MATHEMATICAL MODELING OF DISTRICT HEATING AND ELECTRICITY LOADS

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Abstract

In recent years it has been more common to use linear or mixed integer programming methods for finding optimal solutions to the complicated operating options in modern Combined Heat and Power, CHP, networks. Electricity may be bought from the national grid or it may be produced in ordinary condenser or CHP plants owned by the utility. In the same manner district heat can be produced by use of waste heat from industries or from the CHP plant. Other options are burning garbage in an incineration plant, using heat pumps in a sewage water plant or just burning fuels in an ordinary boiler. Combining these options and including the possibility to use conservation measures in the industry or in the housing stock will result in a very complex situation if one tries to find the optimal solution characterized by the lowest Life-Cycle Cost, LCC. Load management equipment, such as hot water accumulators, will aggravate the problem even further. By the use of modern computers, very large problems can be solved within a reasonable measure of time. The bases for the mathematical models are the thermal and electrical loads. Splitting these loads in finer and finer segments will yield a model that more closely will depict the reality. Two methods have frequently been used, one where the high and low price hours in each month have been lumped together, resulting in 24 segments except for one segment showing the influence of the maximum electricity demand. The other method tries to model the loads by lumping the energy demand in six electricity tariff segments but also using about fifteen elements for a more versatile picture of the district heating load. This paper describes the two methods using monitored data for 1990 from Kalmar in the south of Sweden. It is also discussed if one of the methods is preferable or if a combination must be elaborated in order to model the reality close enough for practical use.

INTRODUCTION

Mathematical modeling by use of linear or mixed integer programming methods has found an increased interest in recent years. This is mostly the result of the common use of computers and you can nowadays find a machine with high calculation capacity on almost every desk . Previously, solving optimization problems with the linear programming technique has been very tedious, and neither the speed nor the memory capacity have been sufficient in small desktop machines. These problems, at least to a part, have been solved by the introduction of 386 and 486 processors running in protected mode. This paper, however, does not deal with how to find the optimal solution for a linear program and therefore the interested reader is referred to Refs. [1] or [2] for more details. One of the major drawbacks with this technique is that the entire mathematical problem must be linear. Only expressions of the type:

$$C_1 \times A + C_2 \times B + C_3 \times D = C_4$$

where $C_{1...}$ = different constants and A, B, D = different variables can be dealt with. Note that the = sign could be $\leq$ or $\geq$ instead. On the other hand, several thousands of equations could be included in the problem where some of the variables only may take the values 0 or 1. This means that nonlinear problems many times can be transformed to linear ones and solved by use of the traditional Simplex and Branch-and-Bound methods see Ref. [3]. In linear programs there is an objective function that is to be minimized or maximized. In the case of energy system optimization, this function mostly includes the total LCC of the system and the problem will be to find the minimum LCC. The thermal and electrical loads will be part of the objective function because the total cost is a result of, among other things, using different fuels etc. in the boilers. One detailed example of how to design the objective function, and the constraints, in a mixed integer program can be found in Ref. [4].

THE DISTRICT HEATING LOAD

In order to enlighten the situation, we will use an example from Kalmar in the south of Sweden. The district heating load have been monitored once an hour for almost a full year, 1990-03-19, 14.00 to 1991-03-01, 07.00. In Figure 1, a duration graph , i.e. the values of the load have been sorted in falling magnitude, is shown.

The maximum load in the district heating net is 78.2 MW but this load did only emerge during one hour. The peak is very steep and has a very short duration, see Figure 1. The amount of energy that must be produced above 60 MW is minute, and could therefore not influence the cost very much. One important fact, however, is that the utility must be able to produce 78.2 MW or there will be a shortage of district heat. This means that it is essential to install boilers et c. that are able to produce the total demand and while there might be a substantial cost for this equipment, it must be included in the objective function.

