LINEAR PROGRAMMING OPTIMIZATION IN CHP NETWORKS

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Abstract

This paper shows how to simulate a CHP network, CHP = Combined Heat and Power, using the method of linear programming. This method makes it possible to optimize the mathematical model and subsequently find the very best combination of electricity production, electricity purchase and heat production in a district heating system. The optimal solution is characterized by the lowest possible Life-Cycle Cost, LCC, i.e. the sum of operating, maintenance and building costs during the project life of the plant. The paper shows the design of the mathematical model and further a case study is presented using the district heating net in Malmö, Sweden, as an example.

INTRODUCTION

When electricity is produced in an ordinary condenser plant, the steam is used in the turbine, which in turn runs the electricity generator. The turbine rotates because of the difference in steam pressure before and after the turbine. Subsequently it is important to make this difference in pressure as high as possible and cold sea water is commonly used for this cooling purpose. Unfortunately, most of the heat, originally embedded in the fuels, is then wasted as luke warm water into the sea. Using a district heating grid as a cooling device makes it possible to utilize this heat in the waste water. However, the temperature of the cooling water must be increased for use in a district heating grid. An average temperature in the cooling water of about 10 degrees Centigrade in the sea must be increased to about 90 degrees in the district heating net during the winter season. The amount of electricity produced in a plant will then be reduced, but the total fuel efficiency will be increased in a significant way. Under winter conditions in Sweden it is profitable to produce electricity in district heating grids because there is a need both for electricity and heat. During the summer this need is reduced and it is cheaper to buy electricity from the national electricity grid because the production is based on hydro and nuclear power. The district heating plant will, during the summer, use almost all of the energy in the fuels for heating the water to about 70 degrees C and no electricity will be produced. If perfect pricing, i.e. short range marginal cost, of the electricity and heat is utilized, the heat from the plant would be cheaper during winter conditions because the marginal price for the heat is then very low. The subscriber should only pay the cost for the utility to raise the outlet temperature of the water from about 10 to 120 degrees, which is the necessary hot water temperature in the grid. During the summer, however, the marginal cost for the utility will be the cost for fuels in the boilers. Today the opposite is valid, at least in Swedish municipalities, which depends on the use of alternative cost pricing. This method implies, however, that the production units are used less then optimal, at least as long as the marginal cost is lower than the alternative. In Refs. [1], [2] and [3], CHP and pricing theories are shown in further detail.

LINEAR PROGRAMMING

In a linear program there is a mathematical function to be maximized or minimized. This function is called the objective function. Often, there is a set of constraints that influence the mathematical model and all these constraints must be satisfied at the same time. Linear programs is often solved by use of the Simplex method which is implemented in most commercial programs for optimization. Closer details about linear programming can be found in e.g. Ref. [4].

THE MATHEMATICAL MODEL

When a linear program is created it is feasible to start with the objective function. In this function the cost for producing the electricity and heat must be shown. To start with the cost for electricity, the tariff used by the Sydkraft company, from which Malmö is buying electricity not produced in the CHP plant, is split in several time segments. During weekdays between November and March from 0600 to 2200 the price has its highest level, while the price is the lowest during off-peak periods in the summer. It is thus necessary to implement the same time segments in the objective function. There is also a tariff element showing the cost for the maximum one hour demand during the year. This value is monitored under high price conditions shown above. Further, the electricity and heat load varies due to the climate and here monthly mean temperatures have been used for showing this influence. In this case study 25 time elements has been used, 12 for high price periods under each month over a year, 12 for low price conditions and 1 element for the monitored maximum demand price. The aim of the study is to find out how much heat and electricity that are to be produced in the CHP plant, and how much to by from the Sydkraft utility, if the total cost for the municipality is to be minimized. Suitable variables are subsequentely the electricity power provided by the CHP plant during high and low price conditions over a year. Another variable is the maximum demand. In order to solve the problem, it has also been necessary to implement variables for the thermal load satisfied by waste heat from the CHP plant, and from ordinary boilers in the district heating facility. All these variables and several others, must be connected to the cost for using them. The constraints in the model provides that the variables are lower, higher or equal certain values. For example, it is necessary that the sum of purchased electricity and electricity produced in the CHP plant, is higher or equal to the need, and this in each time element. The total amount of district heat must also exceed or equal the nead for heat and so on. Minimization of the objective function, which shows the total cost, will provide that not more than necessary of the need is produced.

