Insulation and bivalent heating system optimization considering residential housing retrofits and time-of-use- tariffs for electricity

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Abstract

Time-Of-Use tariffs, which reflect the cost for producing one extra unit of electricity, will be more common in the future. In Sweden the electricity price will be high during the winter while it will be cheaper during summertime. A bivalent heating system, where an oil-boiler takes care of the peak load and a heat pump the base load, may therefore decrease the cost for space heating in a substantial way. However, insulation retrofits may as well reduce the peak load in the building. This paper shows how a bivalent heating system is to be optimized while also considering insulation measures. The optimization is elaborated by use of a mixed integer programming model. The result is also compared with the derivative optimization in the OPERA model. Both methods use the Life-Cycle Cost as a ranking criterion, i.e. when the lowest LCC for the building is achieved, no better retrofit combination exists for the remaining life of the building.

(Note! The text below is in an early version of the paper)

INTRODUCTION

Since 1985 a research project has run, financed by the Swedish Council for Building research and the municipality of Malmö, Sweden. The aim of the project was to develop a method of building retrofit optimization, i.e. how should an existing building be retrofitted in order to acheive the best possible solution. The project resulted in the OPERA model (OPtimal Energy Retrofit Advisory) which enabled the user to study a unique multi-family building with a number of possible, building, ventilation and heating equipment retrofits. The OPERA model is dealt with in detail in Ref. [1]. Experience from a number of OPERA runnings showed that bivalent systems were very often the best solution combined with some cheap envelope and ventilation retrofits, such as attic floor insulation and weatherstripping. The bivalent heating system, in this case an oil-boiler combined with a ground water coupled heat pump, however, must be optimized simultaneously with the insulation optimization. The optimal sizes of the oil-boiler and the heat pump depend on the level of insulation in the building, or more accurate, on the thermal load in the building. In Sweden, as in many other countries, the electricity tariff depends on the time of the year. During peak conditions, considering the utility, the electricity price is set high, while during off peak conditions, the price is low. OPERA dealt with this situation by calculating a normalized price for one year. Using this normalized price, implies that the utility gets the same income from the consumers for identical thermal loads. In Refs. [1] or [2] the OPERA optimization, which uses a derivative method, can be studied in detail. However, as is shown in [3] this might lead to some misoptimization. In Ref. [3] a bivalent system was optimized but no insulation measures were dealt with. The optimal size of the heat pump differed by approximately 5 %, when OPERA and a linear programming method was compared to each other. Using linear programming, see Ref. [4] for a detailed description of the method, enables the user to find the optimal solution for discrete mathematical problems, while OPERA more easily deals with continuous functions.

THE OPERA OPTIMIZATION

The OPERA model uses derivative methods in order to find the optimal solution, i.e. the lowest LCC for the building during its remaining life. The model is described in detail in Refs. [1], [2] and [4], and thus only a brief review is shown here how a bivalent heating system and building envelope insulation can be optimized, using derivative methods. Adding insulation to e.g. an attic floor will decrease the U - value and thus also decrease the thermal flow through it. The new U-value may be expressed as:

$$U_{new} = \frac{k_{new} \times U_{exi}}{k_{new} + U_{exi} \times t} \quad (1)$$

where:

- U_{new} is the new U - value,
- k_{new} is the conductivity for new insulation,
- U_{exi} is the existing U - value and
- t the thickness of extra insulation.

The cost for the new insulation is expressed as:

$$C_{ins} = A + B \times t \quad (2)$$

where C_{ins} equals the cost for extra insulation in SEK/m^2, A shows the initial cost in SEK/m^2 and B is the direct insulation cost in SEK/m^2,m.

