MIXED INTEGER LINEAR PROGRAMMING AND BUILDING RETROFITS

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Abstract

When a building is subject for refurbishment it is important to add only such measures that will reduce the Life Cycle Cost, LCC, for the building. Even better is to add measures that will, not only reduce the cost, but minimise the LCC. One means for such an optimisation is to use the so called Linear Programming, LP, technique. One drawback with LP models is that they must be entirely linear and therefore two variables cannot be, for example, multiplied with each other. The costs for building retrofits are, however, not very often linear but instead ”steps” are present in their cost functions. This calamity can, at least to a part, be solved by introducing binary integers, i. e. variables that only can assume two values, 0 or 1. In this paper it is described how to design such a Mixed Integer Linear Programming, MILP, model of a building and how different cost elements of the climate shield influence the optimal solution.

INTRODUCTION

In recent years LP and MILP programming have found an increased interest among researchers in applied engineering. The reason for this is to a part the introduction of fast personal computers one everyone's desk. Problems that took hours, ore even days, to solve can nowadays be solved in minutes or even seconds. It is therefore possible to design models with several thousand variables without having to wait for hours to see the optimal result. This is especially valid for MILP problems because the so called branch and bound method must solve two LP problems for each integer that is introduced. The model is initially optimised by assuming that no variables at all are integers. When this have been done the problem is split in two LP problems. One problem where one of the integers are bound to a value less than or equal to zero and one problem where the integer is set to a value greater or equal to 1. A MILP problem therefore needs substantially more time when it is solved than an ordinary LP ditto.

There are numerous papers about LP and MILP programming present in scientific journals, see e. g. References [1], [2], [3], [4] and [5]. Papers about MILP and buildings, however, are not very common but some have been presented in recent years, see e. g. [6] and [7].

THE MILP MODEL

All LP and MILP problems have a mathematical expression, called the objective function. In our case this function shows the total LCC for the building and the expression is therefore to be minimised. One way to achieve this is to set all variables to zero but if that is the case no heating or building activity is present. A number of constraints must therefore be introduced. One constraint, for example, ascertains that enough heat is supplied to the building while others are used for finding proper thermal sizes of different heating equipment which are possible to install in the building. In Reference [7] the method is shown in detail for the heating equipment and insulation measures and therefore only a brief description is made here.

The need for space heating in a building depends on the climate. It is not possible, or at least very impractical, to use the outdoor temperature in every moment and from this calculate the energy cost for a long period of time. There is therefore a need for splitting one year into several segments and use monthly mean temperatures as a base for the calculations. In Sweden, the electricity rates sometimes make it profitable to use heat pumps for space and hot water heating. The electricity price is high during the winter and low at summer. The price also differs according to the time of day. Weekends have a low price during some of the months. We have therefore found it practical to divide the year into 22 segments where the months November to March are split into three segments each while April to October only holds one segment each. The need for space- and domestic hot water heating is presented in Figure 1.

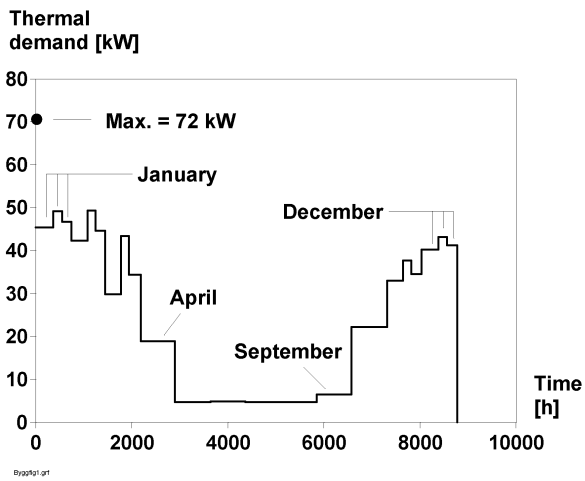

Figure 1: Thermal demand for the studied building, see Reference [8]

