INDETERMINED CHAIR FRAMES OF ASH WOOD

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Abstract

During recent years more interest has been emphasised on wood as a construction material. This is so because wood is a renewable resource and also because problems with waste do not emerge when the wooden structure is taken out of operation. On the contrary this waste is still a resource even if the structure is demolished. Wood could always be used as a biomass fuel which is not expected to contribute to the greenhouse effect. In Sweden most of the interest has been emphasised on our conifers while broad leaved species are much less examined. This paper shows the result from the Finite Element Method applied on indetermined chair frames and compares these findings with actual testing in our laboratory. The conclusion is that there are considerable discrepancies between calculations and real behaviour even for relatively simple structures such as a chair frame. It seems that the real chair is stronger than expected even if the joints between the furniture members must reduce the overall strength found by the FEM calculations.

INTRODUCTION

Furniture, such as chairs, bookshelves, sofas and so forth, are seldom designed by use of modern solid mechanics. Instead, handicraft and aestethical experience decides how the products are constructed. Literature about solid mechanics and furniture is scarce but some research groups have emphasised this field. One of the earliest papers we have found was published in 1966, see Reference [1]. The author claims that much information could be found by dealing with furniture in the same way as other structures and gives examples of the stress found for a chair in use. Several other papers have been published by the same author and the most recent we found is referenced in [2]. Polish researchers have also contributed to this field and one paper of interest is shown i n Reference [3]. We have also taken part in this topic and one of our papers is [4]. Our interest has mostly been faced towards the construction and design of chairs made of massive wood. Further, the studies have been stressed upon Swedish broad leaved species, in this case ash, *Fraxinus excelsior*.

CASE STUDY

Experience from our earlier studies showed that a chair should be designed as found in Figure 1.

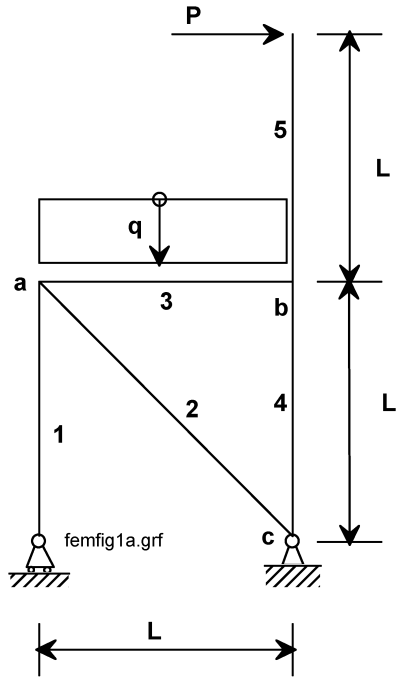

Figure 1: The tested chair frame

The main interest has been focused at the moment found at the joint between the seat and the back rails, see point b in Figure 1, which is decisive for the cross area of these furniture parts. We have now calculated the stress at certain other points as well, by use of a small FEM computer program, PCFEMP, and also tested the stress for a number of combined loads, see P and q in Figure 1.

Our design work started by assuming that the chair must endure a person with a weight of 90 kg, who at the same time will lean backwards with a load, P, of 300 N. The length, L, is 0.4 m. The distributed load, q, will therefore become $90 \times 9.81/(0.4 \times 2) = 1{,}104$ N/m. Note that there are two seat rails, one of each side of the seat. Calculations, see Reference [5], showed that member no. 4 and 5 should at least have a cross sectional area of 0.015×0.03 m, member no 2, 0.01×0.01 m, and no. 3, 0.005×0.02 m. The no. 1 member was from practical reasons manufactured with a cross sectional area of 0.01×0.015 m. Note that the joints are firmly glued which means that the frame is indetermined and hence must be analysed by considering the deformations of the structure. In Reference [5] it is shown in detail how this is done using the method of displacement. The number of cases dealt with now made it, however, necessary to use the FEM program instead.

Consider, for a start, that only the distributed load q is present. Our testing equipment did not include a weight of exactly 90 kg and therefore a combination of smaller weights added up to 93.2 kg. This resulted in $q = 1{,}142.9$ N/m. If it was assumed that the member no 3 had been supported by two hinges the moment in the middle would have been $\frac{q \times L^2}{8}$ or 22.9 Nm. The firm joints however, reduce this moment to only 11.75 Nm according to the FEM calculations, see Figure 2.

