L^2}{12}$ acting clock

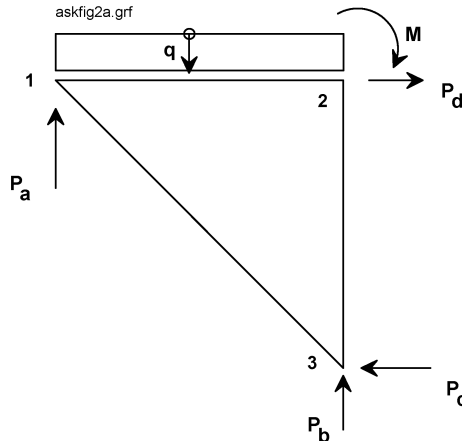


Figure 5: Simplified structural problem of a chair

wise in the right corner, and in the opposite direction in the left corner. We assume that the wooden material in the chair is identical in elastic properties and therefore we, for now, set the Young's modulus to E . The moment of inertia for the rails do not have the same values and, hence, it is convenient to calculate the relationship between them, see Table 1.

Member of the chair	Moment of inertia [m ⁴]	Relation
The back rail	$3.38 \cdot 10^{-8}$	40.6
The seat rail	$3.33 \cdot 10^{-9}$	4.0
The stretcher	$8.33 \cdot 10^{-10}$	1.0

Table 1: Moment of inertia for the wooden members of the chair and their relationship

Now it is time to elaborate the stiffness matrix for the frame. The details of how to achieve this is shown in [10] and therefore it is not repeated here. Only the resulting matrix is shown below:

$$\begin{pmatrix} 18.8 & 8 & 1.41 \\ 8 & 176 & 80 \\ 1.41 & 80 & 162.8 \end{pmatrix} \times \frac{EI}{L} \times \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 14.7 \\ 105.3 \\ 0 \end{pmatrix}$$

The right hand side of the matrix equation shows the moments in the complementary structure. The value 14.7 is the moment in the upper left corner, while 105.3 is the moment emanated from P in Figure 4, which equals 300 N and the length, L , which is 0.4 m. The resulting moment, M in Figure 5, is therefore 120 Nm, which in turn is subtracted by 14.72, emanating, likewise, from the complementary structure. Solving this small equation system results in the unknown rotations, q , where $q_1 = 0.1975$, $q_2 = 0.2975$ and $q_3 = -0.1479$.

The left and right seat rail corners rotates clockwise, while the bottom back rail corner rotates in the opposite direction.

We must now calculate the resulting moments in the beam members of the chair. The method for doing so is shown in Reference [8] and therefore only the result is shown here. M_{12} shows the moment in the left upper end of the seat rail, point 1, and in the direction to point 2, see Figure 5, while M_{21} represents the moment in the right end of the seat rail and so forth. In Table 2 the complementary solution is added to the particular one, which gives us the final result.

Moment	Complementary	Particular	Total
M_{12}	13.85 (T.I.)	14.72 (T.O.)	0.87 (T.O.)
M_{21}	15.85 (T.O.)	14.72 (T.O.)	30.57 (T.O.)
M_{23}	89.42 (T.I.)	0	89.42 (T.I.)
M_{32}	0.34 (T.O.)	0	0.34 (T.O.)
M_{13}	0.87 (T.O.)	0	0.87 (T.O.)
M_{31}	0.35 (T.O.)	0	0.35 (T.O.)

Table 2: Resulting moments in Nm from the calculations

We must also keep track of the direction for the moments and in Table 2, "T.I." means tension at the inside of the frame while "T.O." means tension at the outside.

In Figure 6 these moments have been described in a principal sketch.