In January the first segment includes 368 hours where the electricity rate is high. The total amount of energy in this segment has been calculated to 16,697 kWh and the demand is therefore 45.37 kW, see Reference [8] for all details. The need for heat must be covered by district heating, a heat pump, an oil-fired boiler or a mix of these systems. The thermal sizes of these heat sources are not known and therefore three variables P_{DH01}, P_{HP01}, and P_{OB01} are introduced. The index 01 shows that the first segment is considered. The cost for district heat in Linköping, Sweden, is 0.26 SEK/kWh, the running cost for the oil-fired boiler is 0.39 SEK/kWh, while the electricity cost is 1.01 SEK/kWh in this high cost segment. (1 SEK = 7 US$.) Each system has an efficiency which is set to 0.95 for district heating, 0.75 for the oil boiler and 3.0 for the heat pump. It is assumed that the system is used during the next 50 years and that the real discount rate is 5% which leads to a present value factor of 18.26. The first small part of the objective function can now be elaborated:

$$[0.26\times\frac{1}{0.95}\times P_{DH01}+0.39\times\frac{1}{0.75}\times P_{OB01}+1.01\times\frac{1}{3.0}\times P_{HP01}]\times 368\times 18.26 \quad (1)$$

The other 21 segments are added in a similar way. The equipment must also be installed and purchased. It is assumed that the different systems costs are:

- 40,000 + 60 $\times P_{DH}$ for district heating
- 55,000 + 60 $\times P_{OB}$ for the oil boiler
- 60,000 + 5,000 $\times P_{HP}$ for the heat pump

The costs, however, must also be calculated as present values. The practice life for the district heating system is assumed to be 25 years while the oil boiler and the heat pump is thought to be 15 years. Further, assuming a total project life of 50 years and a real discount rate of 5 % the present value for the district heating system will become:

$$(40,000 + 60\times P_{DH})\times(1+(1+0.05)^{-25}) = 51,812 + 77.72\times P_{DH} \quad (2)$$

Note that P_{DH} et c. now are presented without indices and therefore shows the maximum thermal size of the equipment. The model must therefore include expressions for finding the maximum need for e. g. district heat in all the 22 segments. This is implemented by use of 22 constraints for each heating device, and one is shown here:

$$\frac{1}{0.95}\times P_{DH01} - P_{DH} \leq 0.0 \quad (3)$$

As can be seen above the cost for the district heating equipment starts with a step, i.e. 40,000 SEK. This cost must be present in the objective function but only if district heating is optimal to use. This behaviour is achieved by implementing a binary variable A_1 which only can assume the values 0 or 1, and by introducing one more constraint:

$$A_1\times M - P_{DH} \geq 0.0 \quad (4)$$

M is set to a value higher than P_{DH} might ever take e. g. 200, note that the maximum demand is about 72 kW in Figure 1. If P_{DH} has a value greater than zero, A_1 must become equal to 1. A_1 must also be present in the objective function and then multiplied by the cost 51,812. Because of the minimisation

A_1 turns to 0 if P_{DH} equals zero as well. This part of the objective function therefore becomes:

$$A_1 \times 51,812 + 77.72 \times P_{DH} + A_2 \times 56,260 + 61.33 \times P_{OB} + A_3 \times 105,933 + 8,827 \times P_{HP} \quad (5)$$

A sufficient amount of heat must be supplied to the building. In the first time segment this amount has been calculated to 16,697 kWh. By use of 22 constraints of which the first is:

$$(P_{DH01} + P_{OB01} + P_{HP01}) \times 368 \geq 16,697 \quad (6)$$

this is achieved.

It is also possible to affect the energy need in the building by applying extra insulation and better windows. The method used for extra insulation is to a part presented in Reference [9] and is therefore only described in brief here. The new U-value for an extra insulated wall can be calculated as:

$$U_{NEW} = \frac{k_{NEW} \times U_{EXI}}{k_{NEW} + U_{EXI} \times t} \quad (7)$$

where k_{NEW} is the conductivity for the new insulation in $\frac{W}{m \times {}^\circ C}$, U_{EXI} the existing U-value in $\frac{W}{m^2 \times {}^\circ C}$, and t the thickness of the added insulation in metres. Unfortunately, equation (7) is not linear and thus so called stepwise linearisation must be used. A number of binary integers, in our case 11 variables have been used, must therefore be introduced. The first integer, IS_0, is applied for 0.02 m of extra insulation, the second one for 0.04 m and so on. Only one of the integers can be 1 while the others must be 0. If all integers are 0 it is not optimal to add insulation at all, see equation, i. e. constraint, number (8).

$$IS_0 + IS_1 + IS_2 + IS_3 + \ldots + IS_{10} \leq 1 \quad (8)$$

The integers IS are after this coupled to the cost for extra insulation and are added to the objective function. By use of equation 7 they are also coupled to the decrease of the energy demand in the building and are added to expression 5. Insulation is of no use outside of the heating season. An energy balance for the building shows that four segments, viz. 11-14, only need energy for domestic hot water heating. The IS variables are therefore not present for those segments.