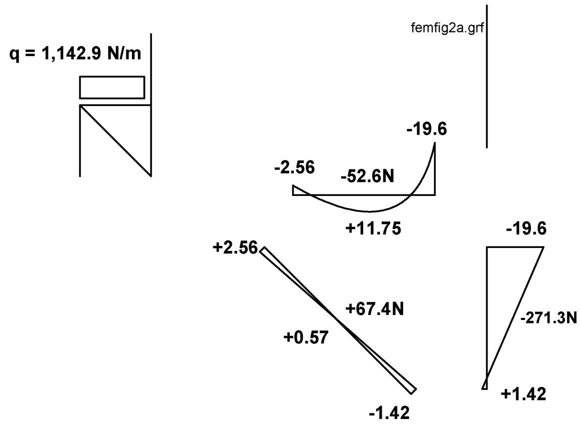

Figure 2: Moment graph and axial forces for the loaded chair. FEM calculations.

In Figure 2 the moments in Nm are found as well as the axial forces in N for different members of the chair. The distributed load results in a moment of 11.75 Nm in the middle of member no 3 and at the same time an axial force, compression, of 52.6 N. One of our strain gauges was located, in the middle of, and under, the seat rail and therefore the moment would result in tension at this point. Classic theory, and our cross sections, now says that for member no. 3:

$$\sigma = -\frac{N}{A} + \frac{M \times z}{I} = -\frac{52.6}{0.02 \times 0.005} + \frac{11.75 \times 0.01 \times 12}{0.005 \times 0.023} = 34.7 \quad \text{MPa}$$

$$\epsilon = \frac{\sigma}{E} = \frac{34.7 \times 10^6}{9,970 \times 10^6} = 0.003,480 = 3,480\ \mu_{strain} \quad \text{(tension)}$$

The modulus of elasticity, E above, has been monitored in our laboratory. The value 9,970 MPa is lower than the one found in literature, i.e. about 13,000 MPa, see Reference [6] page 164. Now this calculated elongation must be compared to the one actually monitored, which was 1,424 μ_{strain}, see Table 1, gauge no 3.

In Table 1, "-" signs show tension while "+" signs equal compression. The monitored value is therefore less than half the one calculated. Using 13,000 MPa would improve the situation but still the values would not coincide.

	Distributed load on seat rail in N/m							
	308.6		616.8		924.6		1,142.9	
Strain gauge no.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.
1	0	-3	0	-16	0	-22	0	-13
2	+253	+84	+505	+156	+753	+225	+934	+251
3	-919	-430	-1,881	-799	-2,819	-1,170	-3,480	-1,424
4	+72	+31	+150	+53	+222	+93	+276	+128

Table 1: Monitored and calculated elongation in μ_{strain} for different chair members and loads. Only the distributed load was present.

We have also monitored the strain at the upper and inside of member 4, gauge no. 2, the lower and left side of member 5, gauge no. 1, the back rail, and in the middle and upper side of member 2, gauge no. 4, the stretcher. Gauge no. 3 is, as mentioned above, located under member 3 at the middle of the beam. In Table 1 the calculated and the monitored strains for the different gauges are shown.

Strain gauges are very sensitive devices which could be seen on the first line in Table 1. Even if the distributed load is applied under gauge no. 1 it reacts due to internal forces in the back rail. Strain gauge no.2 is compressed due to both the axial force and the moment in the upper part of member no. 4. The calculated values for member no 2 are more than three times larger than the monitored ones and the situation aggravates when the load gets larger. Interesting to note is also that the monitored values do not increase in a linear way. Most of the compression should emanate from the moment in the upper part of member no. 4. The joint between the seat and back rails is, as told above, glued firmly together. Because of our very thin seat rail, only 0.005 m, this member is inserted right through and into a long hole in the back rail. The node therefore works like a mortise and tenon joint which is said to be the strongest type for transferring moment, see Reference [7] page 6-32. Nonetheless it seems that the transfer of internal forces from one member to the other is not perfect, at least, if Table 1 is considered. Gauge no. 3 shows a better behaviour but still the calculated values are twice the monitored ones. The monitored strain for gauge no 3 follows almost a perfect linear behaviour, i.e. when the load is doubled so is the strain. The discrepancies here could be the result of the rather stiff seat plate made of birch wood which is of 0.01 m thickness. The plate was only laid upon the seat rail and no glue or screws were used. If the plate had had an infinite stiffness all the distributed load would be transferred down the front and back legs of the chair and the member no. 3 would not be bent at all. The fourth gauge is applied in the middle and on the upper side of member no. 4. Also here the calculated strain is more than twice the monitored one. In Figure 2 it is shown that the stretcher, i.e. member no 2, is bent by rather small moments in each end. If the seat rail is not bent as much as calculated, which was indicated by gauge no. 3, these moments would be even smaller. This results in less compression on the upper part of the stretcher.