CASE STUDY

The case study in this paper has earlier been described in Ref. [1], and all details will not be presented once again. Subsequently, only the necessary parts for understanding how the model is designed are shown here. The district heating load has been designed by use of a gigantic building with a total transmission coefficient of 14.39 MW/K. By use of monthly mean outside temperatures for Malmö and by adding a domestic hot water demand of 350 GWh/year, the thermal need has been calculated to the values shown in Table 1. See also Ref. [1].

Month	Heat (GWh)	Month	Heat (GWh)	Month	Heat (GWh)
January	326.0	May	159.5	September	120.2
February	309.5	June	99.2	October	189.9
March	298.5	July	69.7	November	240.7
April	225.3	August	77.0	December	289.8

Table 1: Energy demand in the district heating grid

The electricity load is split into 24 segments showing the high and low price consumption. The values are monitored during 1988 and are presented in Table 2. The maximum demand for the months of interest have been monitored to 443.1, 419.2, 407.5, 433.5 and 455.3 for January to March and November to December, respectively. The electricity can to a part be produced in the existing CHP plant which is of the size 120 MW_{el}.

Month	High (GWh)	Low (GWh)	Month	High (GWh)	Low (GWh)
January	117.9	103.5	July	68.1	56.7
February	122.1	94.9	August	96.7	70.9
March	131.0	98.5	September	107.2	81.0
April	105.7	94.1	October	111.5	99.5
May	87.9	69.6	November	129.9	98.4
June	88.6	65.1	December	135.6	111.2

Table 2: Electricity demand in Malmö, 1988, see Ref. [1]

The CHP plant can be used down to about 40 per cent of its maximum capacity and if the load is lower the equipment must be turned off. The fuel for electricity production is natural gas imported from Denmark and the cost is 85 SEK/MWh fuel. 1 US \$ = 6 SEK. If the efficiency of the plant, which is 0.85, is included the price will become 100 SEK/MWh produced energy. Further, the heat from the plant is taxed by the government at a rate of 29 SEK/MWh. Note that electricity production is also taxed, but the tax is paid by the end user. The electricity demand that cannot be provided by the CHP plant must be purchased from the Sydkraft company. The tariff is shown in Table 3.

	Electricity price [SEK/kWh]	
Month	Peak	Off-peak
January - March	0.240	0.165
April, September - October	0.165	0.130
May - August	0.112	0.092

Table 3: Electricity prices from Sydkraft, see Ref. [1]

There is also a power fee of 175 SEK/kW for the maximum demand during high price periods for the months November - March. The heat demand that cannot be covered by use of hot water from the CHP plant must be provided by burning fuels in the district heating system. There are several different sources for this heat. The lowest price has the 65 MW incineration plant where garbage is burnt providing heat at a price of 54 SEK/MWh. The second lowest cost has waste heat from two industries providing 30 MW at a price of 100 SEK/MWh while a coal fired boiler is able to provide 125 MW to a cost of 107.5 SEK/MWh. The taxation and efficiencies are included in these prices. There is also a heat pump, 40 MW at a cost of 116 SEK/MWh, taking heat from warm sewage water and further, possibilities to use natural gas and oil fired boilers. The cost for operating the last two boilers are higher than the costs above and in this study they have never been optimal to use.