The same procedure is valid for windows but now different window constructions are coupled to a set of binary integers. The model must deal with insulation measures for the attic, the floor and the walls which can be insulated both on the outside and on the inside of the house. Three types of window retrofits are dealt with, triple-glazed windows, windows with low emissivity coatings and gas filled windows. Four different orientations, north, east, south and west is also present. There are therefore about 75 binary integers present in the model.

One very important part to deal with is the present state of the existing windows or the facade of the building. If the windows for example, are affected by rot they must be changed immediately to new ones and the remaining life of the existing windows is therefore set to null. If this is not the case they have a salvage value which must be considered. This is dealt with by use of a so called unavoidable, or inevitable, retrofit cost. In our case study 27 windows are oriented to the east. Each window has an area of 2.8 m^2 and the cost for a new window of the same type that is present today is assumed to be 1,100 SEK/m^2. If no measures are made for thermal reasons 83,160 SEK must be invested in order to change the bad existing windows to new ones of the same type. Assume that new windows last 30 years before they have to be changed again. A present value calculation for 50 years and a 5% discount rate shows:

$$83,160 + 83,160 \times (1 + 0.05)^{-30} - \frac{83,160}{3} \times (1 + 0.05)^{-50} = 99,984 SEK$$

If the original windows were perfect in condition no investment have to be made for 30 years and the expression would have looked like:

$$83,160 \times (1 + 0.05)^{-30} - \frac{83,160}{3} \times (1 + 0.05)^{-50} = 16,824 SEK$$

If triple-glazed windows, with a cost of 1,300 SEK/m^2 are installed at year 0 the present value becomes 118,162 SEK. In the first case with poor windows the better thermal behaviour must save 18,178 SEK before triple-glazed windows are profitable while they have to save 101,338 SEK if the original windows are in perfect shape. The same procedure must be considered for all the building measures in the model. In Table 1 the inevitable costs for the building are shown both if no thermal improvement is made and for the cases where better windows and added insulation are applied.

Measure	Cost [SEK]
No retrofit	407,632
Attic floor insulation	407,632
Floor insulation	407,632
External wall insulation, outside	222,832
External wall insulation, inside	376,832
Windows,	
north	407,632
east	307,648
south	407,638
west	315,584

Table 1: Unavoidable or inevitable retrofit costs in SEK for the building

To the unavoidable cost above the actual cost for the retrofit must be added which in turn depends on what solution that is optimal. If none of the ”thermal” retrofits are optimal 407,632 SEK must be added to the objective function. If triple-glazed windows oriented to the east are optimal the unavoidable cost is 307,648 SEK while the new windows cost is 118,162 SEK or a total cost of 425,810 SEK. This sum must therefore be compared with the unavoidable cost when no retrofits at all are present and the difference, i. e. 18,178 SEK which is coupled to a binary variable and added to the objective function. The model must therefore include a new set of constraints where the first sets the unavoidable cost if no retrofits are optimal:

$$IS_0 + IS_1 + \ldots + IS_{10} + F_{00} + F_{02} + \ldots + F_{23} + NOR \geq 1 \quad (9)$$

The binary integers F_{00} to F_{23} shows that window retrofits are optimal if the values equal 1. If all the IS and F integers equal 0, NOR (for no retrofit) must be 1 and this binary integer is then coupled with the cost 407,632 SEK above and inserted in the objective function. The second constraint adds the same value if one of the retrofits are chosen:

$$IS_0 + IS_1 + \ldots + IS_{10} + F_{00} + F01 + \ldots + F_{23} + M \times R \geq M + 1 \qquad (10)$$

M is here set to a value higher than the possible sum of all retrofit integers, in our case 200. If one or more retrofits are optimal R (for retrofit) must therefore become 1 and the R variable is coupled to the unavoidable cost and inserted in the objective function. This awkward way is needed because it is not possible to add just a value to the objective function. The unavoidable cost is needed here in order to achieve the accurate LCC but it does not affect the optimal solution. The next four constraints are needed because it must not be possible to add both triple-glazed windows and windows with better thermal performance in the same orientation at the same time. One constraint is:

$$F_{00} + F_{10} + F_{20} \leq 1 \qquad (11)$$

The first figure in the index 00 shows that it is triple-glazed windows while the second figure shows the orientation where 0 means north, 1 means east and so on.