In our next experiment we applied a load, P, of 5 kp, or 49.05 N, at the top of member no. 5. The load were, after reading all the meters, increased in steps of 5 kp up to 30 kp. The experiments had to be cancelled at this point because the chair started to rotate and the front legs were bent in the wrong direction with plausible hazardous complications. The maximum applied load at the top of the back rail was therefore 25 kp, or 245.3 N. This introduced a moment of 0.4 × 245.3 = 98.1 Nm just above the seat rail. The FEM calculations show that the moment in the upper part of member no. 4 now will be positive instead of negative, compare Figures 2 and 3.

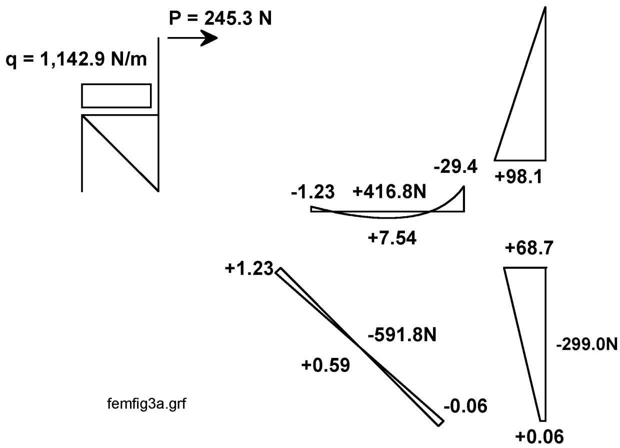

Figure 3: Moments and axial forces in the chair frame. FEM calculations

The maximum moment is found in the lower part of member no. 5, 98.1 Nm. In the top of member no. 4 the moment is 68.7 Nm and at the right side of the seat rail 29.4 Nm is present. The other moments are much smaller and interesting to note is also the decrease of the moment at the middle of the seat rail, from 11.8 to 7.5 Nm.

We have also monitored the strain for the five cases where the top back rail load varies from 49.1 to 245.3 N. In Table 2 the first of these is shown.

	Distributed load on seat rail in N/m							
	308.6		616.8		924.6		1,142.9	
Strain gauge no.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.
1	-875	-584	-875	-608	-875	-609	-875	-576
2	-533	-375	-307	-306	-29	-213	+150	-169
3	-782	-309	-1,719	-737	-2,660	-1,113	-3,327	-1,391
4	+210	+107	+282	+142	+354	+183	+410	+224

Table 2: Monitored and calculated elongation in μ_{strain} for different chair members and loads. Load at the top of the back rail equals 49.1 N.

Firstly, note that for strain gauge no.1 the monitored and calculated values are of the same magnitude even if the monitored values are slightly smaller. The reason for the relative coincidence might be explained by the simple structural element, i.e. the top of the back rail. No joints etc. are involved here.

Gauge no. 2 shows that the top part of member no. 4, i.e. the part of the back rail that is under the seat, is tensed at its left side. The calculations, however, predict that compression should prevail for the highest distributed load. The monitored values shows that tension decrease but not as much as calculated. The next line in Table 2 shows the largest discrepancies. For the highest distributed load the calculated value is almost three times the monitored one. Also for gauge no. 4 the rates between calculated and monitored values are large, approximately two, but in absolute values the differences are rather small, 100 - 200 μ_{strain}. In the next table, number 3, the back rail load is doubled to 98.1 N.

	Distributed load on seat rail in N/m							
	308.6		616.8		924.6		1,142.9	
Strain gauge no.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.	Calc.	Mon.
1	-	-	-1,747	-1,114	-1,747	-1,191	-1,747	-1,054
2	-	-	-1,067	-775	-815	-753	-636	-587
3	-	-	-1,560	-561	-2,501	-988	-3,169	-1,323
4	-	-	+324	+233	+491	+293	+544	+313

Table 3: Monitored and calculated elongation in μ_{strain} for different chair members and loads. Load at the top of the back rail equals 98.1 N.