The cost for new heating equipment is expressed in the same way but with P as a varable showing the thermal power of the equipment. The expressions above, however, must be evaluated as present values, i.e. costs for future changes of the equipment are to be transferred to a base year. This is also the situation for the operating cost. Adding more insulation to the attic floor, will decrease the need for heat in the building and subsequently decrease the cost for both heating and heating equipment acquisition. The decrease will emerge in the future and thus present value calculations are necessary. In Ref. [5] the methods for doing this are presented in detail. Adding all these costs provides the operator with an expression showing the LCC for the building and its possible retrofit measures. In Ref. [2] it is shown that the cost may be expressed as:

$$C_1 + \frac{C_2}{C_3 + C_4 \times t} + C_5 \times P_{hp} + \frac{C_6 \times P_{hp}^2}{C_7 + C_8 \times t} + \frac{C_9 \times P_{hp}^2 \times t}{C_7 + C_8 \times t} + C_{10} \times t \quad (3)$$

where P_{hp} is the thermal power for the heat pump and $C_{1,2,...}$ are different constants.

The optimal conditions are acheived when the derrivative with respect to P_{hp} and t equals 0 simultaneously. However, this is not easily solved in a strict mathematical way and thus OPERA is provided with a numerical optimization process which examines the derivatives for different values on P_{hp} and t. When the derivatives are close enough to 0 the process is terminated.

THE LINEAR PROGRAMMING METHOD

The derrivative method works well as long as the LCC is made up of continuous functions. When a time-of-use tariff for electricity is introduced this is not longer the situation. The tariff is designed in discrete steps and thus the derrivative method is no longer suitable. In the OPERA model this is solved by use of a normalized energy price calculated from the actual time-of-use rate. The procedure in linear programming is to start with an objective function which in this case is to be minimized. This function shall express the total LCC for the building. The difference from the derrivative method is that the LCC function does not have to be continuous. The linear program also contains a set of constraints. These constraints may show the range where some of the variables are to be located. The mathematical model must, however, be linear which is a major disadvantage with the method. All nonlinear functions in the program must thus be approximated with linear pieces in order to solve the problem. It is not possible, or worthwile, in a paper of this kind, to show how to solve linear programming problems. The methods are described in detail in e.g. Ref. [6]. Instead it is shown how the mathematical model is designed using a case study from Malmö, Sweden.

CASE STUDY

The building under consideration is located in the block Ansgarius in Malmö, Sweden. The building envelope is in a poor condition and renovation is necessary in one way or another. The building has thus been subject for an extensive analysis using the OPERA model which is used by the municipality. In this paper, however, only a part of the OPERA calculations are shown and they are also simplified in order to enlighten the use of the two different optimization methods shown here. The OPERA model showed that the best retrofit strategy was to change the original oil-boiler heating equipment to a bivalent oil-boiler heat pump system and combine this with attic floor insulation. This solution will thus be shown here in more detail.

Heating equipment costs

Information from contractors in Malmö showed that the oil-boiler cost could be expressed as:

$$Cost_{ob} = 55000 + 60 \times P_{oil} \quad \text{SEK}$$

The economic life of the boiler is set to 15 years. There is also a cost for installation, here assumed to be 200 $\times P_{oil}$ SEK, which has an economic life of 50 years. (1 US \$ = 6 SEK) For a project life of 50 years and a real discount rate of 5 % the LCC for the boiler can be calculated to:

$$LCC_{ob} = 97000 + 305.93 \times P_{oil} \quad \text{SEK} \tag{4}$$

The heat pump has an acquisition and installation cost of:

$$C_{hp} = 60000 + 5000 \times P_{hp} \quad \text{SEK}$$

and the cost for installation et c. is assumed to be 1 500 $\times P_{hp}$ SEK. The first cost is assumed to occur only once during 50 years while the second cost emerges each 10 years. The LCC for the heat pump will thus be:

$$LCC_{hp} = 60000 + 8546.34 \times P_{hp} \quad \text{SEK} \tag{5}$$

Operating costs

The building is in a poor thermal shape. The total transmission loss has been calculated to 4.780 kW/K, including losses from the ventilation system. The peak load in the building is according to the Swedish building code 167 kW.

Climate conditions

In the OPERA model the climate is described as monthly mean temperatures for different sites in Sweden. This is suitable also in the other optimization method and the need for heat is shown in Table 1.