Normally it is estimated that the peak load and its adjacent values emerge in the same time interval, e. g. between 06.00 - 22.00 in December. A closer look at the peak is presented in Table 1.

The highest value emerged 90-10-30 at 07.00 and the next highest 91-01-31 at 08.00. Unfortunately, the first value seems to be an error. The two adjacent values in the original data file are much lower so the peak load must probably be set to 76.89 instead of 78.2 MW. Fortunately, the error for this hour does not influence the model load in a significant way.

Most important to note is that the peak contains values from very different days of the year. The second highest value in Table 1 occurred at 8 A.M.

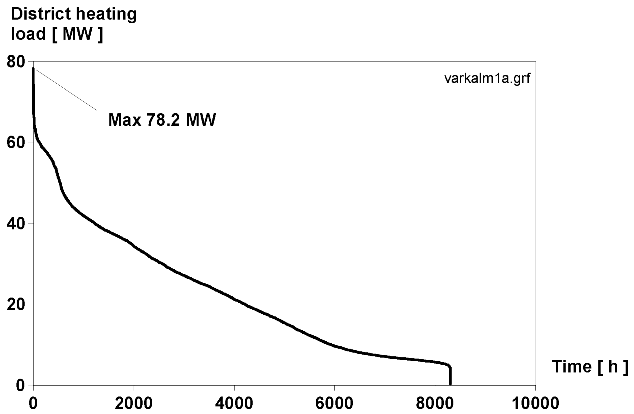

Figure 1: Duration graph of district heating load in Kalmar, monitored values

Date	Hour	Load [MW]	Date	Hour	Load[MW]
901030	7	78.2	910122	7	66.54
910131	8	76.89	910118	7	66.18
910228	16	71.84	910122	8	66
910118	8	70.46	910127	7	65.42
910126	8	70.17	910130	15	65.35
910201	9	67.59	910201	8	65.25
910129	10	67.06	910130	14	65.08
910228	10	66.8	910124	6	64.42
910213	16	66.77	910218	7	64.3
910228	8	66.76	910121	9	64.27

Table 1: Peak load details for district heating grid, Kalmar Sweden

January 3, while the third value emerged almost one month later. However, all values in Table 1 except for the first one which is probably wrong, are found in January or February. A closer look at the data set for the district heating load reveals that not all hours during the full year were included in the file. Of the 8760 hours that should be present, only 8312 exist. Most of the lacking hours occur between March 01 at 01.00 to March 19 at 14.00, or 439 hours, but still about 10 hours are missing. It is important that the model will be consistent and subsequently contain 8760 hours so in order to achieve this, about nine days before March 01 and nine days after March 19 are copied and added instead of the missing interval. The peak load, as found in Table 1 above, will not be affected by these added hours, and thus those values could be used to examine how they will fit into the two types of models frequently used. In the first type of model it is assumed that the climate is of major importance for the district heating load. One example of such a model, used for a study of Malmö, Sweden, is shown in Figure 2, see Ref. [5].

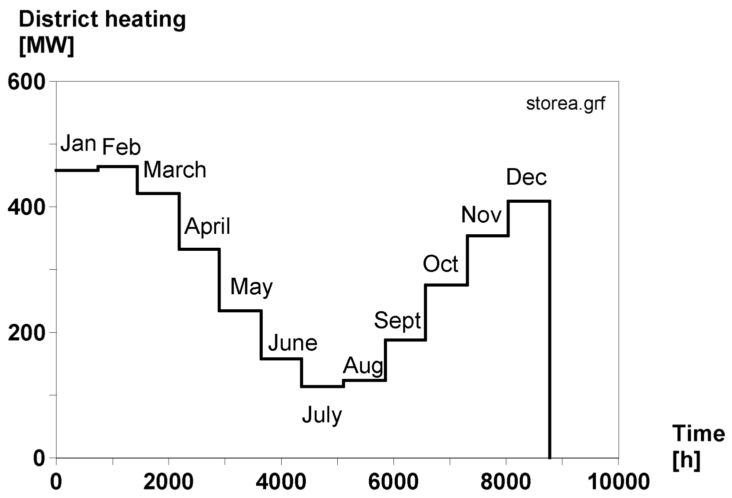

Figure 2: District heating load for 12 months, Malmö Sweden, see Ref. [5]