The increased load on the back rail made the chair flip over for the lowest distributed load on the seat. No values are therefore presented for those columns. For gauge no. 2 the monitored and calculated values comes closer when the distributed load increases while no such phenomenon can be found for gauge no. 3.

In Figure 4 the strains for the gauges are depicted for a load at the seat of 1,142.9 N, i. e. the maximum load that we examined.

For gauge no. 1 the calculated and monitored values differs more and more when the load at the back rail increases. The same is valid for gauge no. 2 as well but the lines cross each other as well. Gauge no.3 shows a different behaviour. The lines are almost parallel but the values differs a lot. For gauge no. 4 a relatively good coincidence is experienced, i.e. the lines are parallel and they do not differ much in magnitude.

The conclusion of the above discussion must be that our calculated values cannot be trusted if they are compared with real values. Some of the discrepancies shown above must perhaps be blamed on the fact that it is difficult to apply a distributed load which really equals the one assumed in the calculations. Thus we have also elaborated FEM-calculations for the seat plate which is manufactured of glued strands of massive birch wood. As mentioned above, the seat plate was only placed upon the seat rails and no joints exist between the rails and the plate. The model, and elements, we chose emanates from the Kirchoff plate bending theory where triangular elements are used, see Figure 5.

Due to symmetry only one fourth of the seat plate is necessary to include in the FEM model. In node no. 1, see the black dot in Figure 5, the plate is supposed to be supported by a hinge, i.e. the plate can rotate around the x- as

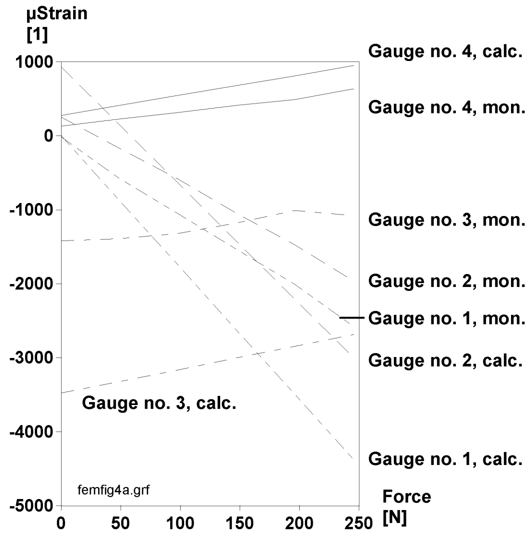

Figure 4: Monitored and calculated strains for a constant distributed load of 1,142.9 N and a varying back rail load, 0 - 245.3 N.

well as the y-axis. No deformation, or deflection, in the direction perpendicular to the plate is possible. In nodes no. 5, 10, 15 and 20 the rotation around the y-axis is prohibited because of symmetry and for the same reason the rotation around the x-axis is not possible for nodes 21, 22, 23 and 24. Node no. 25 cannot rotate at all. In all these points deflection, of course, is possible. The total plate is 0.4 times 0.4 m and thus the distance between node 1 and 2 is 0.05 m. The load on the seat has been calculated to 93.2×9.81 / $(0.4\ times\ 0.4) = 5{,}714$ N/m^2. The question is now how much the plate deflects between nodes 1 to 21, where the seat rail is located and, further, if the seat plate will significantly influence the assumed load on this rail. The calculation shows that the deflections are 0.0, 1.11, 2.04, 2.64 and 2.85 mm for nodes 1 to 5 respectively. The maximum deflection, naturally found for node 25, is 4.08 mm. The maximum moment is found for element 4 which is 134 Nm. This in turn leads to a stress of about 8 MPa which means that the plate is far from breaking. The calculations above therefore show that the stiffness of the plate will probably significantly reduce the assumed distributed load on the seat rail. The next step is to verify if this is the case or not.

In order to test this, the plate has been changed to about 40 strands with a cross section of 0.02×0.01 m. The strands are not glued together and therefore the applied distributed load must be closer related to the one used in the calculations. In Table 4, and Figure 6 the result is presented.

From Figure 6 it is obvious that the discrepancies between the monitored and the calculated values still remain. The profound dip in the curve for the monitored values depends on the weights used during the monitoring process.

