LIFE CYCLE COSTING FOR CONSTRUCTION

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3 Life cycle costing related to the refurbishment of buildings

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3.1 Introduction

This chapter describes the use of life cycle costing when a building is to be retrofitted. The life cycle cost (LCC) includes all costs that emerge during the life of a building, such as building costs, maintenance costs and operating costs. When the LCC is to be calculated, future costs must be transferred to a base year by use of the present value method. Although the LCC includes all costs, this chapter will only consider those costs related to the heating of the building, or the use of energy in one form or another. Retrofits which allow a cheaper form of cleaning or result in a different aesthetic shape are not included. One other constraint is that all the consequences must be expressed in monetary terms. This chapter, however, deals with the implementation of extra insulation on various building parts, changing windows for a better thermal performance, weatherstripping, exhaust-air heat pumps and different types of heating equipment. The basic view is that the building is considered as an energy system and, at least sometimes, all the energy-conserving measures must be dealt with at the same time if an accurate result is to emerge. Another corner-stone of this chapter is that the retrofit strategy shall be the one with the lowest possible LCC, i.e. the situation must be optimized. Derivative, direct search and linear programming methods are dealt with and an extensive reference list is presented showing the state of the art in the middle of 1991. There are also many examples of real cases in order to highlight various aspects of this subject.

When a building is to be refurbished it is important to consider that it already has a life cycle cost (LCC) whether it is rebuilt or is left as it is. If the LCC is to be the ranking criterion for deciding what to do, it is important to compare the new LCC to the old, or existing, LCC. If the new LCC is lower, it is profitable to rebuild; if the opposite is true, the building should not be refurbished at all. One of the basic concepts in life cycle costing is the present value (PV) which is used for transferring future costs to one base year, where they can be compared properly. There are many papers and books concerning the use of the PV for life-cycle costing, e.g. Marshall (1989) and Flanagan *et al.* (1987, 1989), but only the expressions for calculating the PV will be shown here. The first expression, equation (3.1), shows the PV for a

single cost occurring once in the future, while the second expression, equation (3.2), shows the PV for annually recurring costs:

$$PV_s = C_s \cdot (1+r)^{-n}$$
 (3.1)

$$PV_a = C_a \cdot [1 - (1+r)^{-m}]/r$$
(3.2)

where r = the real discount rate, n = the number of years until the signal cost C_s occurs and m = the number of years in which the annual costs C_a occur.

Equation (3.1) is suitable for calculating the PV for, for example, window retrofits or insulation measures, while equation (3.2) is used for energy and other annually recurring costs. Before it is possible to start with the PV calculations it is necessary to find the costs C_s and C_a and proper values for r, n and m. Unfortunately, there are difficulties here, because of uncertainties both for the costs as well as for the economic factors. C_s might be found in certain price lists (see Gustafsson and Karlsson, 1988a, for an example of the calculation), so if these are accurate the problem is partially solved. C_a however, is influenced by the thermal state of the building and large uncertainties due to the fluctuating energy price in the future. The real discount rate, r, cannot be set to an accurate value valid for all investors, and different authors recommend values between 3 and 11%. Van Dyke and Hu (1989) even show that some investors have dealt with negative rates. Note that inflation is excluded from these values. The value for n, the number of years until a retrofit is inevitable, can likewise not be predicted accurately and the same is valid for the projected life of the building, m. Many authors have dealt with this problem, and papers are frequently published in, for example, proceedings from CIB conferences.

From the above discussion it might seem hopeless to calculate anything at all and believe in the result. However, every time an investment is made, values for all the variables are set even if the investor is unconscious of them. A closer analysis will often reveal limits within which the values might move, and then it will be possible to calculate the result using different values for each calculation. Without computers this is a very tedious task and is one of the reasons why life cycle costing has not been frequently used. By using computers, large problems can be solved in a few minutes. It is nowadays possible to calculate the result for a number of different scenarios and then examine the situation in a so-called 'sensitivity analysis'. Several interesting results will then occur and general conclusions can be drawn in spite of the uncertainties in the input data.

