OPTIMAL HEATING-SYSTEM RETROFITS IN RESIDENTIAL BUILDINGS

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Abstract

The optimal heating-system-retrofit strategy for existing buildings differs due to varying prices of energy, building and installation features, climate conditions, etc. We have examined a test building situated in Linköping, Sweden. By using the OPERA model, we were able to arrive at the optimal retrofit strategy, which includes a ground-coupled heat pump using electricity to run the compressor. Unfortunately, the price of electricity differs according to the time of day, month, etc. These variations are not included in the OPERA model. In OPERA, the price should be divided into 12 segments, one for each month of the year since climate data are divided in this manner. Fine tuning of a dual-fuel system (an oil-fired boiler handles the peak load and a heat pump the base thermal load) is optimized using the Mixed Integer Linear Programming (MILP) method. Adding a hot-water accumulator also makes it possible to use low electricity prices for space and domestic hot-water heating. This system competes in the model with traditional heating devices such as district heating. The optimal method of heating the building was found for using the heat pump alone.

INTRODUCTION AND CASE DESCRIPTION

The OPERA model is used for finding the optimal retrofit strategy for an existing building. The model has been described in several international publications, of which Ref. [1] is the latest. The model is therefore not dealt with in detail here. However, the output from the model is shown for our test building, which is a multi-family block with 14 apartments. Table 1 shows the energy use in detail.

MONTH NO	DEG. HOURS	ENERGY TRANSM	HOT WATER	FREE ENERGY	SOLAR HEAT	UTILIZ. FREE	FROM BOILER	INSUL. OPTIM.
1	17782	36.5	3.5	4.2	1.2	5.4	34.7	36.6
2	16272	33.4	3.5	4.2	2.6	6.8	30.2	33.5
3	15698	32.2	3.5	4.2	6.1	10.2	25.5	32.3
4	11304	23.2	3.5	4.2	9.0	13.1	13.6	23.2
5	7440	15.3	3.5	4.2	12.7	15.3	3.5	0
6	4032	8.3	3.5	4.2	13.2	8.3	3.5	0
7	2455	5.0	3.5	4.2	12.9	5.0	3.5	0
8	3422	7.0	3.5	4.2	10.9	7.0	3.5	0
9	6336	13.0	3.5	4.2	7.7	11.9	4.7	13.0
10	10342	21.3	3.5	4.2	4.1	8.3	16.5	21.2
11	13176	27.1	3.5	4.2	1.6	5.7	24.9	27.1
12	15624	32.1	3.5	4.2	0.8	4.9	30.7	32.1
TOTAL	123883	254.7	42.0	50.0	82.8	102.1	194.7	219.1

Table 1: Energy demand in MWh for the test building during the months of a year.

The reason for obtaining constant hot water and free energy is the result of OPERA using only one input value for the full year. This value is then divided into 12 segments, one for each month.

The OPERA model next calculates the optimal method of heating this building. New windows with lower U-values, additional insulation and other retrofits are also taken into account. The optimal solution is found when the lowest possible life-cycle cost (LCC) is achieved. The solution shows that district-heating is the preferred heating system and should be combined with triple-glazed windows and weather stripping. The LCC will thereby be reduced from 2.31 to 1.36 MSEK. The next best solution is to use a dual-fuel system with a heat pump and an oil-fired boiler and to combine this installation with both attic floor and external wall insulation, as well as new windows and weather stripping. Most of the city of Linköping is heated by a district-heating system based on combined heat and power generation (CHP). Therefore, heat is sold to the end user for only 0.26 SEK/kWh, including taxes (one ECU equals approximately 8 SEK). The electricity rate is, however, divided into three segments. The high-cost segment of 0.94 SEK/kWh is applicable from 06.00 to 22.00 on working days between November and March. For the rest of the day, the rate is 0.49 SEK/kWh. From April to October, the rate is 0.38 SEK/kWh throughout the day. By using a heat pump operating only during night time, heat can be supplied for less than 0.17 SEK/kWh, i.e. if the coefficient of performance of the heat pump is equal to 3.0, which might be applicable if a ground-water-coupled heat pump is used. The question is now whether such a dual-fuel system is competitive if the high required cost for heating equipment is considered. MILP has found many applications in recent years since it enables very large and complex problems to be solved and also optimized. The development of faster and cheaper personal computers has contributed to this trend. District-heating systems and insulation measures are dealt with in Ref. [2]. Industrial energy systems are examined in Refs. [3], [4] and [5]. One drawback with introducing integers in linear programming is that the computing time will be significantly longer and that no ranging is possible. However, if steps in cost functions are to be part of the model, integers are necessary.