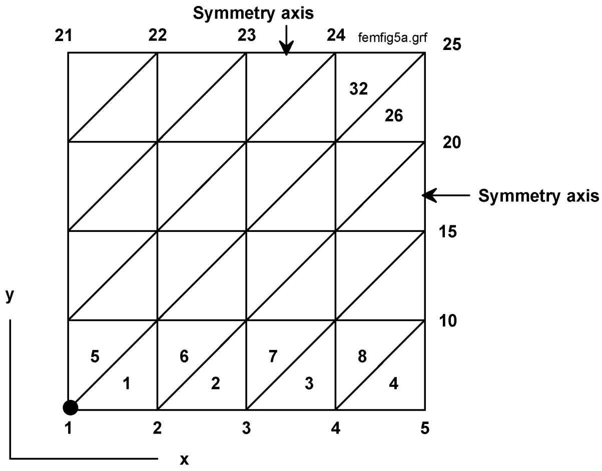

Figure 5: Finite element mesh for one fourth of the seat plate.

We had weights of 10 kg with a diameter of about 0.1 m and therefore only a few of the strands were in operation for transferring the load to the seat rail. When weights of 25 kg could be used, i.e. 306.6 N/m a larger part of the seat rail was exposed for the distributed load and the curve therefore will have a less steep slope. If, however, the calculations would correctly reflect the tension for the rail the monitored values should be located above the calculated one. Why this is not the case is still a mystery to be solved. Further, comparing Table 4 with Table 1 shows us that the seat plate did not influence the result as much as the FEM calculations indicated. A distributed load of 924.6 N/m resulted in a monitored strain of 1,170 μ_{strain} when the plate was used, see Table 1, while a load of 919.7 N/m resulted in 1,054 μ_{strain} when the strands were used. The influence of the plate therefore seems to be minute.

CONCLUSIONS

Finite Element calculations for indetermined chair frames seems to be a useful tool for showing the overall performance of this type of wooden structure. Some of the comparisons between monitored and calculated values showed a fairly good correspondence but unfortunately most of the other showed a poor coincidence. Especially the seat rail which was exposed for a distributed load showed that the ash wood member tensed less than expected. This behaviour was thought to be explained by the fact that the seat plate carried some of the load and therefore made the distributed load lower on the seat rail under the seat. FEM calculations for the seat plate also showed that this could be the case. The experiments showed, however, that this was not so. Replacing the seat plate with about 40 strands of birch wood, which were not glued together, eliminated the influence of internal forces in the plate and therefore a closer resemblance with a distributed load must be ascertained. The monitored values, surprisingly enough, showed that the discrepancies still remained when the

Load		Gauge 3		
[N/m]	Mom. [Nm]	Axial Force [N]	Calc. [μS]	Mon. [μS]
122.6	1.3	-5.7	-373	-310
245.3	2.5	-11.3	-747	-599
306.6	3.2	-14.1	-934	-445
429.2	4.4	-19.8	-1,306	-561
551.8	5.7	-25.4	-1,680	-707
613.1	6.3	-28.3	-1,867	-737
735.8	7.6	-33.9	-2,240	-852
858.4	8.8	-39.6	-2,613	-966
919.7	9.5	42.4	-2,800	-1,053
1042.3	10.7	48.0	-3,173	-1,167
1164.9	12.0	53.7	-3,546	-1,309
1226.3	12.6	56.5	-3,734	-1,386
1348.9	13.9	-62.2	-4,107	-1,614

Table 4: Tension in μ_{strain} on the lower side of the seat rail for a varying distributed load

strand case was examined. For now the results of the experiments can not be explained in a scientific way and therefore our work continues.

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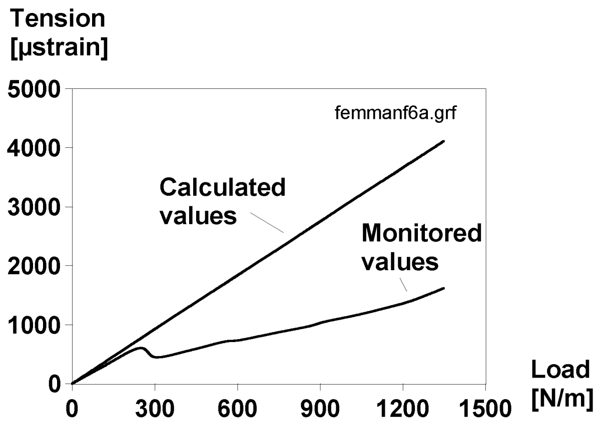

Figure 6: Tension versus load for seat rail. Calculated and monitored values

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