3.2 Insulation measures

The optimal thickness of extra insulation is influenced by a number of variables; the building cost, the climatic conditions, the energy cost, etc. A

suitable way to describe the building cost (BC) is as follows:

$$BC_{ins} = C_1 + C_2 + C_3 \cdot t_{ins} \tag{3.3}$$

where C_1 = the amount in Swedish kroner (SEK) $/m^2$ for scaffolds, demolition, etc., C_2 = the amount in SEK/ m^2 for the new insulation, studs, etc., C_3 = the amount in SEK/ m^2 per metre for new insulation, studs, etc. and $t_{\rm ins}$ = the thickness of new insulation in metres (1 US \$ = 6 SEK approx.).

The reason for splitting up the cost into three parts is the influence of the existing life of the building asset. As an example, consider an external wall. The facade is in a rather poor shape, but nonetheless, the retrofitting of it might not be necessary for, say, ten more years. The C_1 coefficient shows the amount of money to be paid at year 10 whether energy-conserving measures are taken or not. This retrofitting is called inevitable or unavoidable and is very important to take into consideration. Assume that C_1 equals 500 and that the wall must be retrofitted in year 10 when it is unavoidable. The real discount rate is set to 5%, while the project life is assumed to be 50 years. The life of the new facade is assumed to be 30 years. Consequently the PV of the retrofitting, by equation (3.1), will become:

$$500 \cdot (1 + 0.05)^{-10} + 500 \cdot (1 + 0.05)^{-40} - [(30 - 10)/30] \cdot 500 \cdot (1 + 0.05)^{-50}$$

= 349.0

This PV calculation shows the value of the money invested in year 10 and year 10 + 30. Further, the salvage value at year 50 is subtracted. This PV must be added to the LCC of the existing building, because it shows the inevitable retrofit cost. If the wall is retrofitted now, at the present time, the PV calculation will become:

$$500 \cdot (1 + 0.05)^{-0} + 500 \cdot (1 + 0.05)^{-30} + [(30 - 20)130] \cdot 500 \cdot (1 + 0.05)^{-50}$$

= 601.21

From this it is shown that the increase of the cost for retrofitting now, instead of at year 10, is

$$601.2 - 349.0 = 252.2$$

The cost of 601.2 must thus be added to the new LCC. Closer details about PV calculations can be found in Ruegg and Petersen (1987).

After this, the cost for the insulation itself must be included. However, it is assumed that insulation is only installed once, at the base year, so it is not necessary to calculate the PV for the additional insulation. At this point in the examination, it is not possible to determine how much insulation is to be installed and subsequently included in the cost $C_3 \cdot t_{\text{ins}}$ in equation (3.3). It has been shown (Gustafsson, 1986) that the new U-value for an extra insulated asset may be expressed as:

$$U_{\text{new}} = U_{\text{exi}} \cdot k_{\text{new}} / (k_{\text{new}} + U_{\text{exi}} \cdot t_{\text{ins}}) \tag{3.4}$$

where $U_{\rm exi}$ = the existing U-value in ${\rm Wm^{-2}K^{-1}}$, and $k_{\rm exi}$ = the thermal conductivity in the extra insulation in ${\rm Wm^{-1}K^{-1}}$.

Multiplying the *U*-value by, firstly, the area of a building asset, secondly, the number of degree hours for the building site and, thirdly, the energy price, will result in the annual cost for the energy flow through the asset. Further, the annual cost must be multiplied by the PV factor, calculated by using equation (3.2) which will yield the total energy cost for a number of years. Using a real discount rate of 5% and a project life of 50 years makes the PV factor equal to 18.26. In Malmö, in the south of Sweden, the number of degree hours for one year equals 114008. It has been assumed that one degree hour is generated for each hour that the desired indoor temperature, 21°C, is higher than the outdoor temperature. (See Gustafsson (1986) for all details about degree hour calculations.) If the energy cost is 0.40 SEK/kWh, the area of the building asset 200 m², with an existing *U*-value of 0.8 and a *k*-value for the new insulation 0.04 Wm⁻¹K⁻¹, the cost (TC) in SEK for the energy flow through the building asset will become:

$$TC_{\text{energy}} = 114008 \cdot 0.40 \cdot 200 \cdot 0.8 \cdot 0.04 \cdot 10^{-3} \cdot 18.26 / (0.04 + 0.8 \cdot t_{\text{ins}})$$
$$= 5329 / (0.04 + 0.8t_{\text{ins}})$$