THE CASE STUDY MODEL

As mentioned above we have started by implementing the electric demand in MW as variables in the model. There are 12 time elements both during high and low price conditions. It is not feasible to present the complete model and thus only the first elements will be shown below. To start with the objective function, this will show the total cost. The first elements in this function will be:

$$EDH1 \times TH1 \times ECG \times 10^{-6} + EDL1 \times TL1 \times ECG \times 10^{-6} +$$
$$+ HEH1 \times TH1 \times HCG \times 10^{-6} + HEL1 \times TL1 \times HCG \times 10^{-6} \qquad (1)$$

where EDH1 = Electricity high price demand in MW, EDL1 = Electricity low price demand in MW, TH1 = Number of hours in high price element, ECG = Natural gas cost for electricity prod., HCG = Natural gas cost for heat production, TL1 = Number of hours in low price element., HEH1 = The heat from CHP in high price element and HEL1 = The heat from CHP in low price element.

In January, which is the first element, the number of high price hours are 336 while the low price hours equal 408. The natural gas cost is 100 SEK/MWh when electricity is produced while it is 129 SEK/MWh for heat, and subsequently the Expression 1 will become:

$$0.0336EDH1 + 0.0408EDL1 + 0.0433HEH1 + 0.0526HEL1 \tag{2}$$

which shows the cost in MSEK.

The electricity that cannot be produced in the CHP plant must be purchased from the Sydkraft company. In the model this amount is assured by setting two constraints:

$$EDH1 \times TH1 + REH1 \times TH1 \geq ELH1 \tag{3}$$

$$EDL1 \times TL1 + REL1 \times TL1 \geq ELL1 \tag{4}$$

where REH1 = The rest of the high price el. demand, REL1 = The rest of the low price el. demand, ELH1 = The total high price el. need in MWh and ELL1 = The total low price el. need in MWh

Using the values for ELH1 and ELL1 found in Table 2 the constraints will be:

$$336 \times EDH1 + 336 \times REH1 \geq 117.9 \times 10^3 \tag{5}$$

$$408 \times EDL1 + 408 \times REL1 \geq 103.5 \times 10^3 \tag{6}$$

The purchased electricity has a price found in Table 3. In January, which is the first time element, the high price is 240 SEK/MWh while the low price is 165 SEK/MWh. The objective function must subsequently be added with these costs:

$$REH1 \times TH1 \times 240 + REL1 \times TL1 \times 165$$

or in MSEK:

$$0.081REH1 + 0.067REL1 \tag{7}$$

Further, there must be an expression for finding the maximum demand in MW due to the demand fee in the tariff. This maximum, not known in advance, may emerge in any time element. However, there are only five elements of interest because of the tariff, i.e. the high price elements from November to March. A new set of 5 constraints must be designed, but only the first element is shown here:

$$EDH1 + PMAX \geq 443.1 \tag{8}$$

where 443.1 is the maximum demand monitored in January. Note that PMAX is not indexed showing that it is valid for all time elements. There is also a cost for this maximum load, 175 SEK/KW. This cost in MSEK must be present in the objective function:

$$PMAX \times 175 \times 10^{-3} \tag{9}$$

The model must include a connection between the electricity production in the CHP plant and the district heating utility. Also in this case, this is set using a number of constraints, and it is assumed that three units of heat must be produced for each unit of electricity. This is not completely true for the total interval of electricity generation but the approximation will not be of major interest here. The first element constraints will be:

$$3 \times EDH1 - HEH1 = 0 \tag{10}$$

$$3 \times EDL1 - HEL1 = 0 \tag{11}$$

where HEH1 = Heat produced in the high price element and HEL1 = Heat produced in the low price element.