Month	Peak load [kW]	Monthly heat loss [kWh]
January	102.8	76 460
February	103.7	70 326
March	93.7	69 704
April	71.7	51 624
May	47.8	35 563
June	28.7	20650
July	18.2	13 514
August	20.6	15292
September	35.9	25 812
October	57.8	43 031
November	77.0	55 409
December	90.8	67 570

Table 1: Climatic conditions in Malmö, Sweden for the Ansgarius building

Note that the peak load in Table 1 is calculated to 103.7 kW because of the monthly mean values. The ”real” value is still 167 kW.

The electricity tariff

A time-of-use electricity tariff is introduced in Malmö, as follows:

- Fixed fee 5 000 SEK
- Subscription fee 60 SEK/kW
- Power fee 170 SEK/kw
- Energy fee, in SEK/kWh
  - Nov.-March; Mon.-Fri.,06.00-22.00, = 0.392
  - otherwise, = 0.252
  - Apr.,Sept.,Oct.;Mon.-Fri. 06.00-22.00, = 0.252
  - otherwise, = 0.222
  - May-Aug.; Mon. - Fri. 06.00-22.00, = 0.222
  - otherwise, = 0.187

The prices above include taxation of 0.072 SEK/kWh. The prices however, must correspond to the energy need in Table 1 and thus the energy fee is recalculated as:

- Energy fee, Nov. - March, = 0.314 SEK/kWh
- April, Sept. and Oct., = 0.236
- May - Aug., = 0.204

For the OPERA calculation these values must be normalized to a price valid for all the year. The price must give the same income to the utility and thus the total energy cost, for one year, is calculated using the values in Table 1. Dividing this cost with the total amount of energy consumed during one year yields the normalized price 0.28 SEK/kWh.

Optimization with the OPERA model

In Refs. [1] and [2] it is shown how OPERA evaluates the energy cost for the bivalent system and at the same time considers the influence of attic floor insulation. The method is not repeated here but the LCC functions has been evaluated as:

$$LCC_{ob} = 143383 + \frac{196.33}{0.04 + 0.8 \times t} - 305.93 \times P_{hp} \tag{6}$$

$$LCC_{hp} = 60000 + 8546.34 P_{hp} \tag{7}$$

$$LCC_{fee} = 1399.9 P_{hp} \tag{8}$$

$$LCC_{fix} = 91300 SEK \tag{9}$$

$$LCC_{ehp} = 16265 \times P_{hp} - 13.55 \times \frac{P_{hp}^2 + 271.19 P_{hp^2} \times t}{0.192 + 3.457 \times t} \tag{10}$$

$$LCC_{eob} = 2653797 + \frac{11262}{0.04 + 0.8 \times t} - 51194 \times P_{hp} +$$

$$+\frac{42.66 \times P_{hp}^2 + 853.6 P_{hp}^2 \times t}{0.1912 + 3.457 \times t} \tag{11}$$

$$LCC_{ins} = 71625 + 171900 \times t \tag{12}$$

The sum of the functions (6) to (12) is now calculated and the derivatives with respect to P_{hp} and t are calculated and set to 0. The solution from the minimization is:

- Thermal power of the heat pump 77 kW
- Thermal power of the oil-boiler 78 kW
- Heat from the oil-boiler 38 000 kWh
- Heat from the heat pump 468 900 kWh
- Oil-boiler cost, present value 120 900 SEK
- Heat pump cost, present value 718 100 SEK
- Power fee cost, present value 107 800 SEK
- Fixed fee cost, present value 91 300 SEK
- Energy cost, heat pump, present value 797 900 SEK
- Energy cost, oil-boiler, present value 203 800 SEK
- Insulation cost, 0.18 meter mineral wool 102 600 SEK

Adding the costs above together results in a total LCC of 2 142 400 SEK.

Mixed integer programming optimization

When the linear programming method is used it is necessary to describe the energy cost, as well as the other costs, in the objective function. This is done by calculating the energy cost month by month and adding the costs together. In Fig 1 the monthly thermal losses are shown if no attic floor insulation at all is implemented in the building.