THE MILP MODEL

The year has been divided into segments according to the electricity rate, see Table 2.

Month	High-price hours	Medium-price hours	Low-price hours	Total number
January	368	376	-	744
February	336	360	-	696
March	336	408	-	744
April	-	-	720	720
May	-	-	744	744
June	-	-	720	720
July	-	-	744	744
August	-	-	744	744
September	-	-	720	720
October	-	-	744	744
November	336	384	-	720
December	352	392	-	744
Total	1,728	1,920	5,136	8,784

Table 2: Number of hours in different time segments for 1996

With 368 high-cost hours out of a total of 744, 18.1 MWh are needed for space heating during high-cost conditions (Tables 1 and 2). However, heat from appliances, solar gains, et c. is likely to be available only during daytime. Some of this free energy is available from 06.00 to 22.00 from Monday to Friday (which is the high-cost segment). In January, there are 23 working days and, hence, 23/31 of (4.2 + 1.2) MWh are considered to contribute to space heating during high-price hours. Therefore, 14.1 MWh remains for this purpose. Hot-water consumption is also likely to occur during daytime and hence 2.6 MWh must be added, resulting in 16.7 MWh which must be supplied from the heating equipment during high-price hours. Some of this heat could be provided by using a hot-water accumulator coupled to a heat pump. If the accumulator is too small, extra heat must be added by the oil-fired boiler or by the heat pump working on high-price electricity. The price of oil is about 0.39 SEK/kWh in Sweden (1996), so that the cost of oil energy is higher than heat-pump energy even if the pump is used all of the time. A large heat pump is, however, very expensive and such a solution could therefore not be preferable.

All MILP optimization problems have an objective function. In our case, this function shows the cost for supplying the building with heat and this cost must therefore be minimized. The installation cost, in SEK, for heat pumps has been found to be approximately 60,000 + 5,000 $\times P_{hp}$, where P_{hp} shows the thermal power in kW for the pump, see Ref. [6] (1 US$ equals 7 SEK). This equipment must compete with the district-heating system (which costs 40,000 + 60 $\times P_{dh}$) or the oil-fired boiler (with a cost of 55,000 + 60 $\times P_{ob}$). The cost functions therefore start with a major increment. It is important to calculate the present values of all equipment. The heat pump is assumed to have a useful life of 15 years. We calculate the present value for a 50-year life and assume an interest rate of 5%. The present value for the first part of the cost (i. e. 60,000) is

$$60 \times [1 + (1 + 0.05)^{-15} + (1 + 0.05)^{-30} + (1 + 0.05)^{-45}] = 109 \quad \text{kSEK} \tag{1}$$

At year 50, the heat pump has a salvage value corresponding to 10 years of remaining life. Therefore, 3 kSEK must be subtracted, resulting in a present value of 106 kSEK. The life spans are 25 and 15 years for the district-heating and oil-fired boiler systems, respectively. The first part of the function occurs in the objective function only if one of the systems is chosen. Hence, the three binary variables A_1, A_2 and A_3 are introduced, which makes it possible to determine the first part of the objective function as

$$A_1 \times 52 \times 10^3 + 78 \times P_{dh} +$$

$$+ A_2 \times 106 \times 10^3 + 8.9 \times 10^3 \times P_{hp} + A_3 \times 97 \times 10^3 + 106 \times P_{ob}$$

The variables A_1, A_2 and A_3 assume only the two values 0 or 1. If P_{dh} is larger than 0, A_1 must be 1; if P_{dh} equals 0, A_1 must also be zero. The same procedure is valid for the district-heating variable A_2 etc. This behavior is fulfilled by setting

$$A_1 \times M \geq P_{dh}. \tag{2}$$

Here, M is a parameter which must be chosen large enough not to constrain the value of P_{dh}. M is therefore set equal to a value larger than P_{dh} might ever take, e.g., 200 kW; see Ref. [7], p. 179, for further details regarding this fixed charge problem. We next consider the high-price hours of January. Above, we concluded that 16.7 MWh were needed (see the discussion about free gains and high-price hours). If a district-heating system is used, the price will be 0.26 SEK/kWh, no matter what time of day or season the energy is used. For the high-price hours of January, the following constraint was imposed:

$$(P_{1hdh} + P_{1hhp} + P_{1hacc} + P_{1hob}) \times 368 \geq 17 \times 10^3 \tag{3}$$