The district heating load has in that case been calculated assuming that the load origins from a gigantic building. The transmission factor was calculated to 14.39 MW/K, the thermal losses from ventilating the "building" was assumed to be 5.07 MW/K while the heating of domestic hot water was assumed to be 350 GWh for a full year. Using climatic data for Malmö made it possible to calculate the load found in Figure 2.

One very big advantage of the above procedure is that if the "building" is to be affected by retrofit measures, it is possible to calculate exactly how much the load would be decreased by such a measure. If Table 1 and Figure 2 are compared, it is obvious that the finer details of a real load will not emerge in Figure 2. The load for one hour in December might be much higher than one in February even if the model says that this is not the case. On the other hand, the real load for one year will probably not be representative for all future years that are under consideration. This fact speaks for the model in Figure 2 where meteorological mean values for the outside temperatures were used. In Reference [6] the real district and electricity demands of Kalmar have been studied and some statistical investigations have been elaborated. The result shows that there is a fairly good correlation between the outdoor temperature and the district heating load. The correlation coefficient was calculated to 0.895 for the total data set and the correlation was not improved if only certain selected values were used instead of the total set. The "best fit equation" for the district heating demand in [MW] was calculated to:

$$DH_{load} = 5.6 + 1.525 \times t \tag{1}$$

where t equaled the indoor temperature, 20 °C, minus the outdoor temperature. In Figure 3 the calculated values from expression (1) are shown as dots while the monitored values emerge as a line.

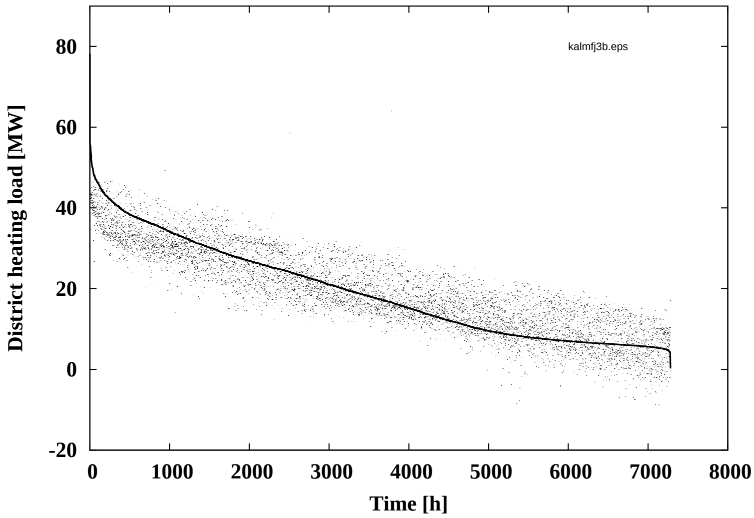

Figure 3: Monitored and calculated values for the district heating load in Kalmar

If Figures 1 and 3 are compared it is apparent that some values are missing, only about 7 500 hours are present in Figure 3, while about 8 200 are dealt with in Figure 1. This is so because the monitored temperatures included some 700 values which could not be used due to registration errors. It must also be noted that the sorting in descending order has been fulfilled so that each monitored value corresponds to the same calculated value for that specific hour. The calculated values will therefore not be sorted in descending order but instead emerge as scattered dots. In Figure 4 also these calculated values are sorted, but two points above each other at the separate curves will now not correspond to the same hour of the year.

However, as Figure 4 shows, the correlation between the calculated values and the monitored values has improved substantially by this procedure.

