OPTIMIZATION OF BUILDING RETROFITS IN A COMBINED HEAT AND POWER NETWORK

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Abstract

We describe a mathematical model of a linear program for optimization of the use of purchased and produced electricity, (both for electricity use and for heat), fuel mix in the district-heating plant and implementation of energy-conservation measures, in the Malmö building stock, of the south of Sweden. We find that energy retrofits will not be profitable compared to producing or purchasing more electricity and heat. The reason for this is the low cost for purchasing electricity from the national grid, even during peak conditions, and the use of waste heat in the district-heating plant.

INTRODUCTION

When electricity is produced in an ordinary power plant with condenser, steam is produced by burning fuels in the boiler. In order to maximize the electricity production, it is important to make the difference in the steam preassure as high as possible between the inlet and outlet of the turbine. This is often accomplished by cooling the condenser with cold water from the sea. However, this means that about 60 % of the energy in the fuel will be wasted as luke warm water while only about 40 % will become useful electricity. By using a district heating grid as a cooling device for the condenser this waste heat can be used for heating e.g. buildings. Unfortunately, it is not possible to use the waste water directly because of its low temperature, about 10 °C. By increasing this temperature to about 100 °C, or many times more, the district heat subscribers could use the heat, but this will also mean that the electricity production in the plant will get lower. The reduction, however, has been calculated to only about 10 to 15 %. Economic theory implies that optimal use of resources prevails if short range marginal costs are used for pricing the resource. This cost equals the amount of money saved if one unit less of the resource is produced, or the cost for producing one extra unit. The cost for scarcity must also be included. The district heat consumer should therefore only pay for the loss of electricity from the Combined Heat and Power, CHP, part of the electricity plant because of the raised temperature in the condenser. If the amount of heat is not sufficient, the cost will of course increase and depend on the fuel mix used for supplying more heat in the district heating plant. If the cost for producing the electricity is higher than the market price the production should of course be terminated, see Ref. [1]. The aim of this study is to find out how to run the different equipment, and how to change the energy system, in an optimal way. In recent years, there has been increased interest in using linear programming techniques for the optimization of energy systems of various sizes, see Ref. [2]. Some journals dealt with energy and economic mathematical modelling in its entirety but there is no publication describing an optimal building-energy system in a detail, although a suitable mathematic technique has been presented, see Ref. [3] and [4] Two authors have described a model used for minimizing the cost of the German energy system. However, buildings and building retrofits were not of special concern in the studies, see Refs. [5],[6] and [7]. A published model deals with building retrofits and bivalent heating systems, see Ref. [8] but the CHP system was not included. The opposite situation applies to the model, see Ref. [1] which forms the basis for our study in which both a CHP plant and a district-heating system are described and analyzed by using linear programming. In a linear program, there is always an objective function which must be minimized or maximized, see Ref. [9]. In this study, the function contains the life-cycle cost LCC of the energy system and this cost is then the subject for minimization. The LCC is calculated as the sum for all building, maintenance and operating costs of the system. The objective function is constrained by a number of equations, e.g. requirements for electricity or heat or the size of a power plant may not be higher or lower than a certain value. One item of special interest is the influence of steps in the cost function, e.g demolition of old boilers etc., before the linear part of the cost function will start. Such problems are solved by implementing binary integers which can only have the values 0 or 1. The same technique is used in making a linear approximation to nonlinear functions, e.g. estimating the influence of additional insulation on external walls. The mathematical problem may then be solved by using an appropriate program for solving a mixed-integer problem, see Ref. [10] We have frequently used the LAMPS and ZOOM programs, Refs. [11] and [12]. The LAMPS system has been implemented on a DEC-2065 machine while ZOOM may be used for various computers.

CASE STUDY

In order to exemplify the method with linear programming, a case study of the municipality Malmö, Sweden, is shown. The electricity load is presented in Table 1.

The load, monitored in 1988, is split in several time segments because of the tariff design for buying electricity from the company Sydkraft. There are high and low price periods and the cost for electricity is shown in Table 2.