When the building is extra insulated there is also a cost for the insulation and for correctly locating it. Assuming that the constant C_2 equals $100 \, \text{SEK/m}^2$ and C_3 equals $600 \, \text{SEK/m}^2/\text{m}$, according to equation (3.3) the result for the building cost in SEK for the asset will be:

$$TC_{building} = 200 \cdot (601.2 + 100 + 600 \cdot t_{ins}) = 140240 + 120000 \cdot t_{ins}$$

The problem now is to minimize the sum of the energy and the building cost; this is done using the derivative of this sum which is set to zero. The way to do this is shown by Gustafsson (1986), but the result is that the optimal level of insulation in metres becomes:

$$t_{\text{opt}} = -(0.04/0.8) + (5329/120000 \cdot 0.8)^{0.5} = 0.186$$

Inserting this value for optimal level of insulation as $t_{\rm ins}$ in the equation above will result in a LCC current of 190 785 SEK. This cost is now to be compared to the LCC if the building is left as it is. For the current asset this is:

$$LCC_{exi} = 200.349.0 + 5329/(0.04 + 0.8.0) = 203025 SEK$$

The existing LCC is thus higher than the new one. Even if the difference is as small as about 13 000 SEK, it is profitable to insulate the asset with the optimal amount of new insulation. In Figure 3.1 the situation is shown graphically. As can be seen, the existing LCC is higher than the optimal new LCC. If, however, the inevitable costs could be decreased, for example by assuming that the remaining life of the asset envelope is increased, the existing LCC will also decrease, and at a certain point it becomes better to leave the

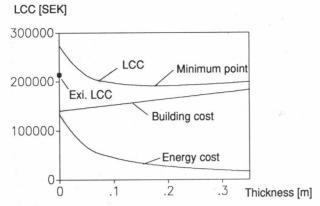


Figure 3.1 Graphic view of insulation optimization.

building as it is. From Figure 3.1 it is obvious that enough insulation must be applied. This limit is in the case above approximately 0.07 m. If less insulation is used, the retrofit will become unprofitable. If too much insulation is installed the same might happen, but if the graph is studied even in detail, this fact could not be observed. It is even better to use a 0.35 m thickness of insulation than not to insulate at all. In Gustafsson (1988) a thorough examination is made of all the parameters concerned.

3.3 Replacing windows

When the replacement of windows is being considered it is not easy to find a continuous function to derive in order to find the best solution, although there have been some attempts to find such a function (Markus, 1979). Instead, it has been shown that it is preferable to compare different sets of windows with each other. The existing LCC is thus compared with the new LCC for a number of alternatives. It is very important to find not just one solution with a lower LCC, but the lowest solution of all. It is also important to consider the fact that a thermally better window normally reflects solar radiation to a higher degree. This fact can be dealt with by use of a so-called shading factor. The situation will therefore differ for various orientations of the windows. The best solution may be, in the Northern hemisphere, to keep the double-glazed windows oriented to the south, while changing to triple-glazed windows to the north. Life cycle costing and windows are dealt with in more detail in Gustafsson and Karlsson (1991). The building cost for windows may be expressed as (Gustafsson, 1986):

$$BC_{\mathbf{w}} = C_1 + C_2 \cdot A_{\mathbf{w}} \tag{3.5}$$

where $C_1 = a$ constant in SEK for each window, $C_2 = a$ constant in SEK/m² for each window and $A_w =$ the area in m² for one window.

Here, BC_w will appear whenever there is a change of a window and the expression is consequently used in a different way from equation (3.3).