The variables HEH and HEL shows the thermal load in MW that is provided by the CHP plant. If this heat load is not sufficient, the equipment in the district heating utility must be used. There are different ways to ensure the necessary amount, burning garbage in the incineration plant, using waste heat from two private industries, using a coal fired boiler or using a heat pump system. The model does not contain any value for the priority, but instead the cost for using the different facilities is implemented in the objective function. When this function is minimized the best way to use the equipment will emerge. In this case study the cost will be:

$$HG1 \times 744 \times 54 + HW1 \times 744 \times 100 + HC1 \times 744 \times 107.5 + HHP \times 744 \times 116$$

where HG1 = The thermal load from garbage, HW1 = The thermal load from waste heat, HC1 = The thermal load from coal heat, HHP = The thermal load from the heat pump.

The number 744 is the sum of hours in January and the other figures shows the cost in SEK/MW described above. The expression, which must be added to the objective function will be in MSEK:

$$0.040HG1 + 0.074HW1 + 0.080HC1 + 0.086HHP \tag{12}$$

However, the model must also include the fact that a certain amount of heat must be delivered to the district heating grid. This is done by setting some extra constraints:

$$\begin{aligned} HG1 \times 744 + HW1 \times 744 + HC1 \times 744 + HHP \times 744+ \\ +HEH1 \times 336 + HEL1 \times 408 \geq 326.0 \times 10^3 \end{aligned} \tag{13}$$

The number 326.0 × 103 is the amount of heat in MWh needed in January, see Table 1.

Further, there must be some constraints showing that the maximum heat and electricity load from the different facilities cannot exceed certain values:

$$HG1 <= 65, HW1 <= 30, HC1 <= 125, HHP <= 40 \tag{14}$$

The figures show the limit in MW for each heating device. It is also necessary to force the model to operate at a maximum of 120 MW, and at at least 40 % of the maximum capacity of the CHP plant or not at all. This is done by the following expressions:

$$EDH1 - INTH1 \times 120 \leq 0 \tag{15}$$

$$EDL1 - INTL1 \times 120 \leq 0 \tag{16}$$

$$EDH1 - INTH1 \times 120 \times 0.4 \leq 0 \tag{17}$$

$$EDL1 - INTL1 \times 120 \times 0.4 \leq 0 \tag{18}$$

where INTH1 and INTL1 are binary integer variables.

The integer variables above can only have values of 0 or 1. This means that if the value equals 0 the CHP plant will not be operating at all, and if they equal 1 the plant will operate at a power between 48 and 120 MW. The model for the time segments in January has now been completed, and the other segments are elaborated in the same way. It is obvious that editing the complete model for all the time elements is a very tedious work. We have therefore developed a FORTRAN program which will print out the necessary input data file.

OPTIMIZATION

The model will contain 145 variables and 161 constraints. It is a linear model with 24 integers included, and thus the Simplex, and branch and bound algorithms can be used for the optimization, see [4] . We have used the ZOOM program, see Ref. [5] which has been slightly modified for implementation in a NORD 570 computer. ZOOM finds the values for the variables that will minimize the objective function and the computer will solve the problem in less than a minute.

THE OPTIMAL SOLUTION

In Table 4 the optimal electricity production is shown in the CHP plant.

Month nr	Variable EDH	Variable EDL
1	120.0	109.5
2	120.0	113.1
3	120.0	84.3
4	120.0	0.0
5	0.0	0.0
6	0.0	0.0
7	0.0	0.0
8	0.0	0.0
9	49.0	0.0
10	118.2	0.0
11	120.0	0.0
12	120.0	78.4

Table 4: Electricity production in MW in CHP plant

As can be found, the maximum capacity should only be used during high price conditions in January to April and November to December. The CHP plant should not be used at all from May to August and only during high price periods in April, September and October. See also Figure 1 for a graphical presentation.