Further, there is a demand charge of 270 SEK/kW during high price periods from November to March. (1 US$ = 6 SEK). The district heating load was not monitored for the same segments of time, i.e. split in time-of-use during the day, and therefore a gigantic building has been designed resulting in a climatic load of the same magnitude as the total monitored annual district heating load. The reason for acting in such a way is also because there must be a consistent influence between the retrofit actions of a building and the decrease of the thermal load. U-values and areas for the different building parts are shown in

Month	High (GWh)	Low (GWh)	Month	High (GWh)	Low (GWh)
January	117.9	103.5	July	68.1	56.7
February	122.1	94.9	August	96.7	70.9
March	131.0	98.5	September	107.2	81.0
April	105.7	94.1	October	111.5	99.5
May	87.9	69.6	November	129.9	98.4
June	88.6	65.1	December	135.6	111.2

Table 1: Electric load in Malmö

	Energy price [SEK/kWh]	
Month	High price	Low price
November - March	0.235	0.142
April, September, October	0.126	0.0997
May - August	0.068	0.057

Table 2: Electricity price Sydkraft 1990

Table 3.

Building part	Area (Mm^2)	U-value ($W/m^2 \times K$)	U×A (MW/K)
Attic floor	3.1	0.5	1.55
External walls	9.7	0.7	6.79
Floor	3.1	0.5	1.55
Windows, 1.2 Mpc × 1.5 m^2	1.8	2.5	4.50
Total			14.39

Table 3: U-values and areas for different parts of the fictional building

Thermal losses because of ventilation is set to 5.07 MW/°C and the heat supply for domestic hot water is calculated to 350 GWh annually. It has also been assumed that the indoor temperature in the building is 21 °C, while the outdoor temperatures for each month are mean values for a thirty year period, monitored by the Swedish Meteorological and Hydrological Institute. The values above result in a district heating load shown in Table 4.

Electricity prices are shown in Table 2, but electricity can also be produced in the CHP plant where natural gas is burnt in the boiler. The model will also contain a gas turbine operating on natural gas as well. The CHP plant exists today and the cost for the equipment must subsequently be considered as so called sunk costs. The gas turbine does not exist but is included in the model in order to examine when new equipment will fall into the optimal solution and to what price. District heat can be produced in a number of different ways. First there can be garbage burnt in an incineration plant. There are also waste heat from some industries, heat pumps in the sewage water treatment plant, coal, oil or natural gas fired boilers. The cost for operating the different facilities, their sizes, efficiencies and so forth are presented in Table 5.

Month	Load	Month	Load	Month	Load
January	340.5	May	173.9	September	134.3
February	323.1	June	113.2	October	204.3
March	312.9	July	84.2	November	254.8
April	239.3	August	91.4	December	304.2

Table 4: District heating load in Malmö in GWh

Equipment Type	Fuel price SEK/MWh	Efficiency SEK/MWh	Taxation SEK/MWh	Heat price SEK/MWh	Size MW
Garbage	54	1.0	-	54	65
Ind. waste	100	1.0	-	100	30
Coal	42	0.8	55	107.5	125
Heat pump	198	3.0	50	116	40
Natural gas	85	0.85	29	129	120
Oil	57	0.8	89	160.3	240
Gas-turbine	85	0.25	-	340	New
CHP-plant	85	0.85	-	100	120

Table 5: Equipment in the district heating plant etc.

There are also costs emerging when retrofitting the building. These are shown in Table 6.

Building asset	Total cost [SEK/m^2]
Attic floor insulation	$0 + 260 + 530 \times t$
New double glazed windows	$0 + 1\ 100 \times A_f$
Triple glazed windows	$0 + 1\ 300 \times A_f$

Table 6: Building retrofit costs

The costs has the form of linear functions where the first constant shows the cost for raising scaffolds and so on, however here set to 0. The two second constants, when insulation measures are considered, show the influence of the insulation as one constant value and one depending of the thickness of insulation. The same method has been used when the so called OPERA model was designed, see Refs. [13] and [14]. The cost for new windows depends on their sizes. There are of course also other retrofits but these are the ones that are implemented in the model because they fell out as candidates in an OPERA optimization.