3.4 Weatherstripping

Mostly it is profitable to decrease the ventilation flow through the building. This can be accomplished by caulking windows and doors. The cost for this measure is often very low compared to other energy retrofits, but nonetheless it is not always the best way to act, especially when exhaust-air heat pumps are part of the solution. It is also important to consider that it is necessary to ventilate the building; too much weatherstripping might make them unhealthy. When using life cycle costing, these facts are often hard to include in the calculations and only the energy costs are dealt with here. Suppose a building has fifty windows and doors to caulk. If the cost for caulking is 200 SEK/item the total cost will become 10000 SEK. Further, assume that the weatherstripping must be repeated after 10 years. The PV cost will thus become approximately 23 600 SEK if a 5% discount rate and a 50-year project life are used. If the volume of the building is 5000 m³ and the ventilation rate is 0.8 renewals per hour, the flow is $4800 \,\mathrm{m}^3/\mathrm{h}$. The heat capacity for air is about 1.005 kJ kg⁻¹ K⁻¹ and the density approximately 1.18 kg/m³. Consequently the heat flow can be calculated to about $5\,700\,\mathrm{kJ}\,\mathrm{K}^{-1}\,\mathrm{h}^{-1}$. If the same number of degree hours as before is assumed to be valid, i.e. 114 008, the energy flow will become 180.5 MWh/year. Using the PV factor 18.26 and an energy price of 0.4 SEK/kWh, as before, the total energy cost will become 451 000 SEK. If the ventilation flow is decreased to, say, 0.2 renewals per hour, this cost will become 338 000 SEK. It is obvious that weatherstripping, in this example, will be profitable.

3.5 Exhaust-air heat pump

One other means to decrease the heat flow from the ventilation is to install an exhaust-air heat pump. This device takes heat from the ventilation air and, by use of electricity, transfers the heat back into the building. One part of electricity may often result in two to three parts of heat. It is, however, very important to install a heat pump of the right size because the amount of heat in the ventilation air is a limited resource. In this chapter no example is presented of how to calculate the LCC for the heat pump. This is because it is very rarely chosen as an optimal retrofit. It must, nonetheless, be emphasized that using a heat pump might make it unprofitable to caulk the windows in the building. Even if weatherstripping is a very cheap retrofit, it

might be even cheaper to use a slightly larger heat pump in order to utilize the increased ventilation flow from not caulking the windows.

3.6 Other building or installation retrofits

Exhaust-air heat exchangers are not dealt with here. This is because of the high cost of distributing the air from the exchangers to different apartments in a building, but the principle for the calculations is the same. Water-heater blankets and the regulation of radiator thermostats can be important measures in decreasing the energy need. However, the blankets are only useful if the water heater is located outside the thermal envelope or if the heating season is very short. Thermal thermostats will ensure that the desired inside temperature stays within defined limits, but they will only be useful if the surplus heat is wasted due to the use of extra ventilation.

3.7 Heating system retrofits

There are a number of heating system retrofits that must be considered. If the building is equipped with an oil boiler, it could be advisable to change it for one with a better efficiency. Or perhaps district heating would be preferable, if such a possibility exists. At least in Sweden, bivalent systems seems to be of interest when larger buildings are considered. A bivalent, or dual-fuel, system has an oil boiler taking care of the thermal peak load while a heat pump is used for the base load. It is important to optimize the sizes of the equipment and it has been shown that the correct level of extra insulation is, in this case, essential for reaching the lowest LCC (Gustafsson and Karlsson, 1988b). However, if the heating system is changed, it will lead to a building retrofit strategy that differs considerably from the one chosen when the original heating system is used. The process for calculation is depicted in Figure 3.2. Figure 3.2 also emphasizes that different retrofits might interact. Suppose an attic floor insulation was found to be profitable. When at the next retrofit perhaps additional external wall insulation is examined, the new LCC is compared to the original one, i.e. without additional attic floor insulation. Suppose also that this retrofit is profitable. The problem encountered is that, if the attic floor insulation is introduced, the external wall insulation might become unprofitable. Using an incremental method as above will overestimate the savings which are actually made. The method for optimization must consequently include an examination which includes the combination of the retrofits. If the difference between the incremental and the combination retrofit is very small the accuracy is satisfied; otherwise the insulation thickness must be changed and the resulting LCC must be recalculated. Perhaps the retrofit considered will drop away totally from the

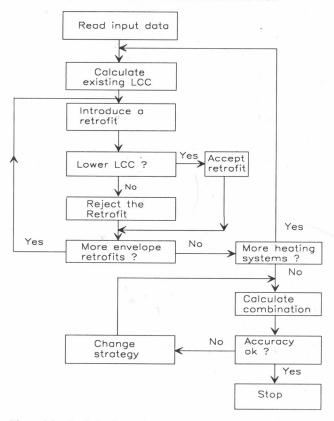


Figure 3.2 Optimization process (Gustafsson and Karlsson, 1989).