In our case the demand charge in the electricity tariff depends on the fuse that must be used, see Table 2.

Fuse size [A]	16	20	25	35	50	63	80	100
Annual cost [SEK]	1,025	1,165	1,375	1,863	2,713	3,475	4,525	6,338

Table 2: Fuse tariff for Linköping, Sweden

The model must therefore include expressions that calculates the current and which set proper values in the objective function. In this case study electricity can only be used for running two heat pumps. The first one, PHP, is used as a normal heating device taking the heat from the ground water while the other one, PEA, takes it from the exhaust air. (It is not plausible that both heat pumps are optimal but we do not now in advance which solution is preferred.) The efficiency for the first one is set to 3.0 and for the second one to 2.0. Therefore the following constraints are used:

$$\frac{1}{3.0} \times P_{HP01} - P_{HP} \geq 0 \qquad \text{Compare with expression no. (3)} \qquad (12)$$

$$\frac{1}{2.0} \times P_{EA01} - P_{EA} \geq 0 \qquad (13)$$

$$P_{HP} + P_{EA} - P_{EL} \leq 0 \qquad (14)$$

$$-\frac{1000.}{380 \times 3^{0.5} \times P_{EL}} + CU = 0 \qquad (15)$$

Equation (15) is only used for calculating the current when we know the demand for electricity and the voltage which is 380 V. If the current is lower than 35 A, but higher than 25 A, an annual cost of 1,863 SEK must be present in the objective function, see Table 2. This is achieved by use of 8 new binary integers, E, and 8 integers Y. In each set only one of the integers can be 1. Eight new constraints must be present in the model and the first one is presented below:

$$CU - 16 \times E_0 + M_1 \times Y_0 \leq M_1 \tag{16}$$

M_1 is a large value, in our case 10,000. If Y_0 is set equal to 1, and CU is lower than 16 which is the first fuse size in Table 2, E_0 finds the value 1. E_0 is then coupled to the annual cost 1,025 SEK which is present in the objective function as a present value. If CU is larger than 16, E_0 must be set to 0.

The model now includes 150 constraints and 182 variables where 74 are binary integers.

OPTIMISATION

The model above is implemented in a Windows 95 program and written in classic C. The program writes the mathematical problem to a so called MPS-file which is an often used standard. Several optimisation codes can read such files e. g. CPLEX or LAMPS, but we have used the ZOOM program, see Reference [10], just because we have some experience in that product. By use of ZOOM it is possible to find the optimal way to heat the building. First an oil-fired boiler, thermal size 21.3 kW, must be combined with a heat pump of 37.8 kW, which add up to 59.1 kW. Two building retrofits were also optimal viz., low emissivity triple-glazed windows and weather-stripping. The first measure decreases the demand from 71.96 to 61.80 kW while the second lowers the demand to 59.1 kW. The heating equipment is therefore sufficient in thermal size. In Table 3 the energy need for the optimal solution is shown in detail.

From Table 3 the average energy cost for each kWh can be calculated, i.e. 0.23 SEK. The value is of that size because of the heat pump. This is also the reason for only two building retrofits being present in the optimal solution. It shall be noted here that time segment number 15 only shows 3.23 kW where instead it should have been 4.86 kW. The higher value must be present in order to provide the building with domestic hot water. The reason for the wrong value is due to the energy balance for the existing building where no retrofits were implemented. The balance shows that the segments 11-14 only are used for hot water heating while segment 15 must use heat also for space heating. The binary variables coupled to window retrofits and weather-stripping for segments 11 - 14 are therefore not present in the in the MILP model. Segment 15, however, has such integers and therefore the model saves heat due to the retrofits even if the value becomes lower than 3,500 kWh/month. The heating season will become shorter if more retrofits are optimal but the phenomenon is here only present in one time segment. The total energy cost in Table 3 which occur every year must now be calculated as a present value. The cost is therefore multiplied with 18.26, see expression (1). The heating equipment cost can be calculated by use of expression (5). It was optimal to choose triple-glazed windows with low emissivity coating. The cost for such windows is assumed to be 1,500 SEK/m^2