It is assumed that the thermal power of the heat pump equals Php and the power of the oil-boiler Poil. The total energy cost for January will thus be:

$$EC_{Jan} = \frac{P_{hp} \times T_{Jan} \times El_{Jan}}{Eff_{hp}} + \frac{P_{ob} \times T_{Jan} \times Oil_{Jan}}{Eff_{oil}} \tag{13}$$

where:

- EC_{jan} = The energy cost in January,
- T_{Jan} = The number of hours in January,
- El_{Jan} = The electricity price in January,
- Oil_{Jan} = The oil price in January,
- Eff_{hp} = The coefficient of performance for the heat pump and

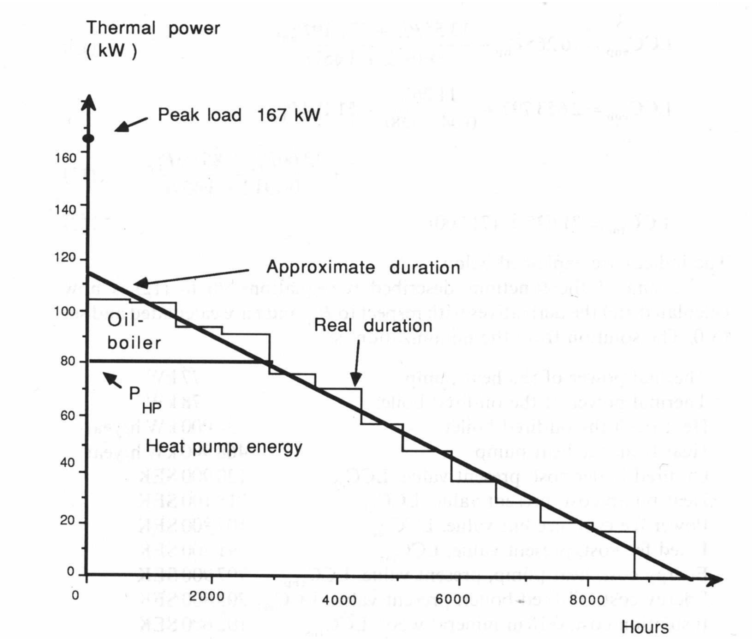

Figure 1: Duration of imposed thermal load according to climatic conditions in Malmö, Sweden

- Eff_{oil} = The efficiency for the oil-boiler.

The annual energy cost is calculated by adding the energy costs for each month together, i.e.:

$$EC_{tot} = EC_{Jan} + EC_{Feb} + EC_{Mar} + \ldots + EC_{Dec} \tag{14}$$

The energy LCC is now easy to calculate, the total energy cost is to be multiplied by the present value factor for annual recurring costs.

Electricity power fee cost

The power fee and subscription fee for electricity are to be paid annually. In this case the total fee, F_{el}, is:

$$F_{el} = \frac{(60 + 170) \times P_{hp}}{Eff_{hp}} \tag{15}$$

Also this cost has to be multiplied with the applicable present value factor in order to achieve the total fee LCC.

Insulation cost

The cost for insulation is presented to the models according to expression 2. In this case study the area of the attic floor is 573 m^2, the initial cost for insulation is 125 SEK/m^2 and the direct insulation cost is 300 SEK/m^2,m. The insulation