The calculated values underestimates the demand for very cold days but will yield higher values during part of the summer. For very warm days, i.e. no real climate load at all, the demand is calculated to negative values, which is of course not logical. These could however be excluded when the final model is designed. The total amount of energy under the two curves are almost exactly the same and thus it seems that the method shown will result in an acceptable model of the district heating demand.

THE ELECTRICITY LOAD

The electricity demand does not show the same high correlation with the climate as does the district heating. In Reference [6] the correlation coefficient has been calculated to 0.326 for the total data set. If only the minimum demand each night was used it reduced the influence of industrial and other activities during daytime, and this increased the correlation coefficient to 0.748.

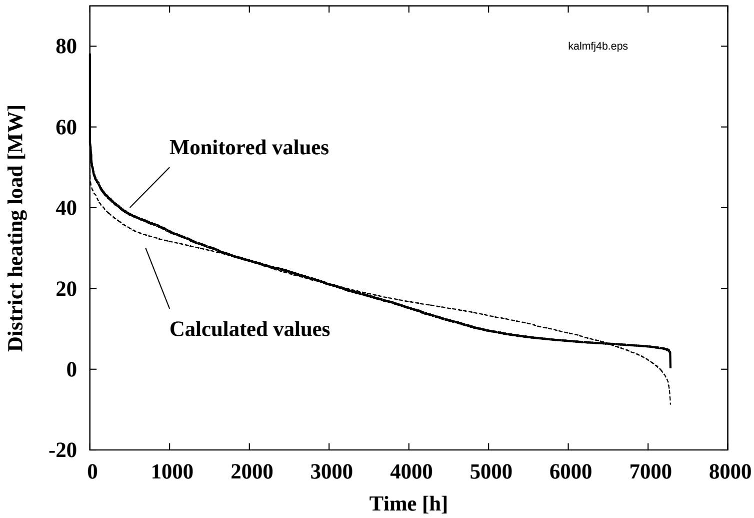

Figure 4: Monitored and calculated values for district heating load in Kalmar

Excluding the summer months, however, did not improve this value. In Figure 5 the electricity demand is shown for the total data set in the form of a duration graph.

The peak load is about 75 MW while the lowest load is about 22 MW. In reference [6] the climatic load was calculated as:

$$E_{load} = 20.58 + 0.809 \times t \tag{2}$$

where t equaled the indoor temperature 20 °C minus the outdoor temperature. If expression (2) is used for calculating the electricity space heating load the result would become similar to that shown in Figure 3, scattered dots in the graph but this time below the curve in Figure 5. The problem will then be the same i.e., should the monitored values be sorted according to their strict order of magnitude or is it better to sort them into separate time segments no matter their level?

THE COST FOR PRODUCTION OR PURCHASE OF ENERGY

The utility can produce electricity and heat in its own facilities or it can buy electricity from the national grid. The tariff that is used is split up in time segments. During working days the high price hours range from 06.00 to 22.00 while a cheaper price is used during rest of the day and on Saturdays and Sundays. The price will often differ depending on the month of the year. During summer the price is lower than it is during winter. The models we build are used for minimization of the life-cycle cost and therefore it is important to use a strong connection between the actual use of energy and the applicable price in the tariff. In Table 2 the monitored demand has been split up according to the month and time of day.