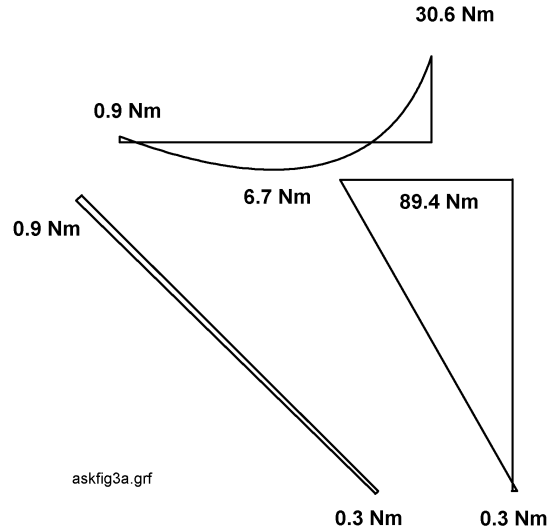


Figure 6: Principal moment graph of chair frame

The scale in Figure 6 is approximate in order to clarify the situation. The maximum moment in the seat rail is about 30 Nm. Using the same procedure as above for calculating the stress in the beam we find that it is about 92 MPa, i.e. somewhat lower than the 118 MPa found in Reference [11]. The maximum moment in the back rail is 120 Nm which implies a stress of 53 MPa which is less than half the allowable load. The joint between the seat and back rails, however, probably makes it necessary to apply the cross section used for the back rail. We also find that the stretcher is exposed to a very small moment.

The axial forces, i.e. compression, in this chair member is calculated to 742 N. By use of the Euler IV case, the critical load for instability equals:

$$P_{critical} = \frac{4 \times \pi^2 \times E \times I}{L^2} = 1,349 \quad N$$

and therefore it seems as if no risk for collapse is present. The value for I is found in Table 1, while L is $0.4 \times 2^{0.5}$ m. We have also used the Modulus of Elasticity, MOE, for E in the expression above, which equals 13,130 MPa, a value found in [11] page 164. If we use our own compression modulus, i.e. 5,600 MPa the critical load will be lower than the one present in the stretcher and there might therefore be a risk of collapse here.

We also have to pay closer attention to the joints. The seat rail had a cross section of 0.005 times 0.02 m while the back rail had a cross section of 0.015 times 0.03 m. In order to join the two members to each other we must make a hole in the back rail. Suppose that this hole penetrates the back rail in its entirety. We will then have about 0.005 m of wood on each side of the hole in the seat rail. The moment of inertia for this part of the back rail will become 2.25×10^{-8} m⁴ and subsequently the stress 80 MPa when 120 Nm is applied. In order to manufacture this joint we must use a so called long hole bore. The edges of the seat rail must therefore be rounded as half a circle in the top and bottom of the rail, see Figure 7.

The same is valid for the stretcher but it will then achieve a circular cross section. With the axial force in mind we think that only half the back rail should be penetrated. This will ascertain that the stretcher could not be punched through the back rail even if the adhesive fails, see Figure 8.

The joint where the stretcher meets the seat rail is only exposed to a very small moment, see Figure 6. The seat rail, however, is thinner than the stretcher and thus the method with a bored hole is not applicable. We suggest that a small notch is sawed out from the seat rail where half the stretcher will fit in. Part of the stretcher is after this glued to the side of the seat rail, Figure 9.

The connection between the seat and back rails have also been studied by use of another FEM method, the one with plain stress elements. The structure was then divided into thirteen elements with 26 nodes, each located in the corners of the elements, see Figure 10 where part of the structure is shown.

The result from the FEM analysis showed that the maximum compression stress in the horizontal direction, i.e. - 38.0 MPa, occurred just above node 2. Maximum tension with almost the same absolute value was found just under node 3 and around node 6, see also Figure 11 where A equals - 30.0, B -20.0 and G +30.0 MPa respectively.

The values from the plain stress method are somewhat lower than those calculated from the beam method. Both calculations show, however, that the stress is lower than the one allowed in literature, and also lower than the one

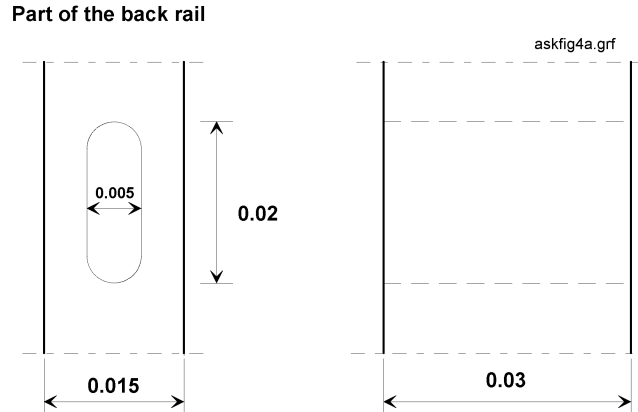


Figure 7: Part of the back rail with hole for the seat rail

found in our own testing equipment, see Figure 1. The value for "stress of rupture" for compression will be the dimensioning criterion with a value of about 51 MPa, see Reference [11] page 164.