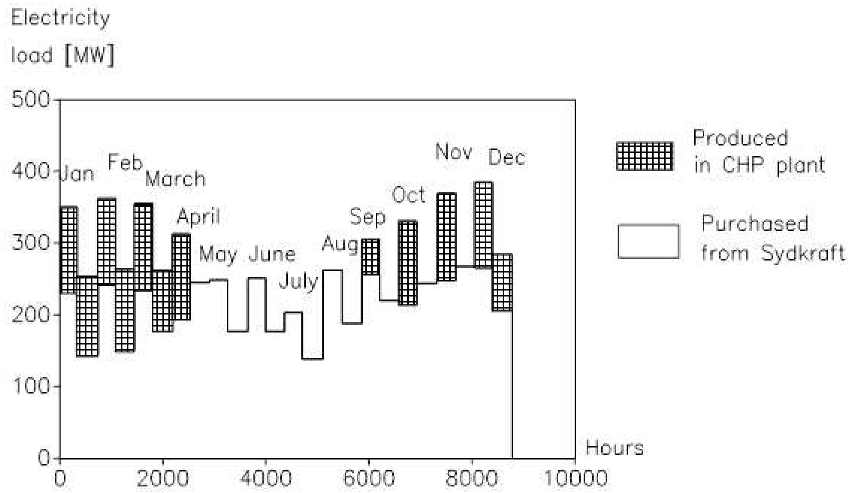

Figure 1: Optimal production and purchase of electricity

Now when the values in Table 4 are known the rest of the variables could be calculated. The optimization technique, however, solves the total problem in one batch and all the 145 variables are presented at the same time as well as the minimal cost which is 656.7 MSEK.

In Ref. [1] a computer program was used for calculating the total cost if a variety of CHP strategies were to be introduced. That program can not optimize the situation without a tedious iterative process but it is suitable for calculating the costs in all of the time elements for all of the equipment used. It has thus been used for clarifying the strategy found optimal above. In Table 5 the resulting electricity production has been calculated as found optimal in Table 4. In January 120 MW electricity is utilized in the high price period. Subsequently $120 \times 336 = 40\ 320$ MWh electricity is produced and further three times more heat. The cost is calculated as shown in the Expressions 2, 10 and 11. Due to approximations of the values the cost will not be exactly the same as shown in Table 5.

The cost for electricity purchased from SYDKRAFT is shown in Table 6.

From Table 5 and 6 it can be found that adding the produced and the purchased electricity will result in the values in Table 2. Under high price periods during the winter, the production in the CHP plant is as high as possible. In the low cost periods there is a combination between purchase and production that is the cheapest solution. In January, for an example, only 109.6 MW of CHP were to be used. If this value is increased, more heat is produced and subsequently less heat is to be produced in the district heating plant. A closer study shows that the load in this element is exactly on the border between waste heat and heat from the coal fired boiler. Producing extra heat in the CHP plant means that waste heat is saved in the district heating utility while producing less heat in the CHP means that the coal fired boiler must be used. The costs for an

Month	Electricity High	Electricity Low	Heat High	Heat Low	Cost [MSEK]
1	40.3	44.7	121.1	134.2	41.44
2	40.3	40.7	121.1	122.3	39.50
3	44.2	31.7	132.6	95.2	36.98
4	40.3	0.0	121.1	0.0	19.65
5-8	0.0	0.0	0.0	0.0	0.00
9	17.2	0.0	51.8	0.0	8.41
10	39.7	0.0	119.2	0.0	19.35
11	42.2	0.0	126.8	0.0	20.59
12	42.2	30.7	126.8	92.3	35.56
Total cost					221.47

Table 5: Optimal electricity and heat production in GWh, CHP plant

Month	Electricity High	Electricity Low	Cost [MSEK]
1	77.6	58.8	28.32
2	81.8	54.2	28.57
3	86.8	66.8	31.86
4	65.4	94.1	23.03
5	87.9	69.6	16.24
6	88.6	65.1	15.91
7	68.1	56.8	12.85
8	96.7	70.9	17.36
9	89.9	81.0	25.37
10	71.8	99.5	24.78
11	87.6	98.4	37.26
12	93.3	80.5	35.67
Total cost			297.23

Table 6: Purchased electricity in GWh

increase in the CHP plant by 10 MW will be:

- For electricity in CHP, $10 \times 100 \times 408 = 0.408$ MSEK
- For heat in CHP $30 \times 129 \times 408 = 1.579$ MSEK
- Less purchase of el. $10 \times 165 \times 408 =$-0.673 MSEK
- Lessuse of waste heat $30 \times 100 \times 408 =$-1.224 MSEK

The total cost will thus increase by 0.09 MSEK. If a decrease in the CHP plant of 10 MW is utilized the situation will be:

- Less cost for el. in CHP $10 \times 100 \times 408 =$-0.408 MSEK
- Less cost for heat, CHP $30 \times 129 \times 408 =$-1.579 MSEK
- Purchase of electricity $10 \times 165 \times 408 = 0.673$ MSEK

- Heat from coal $30 \times 107.5 \times 408 = 1.316$ MSEK

In this case the cost will also increase but only with 0.002 MSEK. The important is, however, that it is shown that the optimal solution is to use only 109.6 MW in the CHP plant. In April, see Table 4, a 120 MW is to be used in the CHP plant during the high price period. The cost for electricity is exactly the same as for the low cost period in January, i.e. 165 SEK/MWh, and it could be assumed that it would be better to use less power in the CHP plant. In this element coal is already used as a fuel and a less use of CHP will increase the use of coal in the district heating grid which will lead to an increased total cost. In Figure 2 the optimal use of heating devices is shown.

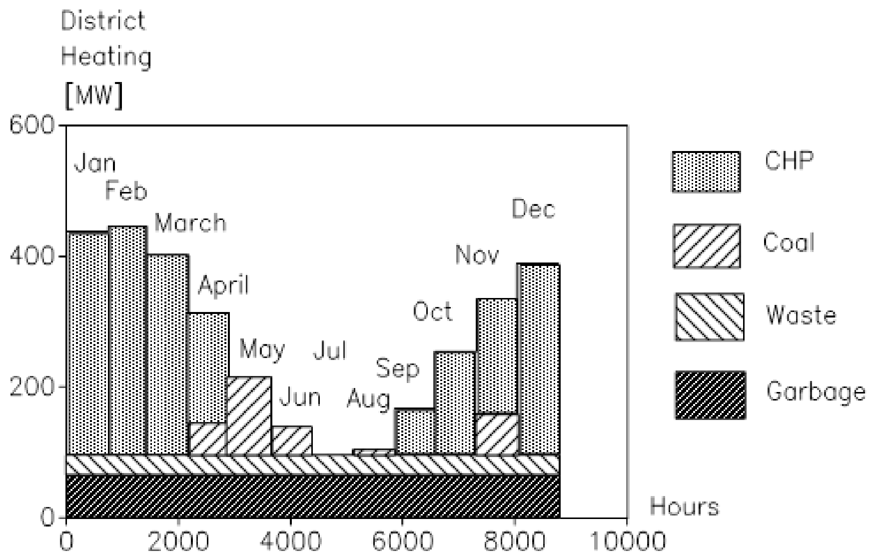

Figure 2: Optimal use of fuels and CHP heat in district heating system

Interesting is also to note that if the model is solved as a total linear program, i.e. no integers, only the electricity production in the low cost element in November will change to 41 MW. As mentioned above, this load is too small and the CHP plant must be turned off instead.

CONCLUSIONS

The paper shows that it is possible, and advantageous, to use a linear programming model for finding the optimal operating strategy when a CHP network is examined. It is also shown that the use for peak load fuels as coal, oil etc. can be held at a very low level while maximum use of heat from garbage and waste heat is utilized. During low price conditions in the electricity grid it is cheaper to buy the electricity than to produce it in the municipal CHP plant.

FURTHER WORK

Our work now continues by implementing energy retrofits in both the electricity grid and the district heating grid. Examples of such retrofits could be extra insulation on attic floors, weatherstripping or heat pumps in the buildings connected to the grids of concern.

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