optimal solution. Fortunately, this interaction is usually very small, at least if the best candidates for an optimal solution are examined. Sonderegger et al. (1983) have calculated the difference to be about 2% in some cases and usually the interaction can be neglected. It must be noted that sometimes the situation is the opposite, i.e. interaction leads to a lower LCC for the combination than for the incremental method. This has been observed for fenestration measures and is discussed in detail in Gustafsson and Karlsson (1991), but the cases where this fact has been observed are rare and probably of academic interest only. In Table 3.1, a case study is presented clarifying the above discussion. The original LCC is calculated to 1.48 million SEK. The computer program used then checks to see if the attic floor insulation is profitable. This is not the case and thus the value '.00' is shown on the line below. External wall insulation, however, was found to be optimal and the amount saved calculated to be 0.05 million SEK for the project life of the building. Triple-glazing and weatherstripping were also candidates in the optimal solution. If the existing heating system is changed to a new oil boiler, the LCC is increased, even if the money saved by retrofitting is

Table 3.1 LCC table from the OPERA model (values in million SEK) (from Gustafsson, 1990)

	Exis. syst.	New oil	Ele. heat	Dist. heat	Gr. w. heat	Nat. gas	Tou. dist	Tou. elec.	Biv. gr. hp	Biv. o. air hp
No. build. retr.	1.48	1.54	1.69	1.45	1.57	1.23	1.45	1.69	1.38	1.48
Savings:										
Attic fl. ins.	.00	.00	.01	.00	.00	.00	.00	.01	.00	.00
Floor ins.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Ext. wall ins.	.05	.05	.11	.04	.06	.00	.04	.11	.00	.03
Ins. wall ins.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Triple-glazing	.06	.07	.09	.06	.08	.04	.06	.08	.05	.06
Triple-gl. l.e.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Trgl. l.e.g.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Weatherstrip	.01	.01	.02	.01	.01	.00	.01	.01	.00	.00
Exh. air h.P.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
Sum. of retro.	1.36	1.41	1.46	1.34	1.42	1.20	1.34	1.48	1.33	1.39
Sum. of comb.	1.36	1.41	1.46	1.34	1.42	1.20	1.34	1.46	1.33	1.39
Distribution:										
Sal. old boiler	.00	.02	.02	.02	.02	.02	.02	.02	.02	.02
New boil. cost	.08	.10	.03	.06	.28	.09	.06	.03	.25	.31
Piping cost	.00	.01	.00	.01	.16	.01	.01	.00	.07	.01
Energy cost	.60	.59	.62	.56	.28	.63	.56	.61	.34	.35
Connection fee	.00	.00	.00	.01	.00	.01	.01	.00	.00	.00
Buil. retrof. c	.43	.43	.54	.43	.43	.19	.43	.54	.40	.44
Inevitable cost	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25

increased, and therefore this is not a good strategy. District heating, a ground water coupled heat pump, and a bivalent heat pump-oil boiler system are other heating systems with a lower LCC, but the best solution was natural gas. The only building retrofit to be implemented was triple-glazed windows because the old ones were dilapidated! It was also shown that the combination of the retrofit LCC and the incremental LCC has the same value for all the heating systems, with the exception of electrical heating with a time-of-use rate, which is of no interest to the optimal solution. More details and a thorough presentation of the input values for this LCC optimization are presented in Gustafsson (1990). Experience shows that it is usually optimal to use a heating system with a very low operating cost. The acquisition cost for the system, however, cannot be too high, as is the case for a heat pump only meeting the total demand in the house (Table 3.1). Note that there are only a few building and ventilation retrofits which are optimal to install and of those that are optimal the cost of them is low or otherwise their remaining life are very short.