The subscript 1h shows that this is month 1 and a high-price tariff applies, while acc indicates that heat comes from a hot-water accumulator. The accumulator will be dealt with in more detail later. An index ob indicates that an oil-fired boiler is used. One such constraint must be provided for each of the specified time segments (Table 2). The cost of heat production must also be included in the objective function. The energy cost is incurred every year and thus a present-value factor must be introduced. For a 50-year project life and an interest rate of 5%, this factor will be 18.26. The objective function must therefore be augmented by

$$(\frac{P_{1hdh} \times 0.26}{0.95} + \frac{P_{1hhp} \times 0.94}{3.0} + \frac{P_{1hob} \times 0.39}{0.7}) \times 368 \times 18.26$$

to include energy prices and efficiencies or COP. The district-heating price is 0.26 SEK/kWh and the efficiency is equal to 0.95. The other values refer to the heat pump and the oil boiler. Installation costs for the accumulator and the oil-fired boiler, as well as more binary variables, must also be included in the objective function. P_{dh}, P_{hp} et c., i. e. quantities without time-segment signs, show the minimum sizes of the heating-system components which provide sufficient energy. In order to find these values, further constraints are needed. For the district-heating system, these will be

$$\frac{P_{1hdh}}{0.95} - P_{dh} \leq 0 \tag{4}$$

P_{dh} is therefore slightly larger than the largest of P_{1hdh}, P_{2hdh}, et c. Constraints must be imposed for all time segments, as well as for the use of other types of heating equipment. The heat-storage system is assumed to store energy in the form of hot water. This heat energy is assumed to be produced by the heat pump during medium electricity-price conditions (see Table 2). Some or all of this heat is discharged when the electricity price is high. The latter case is covered by Eq. (3). During the medium-cost period, the accumulator is charged. There are only 8 hours available for charging the accumulator during any 24-hour working day. In January 1996, there were 23 working days and thus 184 hours for charging. Hence, $P_{1macc}\times$ 184 kWh should be added to the right-hand side of our constraints. There is also a possibility that it is possible to charge the accumulator faster than to discharge it. Table 2 shows that 378 medium price hours occur in January and the complement to Eq. (3) must be changed accordingly. The value on the right-hand side must also be changed because solar and free energy are likely to become available only during daytime. There is an energy need of 18.5 MWh for this time segment. Adding domestic hot-water usage and subtracting free energy from appliances and solar radiation during Saturdays and Sundays decreases this amount to 18.0 MWh. The medium cost constraint will therefore be

$$(P_{1mdh} + P_{1mhp} + P_{1mob}) \times 378 - P_{1macc} \times 184 \geq 18.0 \times 10^3 \tag{5}$$

The cost for a hot-water accumulator depends on the size measured in kWh. In Ref. [8], this cost has been estimated at 1,500 SEK/kWh and the reader is directed to this reference for further information. The maximum thermal demand occurs on the coldest winter day and implies a peak of 71.96 kW. The model must therefore include the expression

$$P_{dh} \times 0.95 + P_{hp} \times 3.0 + P_{acc} + P_{ob} \times 0.7 \leq 71.96 \quad \text{kW} \tag{6}$$

The largest value of the variable P_{acc} is found by using an equation similar to Eq. (4). The specified price for district heating does not include a subscription fee. Depending on the tariff, this fee should be based on heat consumption during one year divided by a category value, which in our case is 2,200 hours. The subscribed thermal power value is then used for the annually recurring cost (4,000 + 260 $\times P_{dhS}$) which, in turn, must be multiplied by the present-value factor 18.26. The model must include expressions which add this cost to the objective function, viz.,

$$P_{dh1h} \times 368 + P_{dh1m} \times 376 + P_{dh2h} \times 336 + P_{dh2m} \times 360 - E_{dh} \leq 0.0 \tag{7}$$

$$P_{dhS} \times 2,200 - E_{dh} \geq 0.0 \tag{8}$$

The cost $(A_1 \times 4,000 + 260 \times P_{dhS}) \times 18.26$ is added to the objective function. The electricity rate subscription fee includes an annually recurring cost of 1,100 SEK, which must also be added to this function. The binary variable A_2 is used for this purpose. The program leads to a problem with 73 variables and 186 constraints. Four of the variables are binary, i.e. they can only take the value 0 or 1.