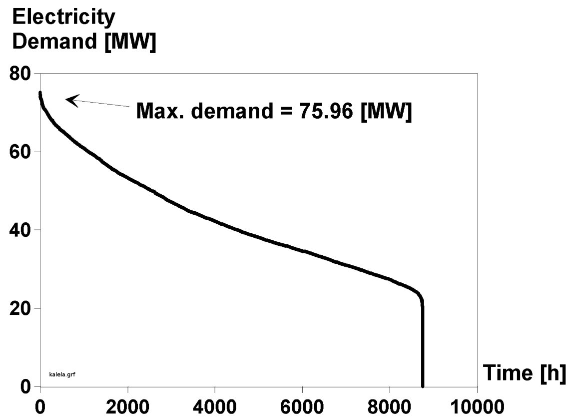

Figure 5: The electricity demand in Kalmar, Sweden, during 1990

In January, for example, there are 368 high price and 376 low price hours. The use of electricity during high price conditions are 22.1 GWh which implies an average demand of 60 MW. For the low price segment the corresponding values are 17.8 GWh and 47.3 MW. The district heating use is 16.5 GWh, and 44.9 MW, for the high cost segment and 14.9 GWh, i.e. 39.5 MW, during the low cost segment.

The electricity demand in Table 2 must now be sorted in descending order so it could be compared to the situation found in Figure 5. This has been done in Figure 6.

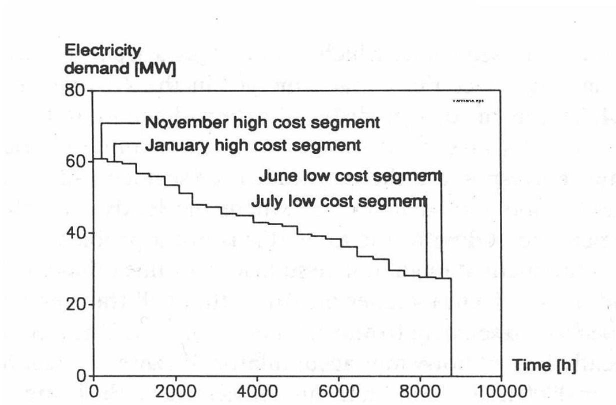

Figure 6: Duration graph of the 24 segment electricity demand, Kalmar, Sweden

Unfortunately, it has not been possible to show both the curve in Figure 5 and the graph in Figure 6 in the same diagram. The calculation process used

Month	Hours		Electricity				District heating			
	High	Low	High		Low		High		Low	
	Energy	Load	Energy	Load	Energy	Load	Energy	Load		
Jan	368	376	22.1	60.0	17.8	47.3	16.5	44.9	14.9	39.5
Feb	320	352	18.1	56.7	14.0	39.7	17.0	53.2	16.3	46.4
Mar	352	392	18.8	53.4	14.2	36.2	12.1	34.3	11.4	29.1
Apr	336	384	16.1	47.9	12.6	32.8	7.9	23.5	8.1	21.2
May	368	376	15.8	43.0	10.6	28.1	5.0	13.5	5.0	13.3
Jun	336	384	14.3	42.6	10.5	27.4	2.7	8.0	2.9	7.6
Jul	352	392	13.8	39.2	10.9	27.7	2.7	7.6	2.6	6.7
Aug	368	376	16.7	45.5	11.1	29.6	2.5	6.9	2.4	6.5
Sep	320	400	16.4	51.2	13.5	33.6	5.4	17.0	5.4	13.6
Okt	368	376	20.6	55.9	14.4	38.4	9.2	25.0	7.8	20.7
Nov	352	368	21.4	60.8	15.6	42.0	12.3	35.0	10.4	28.3
Dec	336	408	20.0	59.5	18.3	45.0	13.1	39.0	13.4	32.8
Sum			214.1		163.5		106.4		100.6	

Table 2: Electricity and district heating demand in GWh and MW during 1990 in Kalmar, Sweden

for elaborating Figure 6 ascertains that the same amount of electric energy is shown in both figures. It is also obvious that Figure 6 does not show the peak load which is apparent in Figure 5. Because of the very small area , i.e. energy, in the peak the difference in electricity cost will not differ very much between the two graphs shown. There is, however, a possibility that the size of the equipment will be misinterpreted if only Figure 6 is used. In Reference [4] this has been dealt with by use of a 25th segment showing the maximum demand for the months November to March, which are the months of concern due to the demand part of the tariff.