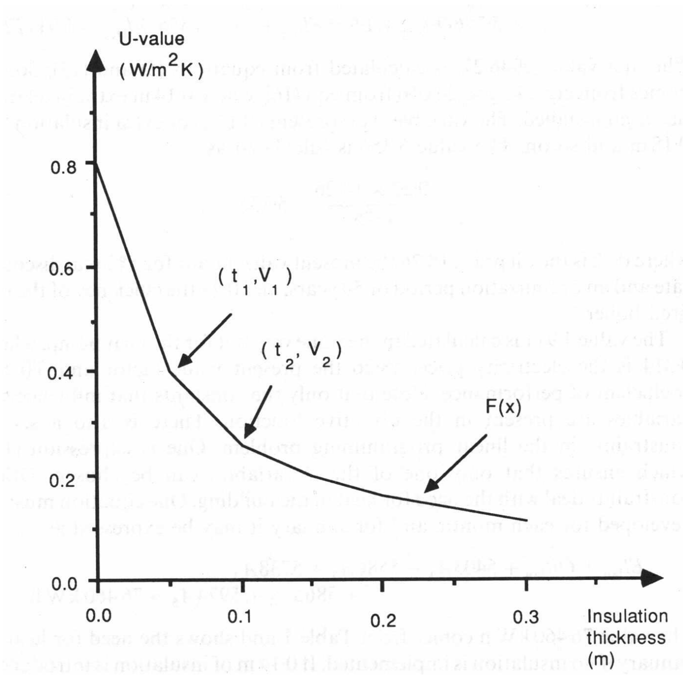

Figure 2: U-value as a function of insulation thickness

cost only occurs once and thus no present value is required. The cost will emerge as:

$$C_{ins} = 71625 + 171900 \times t \quad \text{SEK} \tag{16}$$

The expression (16) may be included directly in the LCC function, which is to be optimized, as long as OPERA is considered. In the linear program however, the expression (16) is substituted with a discrete function which shows the insulation cost for discrete amounts of insulation. The reason for this is the fact that the decrease in energy cost is not a linear function of the insulation thickness.

Piecewise linearization

In Fig. 2 the U-value is shown as a function of the attic floor insulation thickness.

It is obvious that the U-value and subsequently the energy cost and later the total LCC, is not a linear function of t. In order to implement the function in a linear program it is necessary to make it linear. In this case a method from [6] is used where the nonlinear expression is substituted with a linear one of the following type:

$$A_1 \times V_1 + A_2 \times V_2 + A_3 \times V_3 + \ldots = F(x) \tag{17}$$

where $A_{1,2,3...}$ are binary integers, 0 or 1 and

$$A_1 + A_2 + \ldots = 1 \tag{18}$$

One of the values $V_{1,2,3...}$ thus has to be selected by the model. The linear expression is now a function of $A_{1,2,3...}$ and not a function of t as the nonlinear function was before. This also means that also the linear insulation cost function should be transferred to a function of $A_{1,2,3...}$ instead of t. The beginning and end of the objective function in this case will thus be:

$$9946.27 * Php + 305.93 \times P_{ob} + 24066 \times A_1 + 27504 \times A_2 + 30942 \times A_3 + \ldots +$$

$$+5.356 \times Oil_{Jan} + 1.911 \times El_{Jan} + \ldots + 5.356 \times Oil_{Dec} + 1.911 \times El_{Dec}$$

The first value, 9 946.27, is calculated from expression (5) and (15), 305.93 comes from formula (4) and 24 066 from expression (16) where 0.14 meter of extra insulation has been implemented. The variable A_2 represents 0.16 meter of extra insulation, A_3 0.18 meter and so on. The value 5.356 is calculated as:

$$\frac{0.22 \times 18.26}{0.75} = 5.356$$

where 0.22 equals the oil price, 18.26 equals the present value factor for 5 % real discount rate and an optimization period of 50 years, and 0.75 the efficiency of the oil-boiler. The value 1.9112 is calculated in the same way but for the heat pump where 0.314 is the electricity price, 18.26 the present value factor and 3.0 the coefficient of performance. Note that only the constants that influence the variables are present in the objective function. There are also a set of constraints in the linear programming problem. One is expression (17) above which ensures that only one of the A - variables can be chosen. There are also a number of constraints dealing with the need for heat in the building. One equation must be elaborated for each month an for January it may be expressed as:

$$El_{Jan} + Oil_{Jan} + 5403 \times A_1 + 5586 \times A_2 + 5738 \times A_3 +$$

$$+5865 \times A_4 + 5974 \times A_5 > 76460 \quad \text{kWh}$$

The value 76 460 comes from Table 1 and shows the need for heat in January if no insulation at all is implemented. If 0.14 meter of insulation is implemented this value is decreased with 5 403 kWh, i.e. A_1 equals 1. The value is calculated by use of expression (1), where k_{new} equals 0.04 W/m,K, U_{exi} = 0.8 W/m^2,K and t = 0.14 m. U_{new} will thus equal 0.2105 W/m^2,K. The original U - value = 0.8 W/m^2,K and the decrease in U - value is 0.5895 W/m^2,K. The area of the attic floor is 573 m^2 and the number of degree hours in January equals 76 460/4.78 = 15 996. The decrease is thus:

$$0.5895 \times 15996 \times 573 = 5403 \quad \text{kWh.}$$

There are also one set of constraints that ensures the model to choose a heat pump and an oil-boiler with sizes large enough to deliver the applicable amount of heat in the building. Two expressions for each month must thus be elaborated:

$$\frac{P_{hp} - El_{Jan}}{T_{Jan}} > 0$$

$$\frac{P_{ob} - Oil_{Jan}}{T_{Jan}} > 0 \quad \text{et c.}$$

The heating equipment must also be able to provide the thermal peak load of the building. This will yield the last constraint:

$$P_{hp} + P_{ob} + 11.82 \times A_1 + 12.22 \times A_2 + 12.56 \times A_3 +$$
$$+12.84 \times A_4 + 13.07 \times A_5 > 167 \quad \text{kW}$$

The mathematical model shown above contains 85 unique elements and has 175 non zero elements. The LAMPS program has been used to find the solution to the model which is done after 28 iterations. The solution is characterized by:

- The heat pump power equals 84.0 kW
- The oil boiler power equals 70 kW
- 0.18 meter extra insulation is to be implemented
- Heat from the oil-boiler equals 18 500 kwh
- Heat from the heat pump equals 485 500 kWh
- Oil-boiler cost, present value, equals 118 700 SEK
- Heat pump cost, present value, equals 777 900 SEK
- Electricity cost, present value, equals 822 900 SEK
- Oil cost, present value, equals 98 000 SEK
- Power fee, present value, equals 117 600 SEK
- Fixed fee, present value, equals 91 300 SEK
- Insulation cost 102 600 SEK

The total LCC with linear programming optimization will thus be 2 129 000 SEK.

Comparing the OPERA model with mixed integer programming optimization

From the above discussion it is obvious that it is possible to use both methods in order to optimize a bivalent heating system and at the same time considering insulation retrofits. The mixed programming method will solve the problem with a high accuracy but no severe misoptimization is present if the derivative method is used instead. The insulation thickness will be exactely the same with the two methods while the heat pump size will be slightly smaller, approximately 8 %, if a derivative method is used. The total LCC is a little higher if the OPERA model is used, about 0.6 %, and this can almost always be neglected. The optimal oil-boiler size is somewhat larger when the OPERA model is used which also implies that the oil-boiler energy cost will be higher than the true optimal solution. This may to a part be the result of the approximation of the climate condition due to the method of least squares. The ”real” need for heat in the building, without any insulation measures, is calculated to 545 000 kWh while the OPERA model assumes 548 200 kWh. The maximum load, when the energy need is calculated is 115 kW in the derivative method while 103 kW is the "real" value. This results of course in a larger oil-boiler when the OPERA model is used. The OPERA model however, has some major advantages. When the problem is more complex than the one studied above, the number of variables is increased very much. The OPERA model deals with 10 different heating systems and 10 different building and ventilation retrofits. A linear program which solves such a big problem will be very tedious to design and it might not be possible to solve at all with small computers like IBM AT and others. When the mixed integer problem above was designed, the base for it had been elaborated by an OPERA running. It was thus possible to emphasize the work on a much smaller problem than was originally the case. One more drawback with the mixed programming method is that one has to start with a very strict mathematical problem which has to be implemented in a commercial computer program when the problem is to be solved. Further it is not very easy to design the problem and afterwords to interpret the solution into a language understood by a not mathematically skilled building designer. The conclusion from this paper is thus that the OPERA model works well also for bivalent heating system optimization when time-of-use tariffs for electricity are implemented. If very accurate work is necessary the interesting solution must be scrutinized with a mixed integer programming method.

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