OPTIMIZATION OF THE BASIC CASE

We have used the ZOOM optimization software, see Ref. [9] to solve the problem. The MILP model is optimized in just a few seconds and the result shows that it is optimal to use only the heat pump (see Fig. 1).

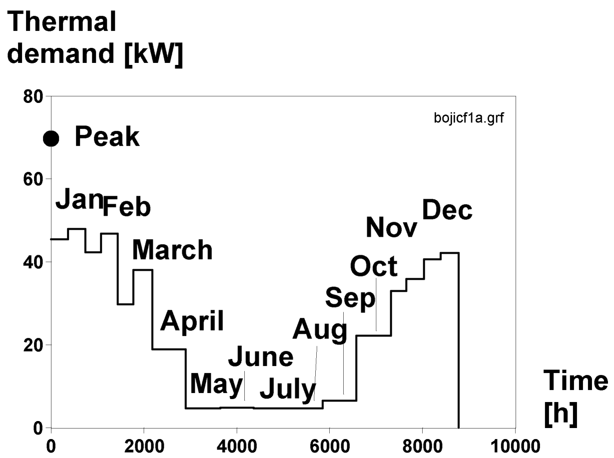

Figure 1: The heat pump is used during all time segments. Optimal solution for the basic case

The oil-fired boiler, district-heating-system or accumulator are never a part of the optimal solution. The minimum value of the objective function is 1.07 MSEK. To clarify the situation, the solution is described in more detail in Table 3.

The heat pump is used throughout the year. For January, during the high-cost electricity segment, the cost is $16.7 \times 10^3 \times \frac{0.94}{3.0} = 5.2$ kSEK; the price for heat is 0.94 SEK per kWh while the COP is 3.0. The resulting annually recurring cost for energy is 40.1 kSEK. We also need the equipment for heat production. The heat-pump system must cover the maximum thermal power in the building, i.e. 72 kW. The optimization results in P_{hp} equaling 23.98 kW. P_{hp} must be multiplied by 3.0 in order to achieve the needed thermal power, which therefore is 71.94 kW. Table 4 shows the total cost for the building when present values are used. The total sum in Table 4 differs by only a few SEK from the cost calculated by ZOOM.

Month	Hours	Power	Heat-pump energy [kWh]	Heat-pump cost [SEK]	Total cost [SEK]
January	368	45.37	16,696	5,231	5,231
	376	47.86	17,995	2,939	2,939
February	336	42.28	14,206	4,451	4,451
	360	46.83	16,858	2,754	2,754
March	336	29.78	10.006	3,135	3,135
	408	38.05	15,524	2,536	2,536
April	720	18.86	13,579	1,720	1,720
May	744	4.70	3,496	443	443
June	720	4.86	3,499	443	443
July	744	4.70	3,496	443	443
August	744	4.70	3,496	443	443
September	720	6.46	4,651	589	589
October	744	22.16	16,487	2,088	2,088
November	336	32.98	11,081	3,472	3,472
	384	35.88	13,778	2,250	2,250
December	352	40.26	14,171	4,440	4,440
	392	42.10	16,503	2,696	2,696
Total	8,784	-	195,522	40,073	40,073

Table 3: Energy cost during one year for an optimal system

Energy cost, 40,073 SEK/year	731,732 SEK
Electricity subscription fee, 1,100 SEK/year,	20,086 SEK
Heat-pump system, electric power, 23.98 kW	317,627 SEK
Total	1,069,445 SEK

Table 4: Present-value cost elements for the studied building

ANALYSIS

If the oil-fired boiler had been used only to cover the peak load of the building, a 24.1 kW unit would have been needed (Table 3). This would cost 99.5 kSEK. At the same time, the cost for the heat pump would have decreased by 71 kSEK. However, this would not be sufficient to change the optimal solution. It is obvious that the increment in the cost function is essential for the optimal solution. We assume that the step for the oil-fired boiler cost is reduced to half its original value, i. e. 27.5 instead of 55.0 kSEK. A new optimization with ZOOM results in an oil-fired boiler, with a thermal size of 33.6 kW and a heat pump with an electric power of 15.6 kW. The boiler is then used only for covering the peak-load and during medium electricity price conditions in February. The LCC is now 1.04 MSEK, which is a small reduction compared to the original case. A further reduction of the same variable does not change the optimal solution. Another plausible solution is to use the heat accumulator. The cost for the accumulator is set equal to the low value 1 SEK/kWh instead of 1,500 SEK/kWh. The LCC calculated by ZOOM now decreases to 0.9 MSEK and an accumulator with a size of 531.2 kWh is optimal. At the same time, the size of the heat pump is reduced slightly compared to the original value because the peak is partly covered by the accumulator. Figure 2 shows the solution in more detail.