The thermal or electric power of the equipment are examples of variables in the model. If these are known all other aspects of the energy system could be calculated. The cost for operating the energy system depends for example on the rate for purchasing electricity from the market, see Table 2. The rate is split in several segments depending on the time of the year and time of the day. The climatic load in the district heating plant depends on the month and so does the electricity load. There is also a rate element showing the cost for maximum electricity demand in SEK/kW. In order to model the energy system cost it is thus necessary to implement 29 time segments, one for high price conditions and one for low price conditions for all twelve months, and one for the maximum electricity load for the five months November to March. Because of the large number of equations the model is presented only for one month, January. The optimal solutions, presented for the various studies, are of course shown for the complete model.

Electricity production

Electricity can be produced by the municipality by burning natural gas in an existing steam boiler. Due to Swedish taxation the electricity production is not taxed at all from the utilities point of view, while the natural gas heat, delivered to the district heating grid, is taxed with 29 SEK/MWh. The natural gas price is 85 SEK/MWh and the efficiency is 0.85. The first part of the objective function, which is to be minimized and note no equals sign, shows the cost for the electricity production by the use of steam:

$$(EDH_1 \times 336 \times 100.0 + EDL_1 \times 408 \times 100.0+$$

$$+HEH_1 \times 336 \times 129.0 + HEL_1 \times 408 \times 129.0) \times 18.26 \times 10^{-6} \qquad (1)$$

where EDH = electricity production during high-price conditions (336 hours) in MWe, EDL = electricity production during low price conditions (408 hours) in MWe, HEH = heat from condenser during high price conditions in MW, HEL = heat from condenser during low price conditions in MW, 1 = number of the month, here January, 18.26 = the present worth factor for 5 % real discount rate and a 50 year project life. There must also be an expression included in the model where the needed electricity from the market, or production in the gas turbine is shown. In Table 1, the need for electricity is shown for various months. Determining variables for the need to purchase, and for gas-turbine production of electricity yields:

$$(EDH_1 + GTH_1 + REH_1) \times 336 \geq 117.9 \times 10^3 \qquad (2)$$

$$(EDL_1 + GTL_1 + REL_1) \times 408 \geq 103.5 \times 10^3 \qquad (3)$$

where REH = the purchased electricity under high price conditions in MWe, REL = the purchased electricity under low price conditions in MWe, GTH = the electricity produced in the gas turbine, high price conditions and GTL = the gas-turbine electricity production during low price conditions. Expressions 2 and 3 show two constraints on the model, i.e. the need for electricity must always be covered. However, if there is a gas-turbine to be used or there shall be a purchase from the market, expression 1 i.e. the objective function, must be changed in order to reflect the cost for doing so. In Table 2 the cost for each kWh is shown and to expression 1 must subsequently be added:

$$(REH_1 \times 336 \times 235 + REL_1 \times 408 \times 142) \times 18.26 \times 10^{-6} \qquad (4)$$

The gas-turbine is not an existing utility and thus this equipment must be bought and installed before it can be used. The cost for installment and operation is assumed to be reflected by the following cost, added to the objective function:

$$3.0 \times GTMF + \frac{85.0 \times 18.26 \times 10^{-6} \times (GTH_1 \times 336 + GTL_1 \times 408)}{0.25} \tag{5}$$

where 3.0 = the cost of a gas-turbine in MSEK/MWe, $GTMF$ = the maximum fuel demand in MW for the gas turbine in any time segment, 0.25 = the efficiency of the gas-turbine, 85.0 = the natural-gas price. It is also necessary to ascertain that $GTMF$ above is the largest value in MWe used during any time segment. This is accomplished by use of the following constraints:

$$\frac{GTH_1}{0.25} - GTMF \leq 0.0 \tag{6}$$

$$\frac{GTL_1}{0.25} - GTMF \leq 0.0 \tag{7}$$

The model will contain 12 equations of type 6 and type 7, two for each month, and because all of them must be valid at the same time GTMF must be set to the largest value for any month. The same technique is used for ascertaining the maximum demand for the electrical charge. Here, only five months are of concern, November to March, and subsequently the following constraint for each month must be added to the model:

$$EDH_1 + PMAX + GTH_1 \geq 443.1 \tag{8}$$

where $PMAX$ = the maximum purchase during any of the five months in MWe, GTH_1 = the gas turbine production in January, high price conditions in MWe and 443.1 = the maximum monitored demand in January. The demand charge, 270 SEK/kW, must be added to the objective function and in MSEK the expression becomes:

$$PMAX \times 270 \times 10^{-3} \tag{9}$$

The existing steam boiler is constrained in electrical size to 120 MW. However, also a lower limit is present. If the electrical load is lower than 40 % of the maximum power the plant is turned off because of loss in efficiency. The model must therefore contain expressions telling that, if electricity production is profitable the electric power must be between 48 to 120 MW, otherwise the plant must be turned off. This is done by use of so called binary integers which can only have values of 0 or 1. The expressions for January become:

$$EDH_1 - INTH_1 \times 120 \leq 0 \tag{10}$$

$$EDL_1 - INTL_1 \times 120 \leq 0 \tag{11}$$

$$EDH_1 - INTH_1 \times 48 \geq 0 \tag{12}$$

$$EDL_1 - INTL_1 \times 48 \geq 0 \tag{13}$$

where $INTH_1$ = binary integer for high price conditions and $INTL_1$ = binary integer for low price conditions. The expressions 10 to 13 tells that EDH_1 must be less or equal than 120 MW if $INTH_1$ equals 1 and at the same time larger than 48 MW. If $INTH_1$ equals 0, this will also imply that EDH_1 will equal 0.

District heat production

The district heat is produced partly by use of waste heat from the electricity production and mainly by use of different fuels in the boilers of the utility. In the first case three units of heat are assumed to be produced for each unit of electricity. This fact must be included in the model and using the variables from expression 1 a set of equations may look like:

$$3.0 \times EDH_1 - HEH_1 = 0 \tag{14}$$

$$3.0 \times EDL_1 - HEL_1 = 0 \tag{15}$$

If the waste heat from the electricity production is not sufficient for supplying the thermal load, fuels etc. must be used in the district heating plant. There are several different sources for this heat, see Table 5, and the cost for using them must be implemented in the objective function. Expression 1 must therefore be added with:

$$(HG_1 \times 54 + HW_1 \times 100 + HC_1 \times 107.5+$$

$$+HHP_1 \times 116 + HGAS \times 129) \times 18.26 \times 744 \times 10^{-6} \tag{16}$$

where HG_1 = the thermal power from garbage incineration, HW_1 = the thermal power from industrial waste heat, HC_1 = the thermal power from the coal boiler, HHP_1 = the thermal power from the sewage water heat pumps and $HGAS$ = the thermal power from the natural gas boiler.

It must be noted that the energy cost for the heat pump, i.e. 116 SEK/MWh, is an approximation of the real cost. The real cost depends on the cost for electricity, which can be produced in the steam-turbine, the gas-turbine or be purchased from the market. The real electricity cost will subsequently depend on the mix of these different ways to supply the system with electricity. Unfortunately, it has not been possible to model this dependence in terms of a linear or mixed integer program and subsequently an approximation, calculated by the municipality of Malmö, has been used for the energy price of the heat pump. The model must also contain expressions about the need for heat in the different time segments. This need is shown in Table 4 and the resulting expression shows that the sum of all the heat produced must exceed the need:

$$(HG_1 + HW_1 + HC_1 + HHP_1 + HGAS) \times 744+$$

$$+HEH_1 \times 336 + HEL_1 \times 408 \geq 340.5 \times 10^3 \tag{17}$$

The different equipment have limited sizes in MW, found in Table 5, which yield the following constraints:

$$HG_1 \leq 65,\ HW_1 \leq 30,\ HC_1 \leq 125,\ HHP_1 \leq 40,\ HGAS \leq 120 \tag{18}$$

The equations above complete the production part of the model, note that only January is presented, and contains about 150 variables and a somewhat larger amount of constraint equations. Before the energy conservation part of the model is presented, this first part will be optimized and discussed in some detail. Due to the large number of variables and the cumbersome way for presenting the mathematical model to the computer program that is used for the optimization, in the so called MPS format, a small FORTRAN program has been developed in order to write the input data file. This program and the MPS input data file cannot be presented here, but they will be published in a separate report, see Ref. [15]. In Table 7, the optimal solution is presented showing the electrical and thermal loads for the various equipment.