3.8 Sensitivity analysis

In the case shown in Table 3.1, there is one solution showing a LCC much lower than the others. This is not always the situation and usually two, or more, of the strategies may be very close to each other, making it hard to

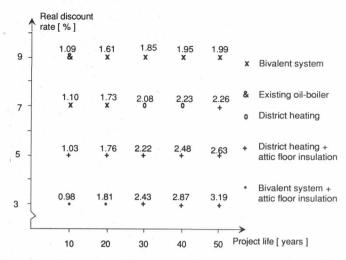


Figure 3.3 Bivariate sensitivity analysis (Gustafsson and Karlsson, 1989a).

know which one to choose. A sensitivity analysis might solve this problem. The aim with such an investigation is to determine if the optimal solution will change substantially with minor modifications of the input data. Of special interest are changes in the discount rate and the project life of the building, as these values cannot be set with total accuracy. Further, variations in energy prices must often be examined, as well as many other items in the input data file. The result may be presented by use of a bivariate diagram (Flanagan *et al.*, 1987). One example is shown in Figure 3.3, presented in Gustafsson and Karlsson (1989a). Note that the two cases in Table 3.1 and Figure 3.3 are not identical.

From Figure 3.3 it is clear that both the project life and the discount rate have a significant importance for the optimal strategy. Note also that the value of the LCC will change for different input values of the rate and project life, but this does not imply that a 3% rate and a 10-year project life are the best values to choose just because this alternative has the lowest LCC. Different strategies must be compared using the same rate, etc. It is important to note that for higher discount rates, less complicated heating systems are chosen, even if they have higher operating costs. For a 3% rate, the bivalent system, which has a very low operating cost but a high acquisition cost, is the best, while an oil boiler is optimal for a rate of 9%. Insulation measures will have an advantage in a long project life, but will be less profitable for a high discount rate. Of mostly academic interest is the fact that the LCC will almost always get lower for higher discount rates, but this fact is not valid for very short project lives. For a project life of 10 years, the LCC is increased when the rate is increased from 3 to 5%. This fact is dealt with in more detail in Gustafsson (1988). In Sweden, district heating is provided by

burning a mix of fuels in the utility plant. During the summer, most of the heat comes from burning refuse in an incineration plant, while oil or coal must be used in the winter. The cost for district heating is consequently lower than the oil price, while, at the same time, the installation cost is higher than the cost of an oil boiler. This is why it is optimal to use district heating for some combinations of discount rates and project lives. It must also be noted that the amount of additional attic insulation is not the same in the optimal strategies shown in Figure 3.3. Longer project lives and lower discount rates imply more insulation. Also, the optimal thickness of insulation is not a continuous function. When it is optimal to add insulation it is often necessary to apply more than 0.1 m or it may be better to leave the building as it is (Gustafsson and Karlsson, 1990). The same reference also emphasizes the importance of the remaining life of the building asset. If this is very short it will often be optimal to add extra insulation, and in that case an extensive amount of insulation should be chosen, say 0.2 m. Such a measure will very much decrease the heat flow through (for example) a wall, and this will imply that if all retrofits are made when they are unavoidable, the thermal state of the building will become better and better, and the cost for achieving this will be lower than leaving the building unchanged.

The influence on input data changes may be split into three different categories: (a) where the LCC will increase for an increase in input data; (b) where the LCC will decrease for an increase; and (c) where the LCC will not change at all for changes in the input data. Some examples of the first category are changes in building costs, installation costs, etc. To the second category apply changes in, for example, the discount rate, the remaining life of a building asset and the outdoor temperature. Some of the input data will apply to more than one of the categories. Consider, for example, a small increase in the cost of the oil boiler. If the oil boiler is part of the optimal solution, the LCC will increase if the cost for the boiler is increased. However, when the cost passes a certain limit the oil boiler will fall out of the solution, and from that point further increases in the oil boiler cost are of no interest. This fact is often used in the practical work with life cycle costing. When a building is analysed for the first time, input values can be chosen without a tedious examination process. The important thing is that the chosen values, at least to some degree, will reflect the real situation. After the first optimization has been elaborated, only the strategies that are close to each other need to be scrutinized. This means that much of the initial work with the input data might not be necessary and that only some of the details must be examined more closely.

In Gustafsson (1988), a sensitivity analysis of all the values used in an optimization is elaborated, but it is not possible to repeat this here. Some of the facts found must, however, be mentioned. For instance, it could be assumed that a small change in the resulting LCC will not be as important as if larger changes are encountered. This is not always true. If a 5% change

in the discount rate was introduced, this could lead to about a 2% change in the LCC, which is one of the largest differences found. However, the LCC for the existing building also changes by approximately the same amount, and implies that the optimal strategy will be almost the same for small changes in the discount rate.