In the vertical direction, maximum compression emerged just to the left of nodes 14, 12 and 11 with a value of 45.5 MPa while the maximum tension value was found immediately to the right of node 13, and around node 6, see Figure 12, where A equals -40.0 MPa, and I is +40.0 MPa.

CONCLUSIONS

We have shown that furniture, such as chairs, can be analysed by use of modern computer programs. In our case two types of Finite Element Methods have been used. The overall structure is preferably analysed by use of so called beam elements while details such as joints can be studied more in detail by plain stress elements. The joint is in the second case divided into small, but finite, rectangular parts while the rest of the structure is divided in larger pieces. The effort is because of this emphasised on points where the stress and strain is of vital interest. Using these methods, show that ordinary chairs are far too strong and material is wasted from the view of solid mechanics. Important is also the fact that much more knowledge is needed about the material wood. Our tests of ash wood in tension, compression and bending shows that values taken from literature are not always relevant.

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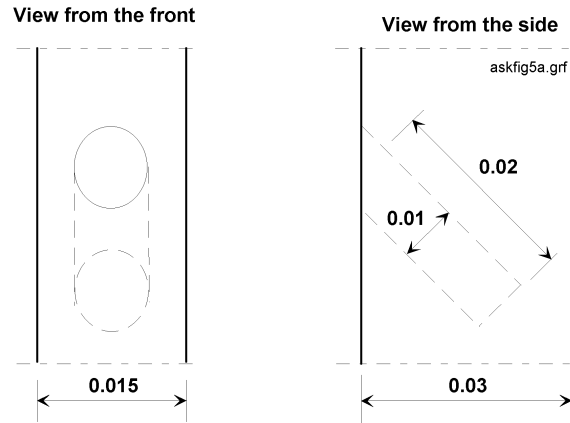


Figure 8: Back rail with hole for the stretcher

Nilsson who provided the FEM-program PCFEMP, which was used for all the calculations. Acknowledgement is also made to Bo Skoog who elaborated all testings of the wood specimens.

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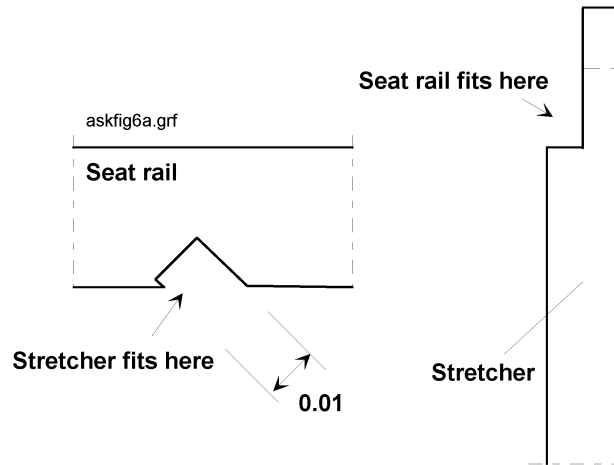


Figure 9: Seat rail and stretcher joint

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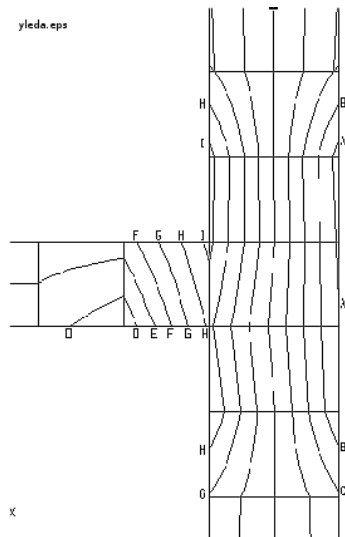


Figure 12: Stress in the vertical direction according to FEM calculations