Month	CHP		Purchase		District heat				
	High	Low	High	Low	Garbage	Waste	Coal	Heat pump	Nat. gas
January	120.0	48.0	230.9	205.7	65.0	30.0	120.8	-	-
February	120.0	48.0	243.4	215.7	65.0	30.0	120.7	-	-
March	120.0	-	236.0	262.0	65.0	30.0	125.0	22.3	-
April	51.6	-	263.0	245.1	65.0	30.0	125.0	40.0	-
May	-	-	249.6	177.6	65.0	30.0	125.0	13.7	-
June	-	-	251.6	176.8	65.0	30.0	62.2	-	-
July	-	-	202.8	139.1	65.0	30.0	-	-	-
August	-	-	262.9	188.5	65.0	30.0	27.9	-	-
September	-	-	304.6	220.1	65.0	30.0	91.5	-	-
October	-	-	331.8	244.0	65.0	30.0	125.0	40.0	14.7
November	120.0	-	248.8	267.4	65.0	30.0	82.6	-	-
December	120.0	-	264.9	283.6	65.0	30.0	125.0	18.5	-

Table 7: Optimal solution when no energy conservation retrofits are present. Values in MW

Production of electricity in the CHP plant is found optimal during high price conditions in November to April but only under January and February during low price conditions. The plant should be operating at its maximum load except for April and at its lowest power under the low price hours. Under all other circumstances the plant should be turned off. Electricity should be purchased in all the time segments but the natural-gas turbine was not optimal to use at all. In the district heating plant the garbage incineration plant and the waste heat should be used at the maximum level all through the year, while the coal fired boiler is to be used at the maximum performance for five months and not at all under July. The heat pump will optimally be operating at its maximum in April and October while it is to be turned off for 7 months. The natural gas boiler is only to be used in October and then only 14.7 MW is necessary. Note that the need for both electricity and heat is covered sufficiently. In January the demand for electricity is approximately 351 MW under the high price period, i.e. $117.9 \times 103/336$ from Tables 1 and 8.

The CHP production and the purchase in Table 8 adds up to 350.9 MW. The total production of CHP electricity is $120 \times 336 + 48 \times 408$ equalling 59 904 MWh, which implies that three times more, or 179 712 MWh, of heat is delivered to the district heating grid. Table 4 states that 340.5 GWh of heat must be delivered. Using 65×744 of garbage heat, 30×744 waste heat and 120.8×744 MWh of coal heat leaves 179.9 GWh which is almost the same as the calculated contribution from the CHP plant. The values in Table 7 could be scrutinized finding reasons for their values but this is not accomplished in this paper. Instead the model will be completed with the energy conservation measures of concern.

Month	High price hours	Low price hours
January	336	408
February	336	360
March	368	376
April	336	384
May	352	392
June	352	368
July	336	408
August	368	376
September	352	368
October	336	408
November	352	368
December	352	392

Table 8: Number of hours in the different time segments

Energy conservation measures

An energy conservation measure, e.g. an attic floor insulation will possibly reduce the need for both electricity and heat. In this study the heat load is calculated by use of a gigantic building, see Tables 3 and 4. If the attic floor in this building is extra insulated the need for heat will be reduced. It has been shown that the new U-value for the attic floor could be calculated as found in Ref. [16]:

$$U_n = k_n \times \frac{U_e}{k_n + U_e} \times t \tag{19}$$

where U_n = the new U-value in W/m$^2\times$K, k_n = the conductivity for the new insulation in W/m$\times$K, U_e = the existing U-value in W/m$^2\times$K and t = the thickness of extra insulation in m. The equation 19 cannot be implemented directly in the model because it is not a linear statement. However, it is possible to calculate a new expression where the nonlinear function has been piecewise linearized. The method is described in Ref. [17] and subsequently only a very brief presentation is made here. The scope is to change Equation 19 so it will no longer be a function of t but instead of some binary variables, A, which correspond to different values for t. The first thing to do is to change the objective function. In Tables 3 and 6 the area and the cost for an attic floor insulation is shown. The cost for the insulation is a linear function of t but as the equation 19 is not, both expressions must be changed. Calculating the cost for extra insulation of some consecutive values for t will yield a new equation. Assume that the optimal insulation thickness is present in the range between 0 - 0.3 m of extra insulation. Further suppose that an approximation in steps of 0.05 metres is sufficient. The cost for 0.05 m of additional insulation is:

$$3.1 \times 10^6 \times (260 + 530 \times 0.05) = 888.15 MSEK \tag{20}$$

For 0.1 m it is 970.3 MSEK and so on. Multiplying these values with the binary integers will result in:

$$888.2 \times A_1 + 970.3 \times A_2 + 1052.5 \times A_3 + 1134.6 \times A_4 + 1216.8 \times A_5 + 1298.9 \times A_6 \quad (21)$$

As before, the binary integers can only have the values of 0 or 1 and setting a constraint:

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \leq 1 \quad (22)$$

will imply that only one, or none, of the values A_1 - A_6 can be chosen. The additional insulation leads to a lower thermal need for heat as shown in 19 and by calculating the difference between the old and the new U-value this decrease in thermal flow can be implemented in the model. Assuming that the existing U-value is 0.5 W/m$^2\times$K, see Table 3, and that the new conductivity for the added insulation is 0.04 W/m$\times$K will result in the values for a thickness of insulation of 0.05, 0.1, 0.15 m etc. shown in Table 9.

Variable t	New U-value	Reduction
0.05	0.308	0.192
0.10	0.222	0.278
0.15	0.174	0.326
0.20	0.143	0.357
0.25	0.121	0.379
0.30	0.105	0.395

Table 9: New, and reduction of U-value in W/m^{2}K for a retrofitted attic floor

Before implemented in the model, the values in Table 9 must be multiplied by firstly, the area of the attic floor, i.e. 3.1 $\times 10^6$ m^2 and secondly, by the number of degree hours assumed to be present in Malmö for each month. In Malmö the outside mean temperature in January is -0.5 °C, the indoor temperature is assumed to be 21 °C, and the number of hours for the same month is 744, which means that the number of degree hours in January is assumed to be 15 996. The decrease in the heat flow for a 0.05 extra attic insulation will subsequently be, 0.192 $\times$ 15 996 $\times$ 3.1 $\times$ 10^6 equalling 9.52 GWh which must correspond to the A_1 value above. The equation 17 must therefore in the right hand side be presented like this, note GWh:

$$340.5 - 9.5 \times A_1 - 13.8 \times A_2 - 16.2 \times A_3 - 17.7 \times A_4 - 18.8 \times A_5 - 19.6 \times A_6 \quad (23)$$

Note also that the function 23 is valid only for January and the model must subsequently contain 11 more equations of the same type. Equation 22 will ascertain that only one, or none, of the A variables is chosen and if one is chosen the appropriate value is added to the objective function and at the same time the need for heat is decreased according to this extra insulation. Another way for decreasing the heat flow is to exchange the existing double-glazed windows for new triple-paned. In Table 6 the cost for changing windows is found. It is assumed that the existing windows must be exchanged immediately. The cost for changing them to new double-pane windows is, see Tables 3 and 6:

$$(0 + 1100 \times 1.5) \times 1.2 \times 10^6 = 1980\, MSEK \tag{24}$$

The cost for exchanging the windows to triple-glazed ones is calculated in the same way to 2 340 MSEK. The difference in cost is subsequently 360 MSEK. Further, it has been assumed that the windows have a new life of 30 years. In this study a project life of 50 years is assumed and subsequently the difference in cost must be transferred to the base year by use of the present worth method. The discount rate is 5 %, and using a binary integer B for window exchange, the cost in the objective function will become:

$$360 \times [1 + (1 + 0.05)^{-30} - \frac{10}{30} \times (1 + 0.05)^{-50}] \times B = 432.8 \times B \tag{25}$$