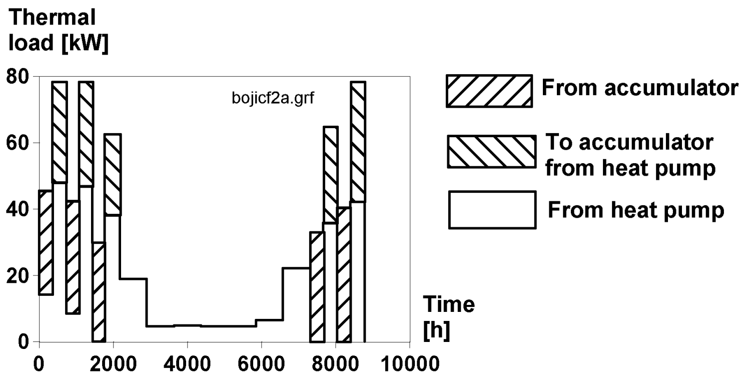

Figure 2: Use of the no-cost accumulator

The rectangles which show the energy amount from the accumulator have the same area as the rectangles showing energy from the heat pump to the accumulator. However, if the number of hours available for storage is considered, the latter rectangles should be narrower and taller. We consider January. ZOOM calculates the thermal load of the accumulator heat for the high electricity cost segment as 31 kW. Adding this value to the heat pump thermal load of 14.3 kW results in 45.3 kW, which is the requirement found in Table 3. The accumulator has thus stored $31.1 \times 368 = 11.4$ MWh. This heat must be produced by the heating system, but now there are only 8 hours each working-day night available for this purpose, resulting in 184 hours. This value is applied for the accumulator, which ZOOM sets equal to 62.1 kW charging power or $62.1 \times 184 = 11.4$ MWh; but it is not used for the heat pump or other heating-system equipment. If this is taken into account, the accumulator must be smaller. At the same time, the total cost increases and the no-cost accumulator will perhaps again be eliminated from the optimal solution. We noted that the subscription fee, among other factors, makes this system unprofitable for district-heating. When the value 260 in the district-heating tariff is reduced to 1.0, ZOOM calculates the total cost as 1.07 MSEK, which is also a small decrease below the original value. The district-heating system is next used during the high-electricity-cost segments and further combined with the heat pump during medium-cost periods. Details are shown in Fig. 3 and Table 5. The district-heating system must have a size of 47.8 kW in order to meet the demand, while the heat pump must have a size of 12.7 kW_{el}.

The three cases show almost identical total costs. This result follows because ZOOM always optimizes the system. If district heating is used, the optimal total cost may be almost the same as for an optimal heat-pump system as long as the strategy is optimal. Choosing too large or too small a heat pump may significantly increase the total cost. In our basic case, a heat-pump system was