A very high existing *U*-value for, say, an external wall in a poor thermal status might be expected to influence the LCC very much and to influence the new optimal *U*-value. This is not so. The new optimal *U*-value is not influenced by the existing one (Bagatin *et al.*, 1984; Gustafsson 1988), and the fact is that as long as the optimal insulation is introduced, the resulting LCC is almost constant. The same is valid for the actual insulation cost. If this cost is increased the optimization results in a thinner insulation which in turn will decrease the new LCC.

Annual increases in energy prices will naturally lead to a more extensive retrofit strategy which will lead to a lower LCC than might first be expected. This will also imply that, if the proprietor knows in advance what the energy prices will become, there is a better possibility of reducing the effects of escalating energy prices than if no action is taken at all. In some way, the optimization leads to a model that is in some regards self-regulating. The optimization makes the best of the situation and the result of a change might not be as bad as first assumed.

3.9 Linear programming techniques

In recent years there has been an increased interest in linear programming. The technique, which was developed in the 1960s, is not in common practice, because of the very tedious calculation procedures, and the use of fairly advanced mathematics. However, now that computers are on every desk, the situation is different, and the design of mathematical software makes the solving of complex linear programs much easier. It must be noticed that linear programming is an optimization technique which is not confined to life cycle costing. The reason for choosing linear programming is that it is possible to prove mathematically that the optimum solution, i.e. the best solution with the lowest LCC, has been found. The method is also suitable when discrete time or cost steps are included in the problem. This might seem to be of only minor interest but the tariffs for energy for tomorrow will probably always be of the time-of-use type where the price changes according to the time of day or year. In traditional methods, such as OPERA, these tariffs must be normalized many times and approximated by a mean value of the real price, which might greatly influence the optimal solution (Gustafsson and Karlsson, 1988). It is not possible to deal with linear programming in detail here and thus only a very brief presentation is made.

The LCC must be expressed in a so-called objective function. This function,

which is the expression to be minimized, must be totally linear, i.e. it is not possible to multiply or divide two variables by each other. A variable must only be multiplied by a constant. The objective function is, after this, minimized under a set of constraints which also have to be linear functions. All of the constraints must be valid at the same time. The procedure for solving such problems includes the use of vector algebra and is not dealt with here. (See Foulds (1981) for the basic concepts, and Murtagh (1981) for deeper insights into linear programming and how to solve such problems.)

In Sweden, it is common to describe the climatic conditions for a site by the use of mean values of the outdoor temperatures for each month of the year. Using twelve mean values instead of a continuous function makes it possible to use the linear programming technique, as it is not possible to take derivatives of functions with discrete steps. The thermal load in kilowatts and the need for heat in kilowatt-hours will consequently also follow the climate function, which implies that the steps are also included when the thermal situation is elaborated. In Table 3.2 the initial thermal load is shown for a building in Malmö, Sweden.

Suppose that only the attic floor insulation is of interest, in order to make the problem shorter and easier to deal with. The new demand for the building is now to be calculated. One variable is thus introduced showing the thermal load, in kilowatts, for the building for each month. Further, suppose that the building is heated by district heating where a time-of-use tariff is used. The cost for heat is assumed to be 0.2 SEK/kWh during November to March and 0.10 SEK/kWh for the remaining months. The first part of an objective function might be presented as:

$$(H_1 \cdot 744 \cdot 0.2 + H_2 \cdot 678 \cdot 0.2 + H_3 \cdot 744 \cdot 0.2 + H_4 \cdot 720 \cdot 0.1$$

$$+ \dots + H_{12} \cdot 744 \cdot 0.2) \cdot 18.26$$
(3.6)

where H = the new optimal heat load in kW for each month, 1,2,... = the number of the month, 744,... = the number of hours in each month, 0.2, 0.1 = the district heat price for various months in SEK/kWh and 18.26 = the present value factor.