Segment	Hours	Oil-Boiler			Heat Pump			Total cost
No	[h]	Demand	Energy	Cost	Demand	Energy	Cost	
1	368	-	-	-	36.62	13,476	4,536	4,536
2	184	2.51	462	240	37.88	6,970	1,308	1,548
3	192	-	-	-	37.88	7,272	1,365	1,365
4	336	-	-	-	33.49	11,253	3,788	3,788
5	168	2.68	450	234	37.88	6,364	1,194	1,428
6	192	-	-	-	35.85	6,883	1,292	1,292
7	336	-	-	-	22.06	7,412	2,495	2,495
8	168	-	-	-	35.66	5,990	1,124	1,124
9	240	-	-	-	26.59	6,382	1,198	1,198
10	720	-	-	-	13.11	9,439	1,428	1,428
11	744	-	-	-	4.70	3,496	529	529
12	720	-	-	-	4.86	3,499	529	529
13	744	-	-	-	4.70	3,496	529	529
14	744	-	-	-	4.70	3,499	529	529
15	720	-	-	-	3.23	2,326	352	352
16	744	-	-	-	17.07	12,700	1,922	1,922
17	336	-	-	-	26.28	8,830	2,973	2,973
18	168	-	-	-	30.93	5,196	975	975
19	216	-	-	-	27.83	6,011	1,128	1,128
20	352	-	-	-	32.58	11,468	3,861	3,861
21	176	-	-	-	35.49	6,246	1,172	1,172
22	216	-	-	-	33.55	7,247	1,359	1,359
Sum	8784	-	912	474	-	155,455	35,586	36,060

Table 3: Optimal demand, kW, energy need, kWh, and cost in SEK for the energy use in the studied building

while weather-stripping has a cost of 14,000 SEK and a life of 10 years. In Table 4 all these costs are presented as present values and the sum represents the total LCC.

The sum in Table 4 differs only with about 2,000 SEK from the value calculated by ZOOM.

It was not found profitable to add extra insulation to the climate shield. Experience from the OPERA-model shows, however, that at least extra attic floor insulation many times is a profitable retrofit, see Reference [11]. When MILP is used it is not possible to use the so called ranging method, i.e. to examine in which interval a variable is optimal. Therefore, it is necessary to change a variable in the input data and optimise the problem once again. One suitable parameter to change is present in the cost function for insulation. The cost for all insulation measures is presented in the following form:

$$C_{ins} = C_1 + C_2 + C_3 \times t \tag{17}$$

where C_1 shows the unavoidable cost in SEK/m^2, C_2 the "step" cost for the insulation in SEK/m^2, C_3 the cost in SEK/(m$^2\times$ m) and t the added amount of new insulation in m. If the cost C_2 is changed it will only affect the possibility for insulation to be optimal, not the amount of insulation that should be added which is the case if C_3 is changed. In the original case C_2 was set to 260 SEK/m^2 and this is now changed to 200. The optimisation now shows that 0.14 m attic

Oil-boiler, (28.4 kW)	58,001
Heat pump, (12.6 kW)	217,153
Fuse tariff, (20 A)	21,273
Energy	658,456
Windows, east	136,341
Windows, west	125,521
Weather-stripping	33,099
Unavoidable retrofit cost	215,600
Life-Cycle Cost	1,465,444

Table 4: Present value costs and LCC in SEK for the studied building

floor insulation must be added. Because of this a slightly smaller heat pump should be used and, further, the total LCC is reduced to 1.454 MSEK. (The shift between extra insulation or not seems to emerge for a C_2 cost of about 240 SEK/m^2.)

Above it was found that triple-glazed windows were optimal. This is so because the original double-glazed windows were worn out and their existing remaining life was set to 0 years. Suppose they have 20 years left before they must be changed. A new optimisation shows that window retrofits no longer are optimal, but instead 0.16 m of extra insulation should be added to the attic floor. Some extra optimisations show that window retrofits falls out from the solution if the original ones have approximately 15 years left of their remaining life.

CONCLUSIONS

It is shown that a building can be described mathematically in the form of a Mixed Integer Linear Program, MILP, model. The integers are very important because "steps" in the cost functions can be dealt with. Small changes in these steps might result in different optimal solutions. Fortunately, the optimisation results in solutions that differs very little from each other in terms of the minimised Life-Cycle Cost. Small errors in input data do therefore not necessarily lead to hazardous solutions as long as the proprietor acts in an optimal way. If however, combinations of measures are chosen that do not fit together the result is likely to be an expensive experience.

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