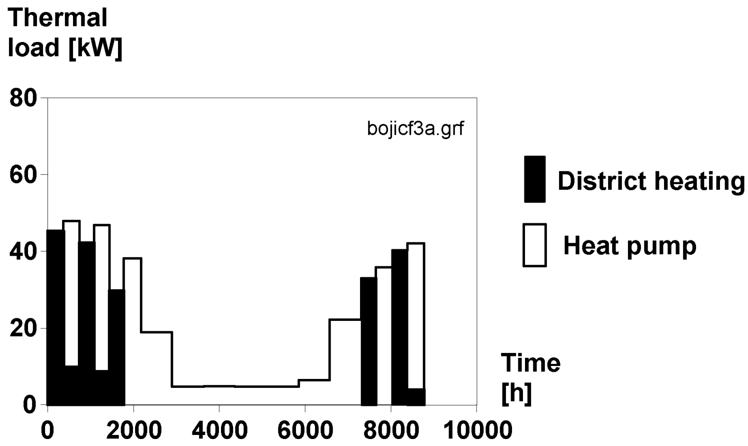

Figure 3: Optimal solution with a low subscription fee for district heating

optimal. If the electricity price in the high-cost segment goes up, the system loses part of this heat source which happens for a segment cost of about 1.1 SEK/kWh and resulting in a total cost of 1.13 MSEK. If the price of district heating is now increased to 0.30 SEK/kWh, the heat-pump system is once again optimal. If both prices are increased, then the oil-fired boiler must be used. It is important to note the influence of the binary integers. ZOOM always solves the linear program first, by assuming that A_1, A_2 etc. are ordinary variables. The optimal solution to this problem is to use both the heat pump (15.1 kW) and the district-heating system (31.3 kW). The total cost becomes 0.9 MSEK. Using integers eliminates the district-heating system and leads to a heat pump with a power rating of 23.98 kW. The total cost is thereby increased because the increment in the cost functions are now properly handled. Integers must be included to find the right solution.

CONCLUSIONS

We have shown how to create an MILP model of a building with three different heating systems and a hot-water accumulator. For the first inputs, the heat-pump system was found optimal. If input data are changed, other systems come into operation but the total cost is only changed by a small value because each set of data leads to new optimal solutions. We have shown the importance of using binary integers in order to model increments in the cost functions. Ordinary linear programming resulted in systems that differ significantly from the MILP solutions.

Month	Hours	District heating			Heat-pump			Total
		Power	Energy	Cost	Power	Energy	Cost	cost
January	368	45.37	16,696	4,569	-	-	-	4,569
	376	9.81	3,689	1,010	38.05	14,307	2,337	3,347
February	336	42.28	14,206	3,888	-	-	-	3,888
	360	8.78	3,161	865	38.05	13,698	2,237	3,102
March	336	29.78	10,006	2,739	-	-	-	2,739
	408	-	-	-	38.05	15,524	2,536	2,536
April	720	-	-	-	18.86	13,579	1,720	1,720
May	744	-	-	-	4.70	3,496	443	443
June	720	-	-	-	4.86	3,499	443	443
July	744	-	-	-	4.70	3,496	443	443
August	744	-	-	-	4.70	3,496	443	443
September	720	-	-	-	6.46	4,651	589	589
October	744	-	-	-	22.16	16,487	2,088	2,088
November	336	32.98	11,081	3,033	-	-	-	3,033
	384	-	-	-	35.88	13,778	2,250	2,250
December	352	40.26	14,171	3,878	-	-	-	3,878
	392	4.06	1,592	436	38.05	14,916	2,436	2,872
Total	8,784	-	74,602	20,418	-	120,926	17,964	38,383

Table 5: Energy consumption and cost for the low subscription fee system

Acknowledgement

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References

[1] Gustafsson Stig-Inge. Does Postponed Retrofitting Save Money? *Heat Recovery Systems & CHP*, 15(5):469 – 472, 1995.

[2] Gustafsson Stig-Inge. Optimization of Building Retrofits in a Combined Heat and Power Network. *Energy - The International Journal*, 17(2):161–171, 1992.

[3] Nilsson K., Söderström M. and Karlsson B. G. MIND optimization reduces the system cost of a refinery. *Energy-The International Journal*, 19(1):45–54, 1994.

[4] Lee B. and Reklaitis G. V. Optimal scheduling of cyclic batch processes for heat integration - I. Basic formulation. *Computers & Chemical Engineering*, 19(8):883–905, 1995.

[5] Ito K., Shiba T., Yokoyama R., and Sakashita I. S. An optimal Operational Advisory System for a Brewery's Energy Supply Plant. *Journal of Energy Resources Technology*, 116(1):65–71, 1994.

[6] Gustafsson Stig-Inge. A Computer Model for Optimal Energy Retrofits in Multi-Family Buildings. The OPERA model. Technical report, Swedish Council for Building Research, Document D21, Stockholm, 1990.

[7] Foulds L. R. *Optimization techniques.* Springer Verlag, New York Inc., 1981.

[8] Gustafsson Stig-Inge. Hot Water Heat Accumulators in Single-Family Houses. *Heat Recovery Systems & CHP*, 12(4):303–310, 1992.

[9] Marsten R. Users Manual for ZOOM. Dept. of Management Information Systems. University of Arizona, U.S.A.