THE LOAD MANAGEMENT PROBLEM

Load management equipment in residential buildings and industry is nowadays available in order to cut peak loads. Examples could be to add hot water accumulators in buildings or disconnecting kilns for short periods of time in an industry. In Reference [7] this is dealt with by use of maximum load days or lumps of days. It should then be possible to ascertain that energy in a cut of a peak in a high cost segment will emerge in the low cost segment for the same day or at least later in the high cost segment. Figure 5 shows that some 200 hours have higher demands than the approximately 60 MW that is found as the peak in Figure 6. It will not be possible in practical use to add all these hours to a model as separate segments and therefore they are lumped in smaller segments as shown in Reference [7]. The method used for elaborating Figure 6 is not sufficient for this fine tuning of the model and thus it would be important to split the left hand side of the load into smaller segments. Likewise important is to ascertain that the electricity and the district heating loads coincide to each other. The selected maximum electricity load segments must therefore correspond to the very same time segments for the district heating load. In Reference [7] or in Reference [8] this is not fulfilled but instead the maximum electricity load segments are assumed to correspond to maximum district heating load segments, which is not necessarily true. See Table 2 where the maximum electricity load emerges in the November high cost segment while the maximum district heating demand is found in the February high cost segment. At the same time, maximum district heating segments must correspond to the very same time segments in the electricity load. In the method shown in Figure 2 where the loads are split up in segments, strictly due to the time this is not a problem but as shown above this method might not be sufficient for fine tuning of the model. Electricity load management includes measures that will transfer some of the energy in a peak load segment to another period of time. One example of this is an electrically heated hot water accumulator. If there is a risk for a peak hour, say from 08.00 A.M. to 09.00, in the use of electricity, the heater should be disconnected a certain period of time, say one hour. The energy that was going to be used must now instead be transferred by the model to the hour between 09.00 to 10.00. The possibility to produce district heat, using CHP, is at the same time reduced during the first hour and increased under the second hour. Everyone understands that the process for evaluating all the possibilities is a major task and we do not think that it will be possible to design a model that is able to accomplish the accurate answer for all different kinds of load management measures. One example can be shown from the figures in Table 1. If we could use a load management system to decrease the load by say 5 MW at 0800 the 31th of January 1991 this will be a fine measure. (The second value is used here because the first one was probably wrong.) These 5 MW could be transferred to any one of the hours coming later that day because the same date does not emerge again in the table. If, however, we want to transfer 5 MW from 08.00 the 28th of February to emerge at 10.00 the same day this will not be a good idea because in that case this hour might result in a new peak.

CONCLUSIONS

We have shown that neither of the two common methods for modeling electricity and district heating loads will be sufficient for proper use in linear programming methods. The first type of model splits the loads due to the different tariff elements. In Sweden this means that the loads are split in high and low cost segments for every month during one year which will lead to 24 time segments. There is also one segment showing the maximum demand for electricity due to the a certain cost for this demand. This first method, however, will calculate the average mean in all segments and thus the finer details in the load will be overwhelmed. One major advantage with the method is that the model is consistent, i.e. each time segment will correspond exactly for the two loads. Load management equipment could not be modeled properly as long as part of the energy in the high cost segment is not transferred completely to the low cost ditto, and vice versa. It will therefore not be possible to model a transfer of energy from one hour to a subsequent one without special measures. The other model splits the loads in a more versatile manner where peak hours and peak days occur. These models are, however, not consistent because it is not ascertained that the time segments for the electricity and the district heating loads are the very same. Instead, it is assumed that the district heating and the electricity loads have their maximums at the same time. We have shown here that this is not always true. Neither in these type of model is it possible to add load management measures because the model does not keep track of subsequent hours in perfect order. The model of the loads will, however, show a better correlation to the real ones. It seems that the first method is the one to prefer because of the consistency in time segments but more time elements should be added to model the peak in a more accurate manner.

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