The window exchange will influence the need of heat, reducing the U-value from 2.5 to 2.0 W/m$^2\times$ K, and the right hand side of equation 17 must subsequently be added with:

$$-15996 \times 1.5 \times 1.2 \times 10^6 \times (2.5 - 2.0) \times 10^{-9} \times B = -14.4 \times B\, GWh \tag{26}$$

Note that the thermal district heating load in MW also is influenced by the building retrofits. The model does not contain any expression for this because of the lack of a suitable cost. The existing equipment can meet the need for heat in every moment and no new boilers etc. must be built. The model must also contain expressions showing the cost and the consequences for conservation of electricity. Unfortunately, it is not clear how an individual retrofit affects the electricity load. It has thus been necessary to design another gigantic building which could be insulated etc. in the same way as the earlier one. Information from Malmö shows that this new building should have a transmission coefficient of about 1.975 MW/K. The use of electricity for space heating has been calculated based on that value and the result is shown in Table 10.

Month	High price	Low price	Month	High price	Low price
January	14.27	17.33	July	2.52	3.06
February	14.03	15.03	August	3.13	3.19
March	14.25	14.56	September	5.21	5.45
April	9.95	11.38	October	8.03	9.75
May	6.95	7.74	November	11.19	11.70
June	4.17	4.36	December	13.21	14.71

Table 10: Assumed electricity used in GWh for space heating in Malmö

Further it is assumed that the district heated and the electrically heated building behave equally. Thus it is supposed that about 35 % of the electric space heating load depends on the ventilation, which implies that 1.284 MW/K is a result from transmission of heat through the walls etc. In order to examine if an attic floor insulation will be optimal it is in the same way assumed that about 11 % of the heat flow depends on this asset, resulting in 0.138 MW/K. If the existing attic has the same U-value, 0.5 W/m$^2\times$K this will imply that the area of the attic floor is 276 000 m^2. There is now sufficient information to design the amendments to the objective, and other functions and constraints. Firstly, expression 1 must be completed with, see the design of expression 21 etc.:

$$79.25\times D_1+86.58\times D_2+93.91\times D_3+101.24\times D_4+108.57\times D_5+115.90\times D_6 \quad (27)$$

where the variables D_1 - D_6 are binary integers. The influence on the electric load is calculated the same way as above, see 23 etc. and thus only the result is shown:

$$117.9-0.384\times D_1-0.56\times D_2-0.65\times D_3-0.71\times D_4-0.76\times D_5-0.79\times D_6 \quad (28)$$

$$103.5-0.47\times D_1-0.67\times D_2-0.79\times D_3-0.87\times D_4-0.92\times D_5-0.96\times D_6 \quad (29)$$

$$D_1 + D_2 + D_3 + D_4 + D_5 + D_6 \leq 1 \quad (30)$$

The two expressions 28 and 29 must be added to 2 and 3 respectively. Note that 28 deals with the high price period and 29 with the low price period. The extra attic floor insulation will also decrease the demand of electricity in MW. This is of interest during the 5 months when there is a cost for the demand due to the tariff of electricity. The dimensioning outdoor temperature is assumed to be - 14 °C and the desired indoor temperature 21 °C.The right hand side of Equation 8 must subsequently be completed with:

$$443.1-1.86\times D_1-2.69\times D_2-3.16\times D_3-3.46\times D_4-3.67\times D_5-3.82\times D_6 \quad (31)$$

Finally, the model contains a heat storage in order to store heat during the low price periods and using it under more costly time segments. The storage is assumed to use a water tank for the accumulation. The capacity of heat in water is about 4.18 KJ/kg×K which implies about 1.16 kWh/m^3×K. Further it is assumed that the temperature range of the water is about 40 K meaning that about 46 kWh can be stored in one m^3. The total cost for the storage has been assumed to 7 000 SEK/m^3 or about 150 SEK/kWh, due to a minor investigation in Ref. [15]. The storage is used for storing heat under the night hours, from 2200 to 0600. Each working day, 8 hours are present while during Saturdays and Sundays it is a long low price period without any interupt. The working days will subsequently be the period which will decide the size of the storage. In January, 336/(24-8) equalling 21 working days are present and there is therefore 21 × 8 or 168 hours available under this month for storing heat, while 336 hours could be used for the discharge. The model must include this fact and the expression is:

$$336 \times HSH_1 - 168 \times HSL_1 = 0 \quad (32)$$

where HSH_1 = the heat flow in MW in January, high price periods and HSL_1 = the heat flow in MW in January for low price periods. It is assumed that it is the electric load that is to be decreased by use of the storage and the equations 2 and 3 must be completed with the following expressions in their left sides respectively:

$$+HSH_1 \times 336 \tag{33}$$

$$-HSL_1 \times 168 \tag{34}$$

The model must also include a statement showing the maximum energy storage in any month:

$$-HSL_1 \times 168 + HSMAX \geq 0 \tag{35}$$

where $HSMAX$ = the maximum energy storage in MWh for any month. Equation 8 is affected and the HSH_1 variable must be added to the left side of that statement. The objective function must include the cost for the storage, i.e. $HSMAX\times$ 150 000 $\times 10^{-6}$ expressed in MSEK. This expression completes the model which now contains 220 variables and 211 constraints. Optimizing the model shows that not a single retrofit action in order to conserve electricity or heat is profitable. Neither is it profitable to produce more electricity in a new gas-turbine nor the heat storage is chosen. The optimization did result in the same strategy as shown in Table 7. However, it is possible to slightly change the model in order to force it to choose e.g. the attic floor insulation retrofit. This could be done by deleting the $<$ - sign in expression 22 which means that one of the A - variables must be set to 1. Optimizing this new situation shows that the value of the objective function is increased by about 747 MSEK, from 11 877 to 12 625, and the smallest amount of extra insulation is used, or 0.05 m. In Ref. [15] a closer examination has been made for different costs for the insulation and it is shown that the cost must be decreased with about 80 % before the insulation will be profitable and part of an optimal solution. It must be noted that the assumption of the original U-value was rather low, 0.5 W/m$^2\times$K which means that profitable extra insulation is very hard to achieve. A higher U-value in the original building, say 1.0 W/m$^2\times$K probably would have changed the situation. The same reason is valid for exchanging the windows, the U- value is only changed from 2.5 to 2.0 W/m$^2\times$K and the savings from this cannot compete with the cost for the exchange. There is also a possibility to change the heating system in an existing building in order to get a lower LCC. In this case, however, the district heated building is heated with waste heat form the electricity production, garbage, industrial waste heat etc. which means that the heat has a very low cost compared to the competing heating systems available. Electricity savings are much more profitable. Deleting the $<$ sign in equation 30 forces the model to choose extra insulation and the optimal situation is to set $D_2 = 1$ which means that 0.1 metre of extra insulation is to be implemented. The value of the objective function will now increase with 49 MSEK which is much less compared to district heating conservation by use of additional insulation. If the demand charges in the electricity rate are increased, this will imply that the gas turbine, the heat storage and the conservation retrofits will become more profitable. The demand charge must, however, be increased to about 700 SEK/kW, compared to 270, before the gas turbine will be profitable. The conservation measures will not emerge as optimal before the cost for the gas turbine exceeds 4 200 SEK/kW. Some experiments with the model show that the heat storage will be optimal if the cost for it is decreased with about 90 %. If the electricity tariff is increased it will lead to an increased production in the CHP plant and further using the gas-turbine, before the implementation of the heat storage will be optimal.

CONCLUSIONS

The paper shows that it is possible to build a mathematical model of a municipality energy system including both new and existing production units, CHP plants, district heating equipment and gas-turbines, as well as energy conservation measures such as attic floor insulation, exchanging windows and heat storages . The model is designed as a mixed-integer program and thus it contains linear expressions using ordinary as well as binary variables. The binary variables are used for solving originally nonlinear expressions. When the model is optimized it is revealed that with the prices for heat, electricity, operating equipment and energy conservation measures valid in Sweden today, none of the energy conservation measures tested in the model, was found to be optimal to implement. Instead it was better to produce more heat or produce more electricity in the existing plants. If the need could not be covered, purchase from the market was likwise optimal.

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