Note that the influence of leap years is considered for February. The demand in Table 3.2 must be covered in one way or another. The model is

Month	Heat (MWh)	Month	Heat (MWh)	Month	Heat (MWh)
January	32.60	May	15.95	September	12.02
February	30.95	June	9.92	October	18.99
March	29.85	July	6.97	November	24.07
April	22.53	August	7.70	December	28.98

Table 3.2 Heat demand for a building sited in Malmö, Sweden

therefore supplemented by twelve constraints showing the situation for each month and the first three of them will be:

$$H_1 \cdot 744 \ge 32.60$$
 $H_2 \cdot 678 \ge 30.95$ $H_3 \cdot 744 \ge 29.85$ (3.7)

The cost for additional insulation was shown in equation (3.3) and the influence that this insulation has on the thermal load in equation (3.4). From equation (3.4) it is obvious that it is not a linear expression, since $t_{\rm ins}$ is present in the denominator. However, it is possible to make it a linear function of $t_{\rm ins}$, but in that case equation (3.3) will be nonlinear. A method described by Foulds (1981), called piecewise linearization, is therefore used. In this method, the value of a function is calculated for a number of discrete values of $t_{\rm ins}$ and each value for the function is coupled with a binary integer variable which can only have the value 1 or 0. All these binary variables are added and the sum is constrained as lower than or equal to 1. This forces the model to choose one or none of the variables. The original nonlinear function of $t_{\rm ins}$ is thus changed to a linear function of the binary variables. The situation is depicted by the following example. The decrease of the heat demand is shown in equation (3.4), and for five steps of insulation thickness, the decrease is as presented in Table 3.3. (See also Gustafsson and Karlsson (1989b).)

Suppose the area of the attic floor is 200 m². The number of degree hours in Malmö for January has been calculated as 15 996 and subsequently the decrease in heat flow, in kWh, through the attic will become:

$$10^{-3} \cdot 15996 \cdot 200 \cdot (0.4 \cdot A_1 + 0.533 \cdot A_2 + 0.6 \cdot A_3 + 0.640 \cdot A_4 + 0.667 \cdot A_5)$$
(3.8)

Equation (3.8) and eleven more expressions for the rest of the year must be added to the left-hand sides of the constraints in equation (3.7). Note also that:

$$A_1 + A_2 + A_3 + A_4 + A_5 \le 1 \tag{3.9}$$

and that the A variables are all binary integers. One or none of them must be chosen according to equation (3.9). Only lacking now is the building cost for the additional insulation. Using the same values as above for derivative

Table 3.3	Decrease in	U-value for	five discrete steps	s of additional insulation
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Added insulation (m)	Variable	Existing <i>U</i> -value*	New U -value*	Decrease in <i>U</i> -value*
0.05	A_1	0.8	0.400	0.400
0.10	A_2	0.8	0.267	0.533
0.15	A_3	0.8	0.200	0.600
0.20	A_{4}	0.8	0.160	0.640
0.25	A_5	0.8	0.133	0.667

^{*} In Wm - 2 K - 1.

optimization, the cost will be as a function of A_1 - A_6 instead of t_{ins} :

$$200 \cdot [(100 + 0.05 \cdot 600) \cdot A_1(100 + 0.10 \cdot 600) \cdot A_2 + \dots$$

$$+ (100 + 0.25 \cdot 600) \cdot A_5]$$
(3.10)

The model is now totally linear and it is possible to use ordinary linear or mixed integer programming methods for optimization. By the use of more binary integers it is again possible to add the influence of the inevitable retrofit cost as well, i.e. when one of the A variables is chosen, a certain amount is added to the objective, and if none is chosen a different amount is added. As can be found from the above example, the number of equations and constraints will become very large for real-world problems. Now, the tedious work of generating equations and constraints is dealt with by separate computer programs which are used for writing the large input data files. More details and a complete model can be found in Gustafsson (1992).

3.10 Summary

Two different methods are shown for optimizing the retrofit strategy for a building.

In the first method the LCC is actually calculated for a number of cases and the lowest LCC strategy is selected. The other method shows how to design a mathematical model in the form of mixed integer programming. The latter method demands a more skilled mathematician because of the use of vector algebra when solving the problem. However, there are advantages using this latter method owing to the possibilities of solving discrete problems, i.e. the functions are not necessarily continuous. One major drawback is that the problems to be solved must be totally linear, but by the use of piecewise linearization this drawback can be dealt with, at least to some extent.

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