# Optimization of bivalent heating systems considering Time-Of-Use tariffs for electricity

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**Abstract**

The cost for producing energy differs a lot due to the load coupled to the distribution grid. In Sweden the load has its maximum during the winter because of the climate. The cost for producing one extra unit of energy is then about 0.50 SEK/kWh, 1US$ = 6 SEK, while during summer the cost can be ten times lower. In order to encourage the consumers to save energy during the winter, when the cost is high, it can be of importance to introduce a time-of-use tariff which reflects the cost for producing the energy. Such a rate is present in Malmö, Sweden. When retrofitting buildings it is of course important to consider the applicable rate for energy in order to decide the optimal retrofit strategy. In the time-of-use rate the peak load is expensive and a heating system that will use less of the peak energy becomes very competitive. A bivalent heating system, where the base load is provided by a heat pump and an oil-boiler takes care of the building peak load, sometimes can be found to be the best solution. In this paper two different methods are used for the optimization of such a bivalent heating system. One method uses derivative considerations, the OPERA model, while the other uses linear programming.

**Key words: Retrofitting, bivalent systems, optimization, linear programming, oil-boilers, heat pumps, time-of-use tariffs, marginal cost pricing**

## INTRODUCTION

The cost for energy production differs due to the conditions when the energy is produced. In Sweden, with cold winters, the peak load emerges during the winter. Subsequently the cost for producing one extra unit of energy is highest during that season. Due to the marginal cost pricing theory the total amount of energy produced at that occasion shall have the same price. In the winter the peak load is produced by gas turbines, if electricity is considered, while the base load is produced using nuclear and hydro electrical plants. The energy price for a unit produced in in a gas turbine can be higher than 0.50 SEK/kWh, 1 US$ = 6 SEK, while hydro electrical energy produced during the summer can be less than 0.05 SEK/kWh. Ordinary tariffs for energy do not reflect this difference in the cost and subsequently a kWh saved during the summer, for the consumer, will result in the same savings no matter when the energy is conserved. For the energy producing utility however, the time aspect is of great importance. If an energy unit is saved during peak conditions the marginal cost is very high, if lack of capacity is present, the cost can be over 5 000 SEK/kWh, or the cost for building new power plants.

In Sweden where the nuclear plants will be out-faced the society has two different options. The production of electricity must be utilized in other power plants or the energy consumption has to be decreased. An introduction of a time-of-use rate, that reflects the cost for the energy production will encourage the desirable behavior from the consumer. In Ref. [1] a more detailed discussion can be found about differential rates and marginal cost pricing.

# BIVALENT HEATING SYSTEMS

One way to decrease the use of expensive peak energy is to install a bivalent heating system. In the system considered here, a heat pump is used for the base load while an oil- boiler is used for the peak load. The heat pump will deliver about three units of heat for each unit of electricity. However the heat pump is very expensive, about 5 000 SEK/kW and thus it is not possible to use the heat pump as the only heating device. When peak conditions emerge, in Sweden during the winter, an oil-boiler is started in order to provide the building with a sufficient amount of heat. The oil-boiler however, has a high running cost and it is thus not preferable to use the boiler as the only heating system as well. A combination of the two systems is the perfect solution. There are of course other heating systems that has to be considered, e.g. district heating, but they are not dealt with here.

# CASE STUDY

Since April 1985 a research project has run funded by The Swedish Council for Building Research and the municipality of Malmö, Sweden. The aim of the project has been to elaborate a method that finds the optimal retrofit strategy for each unique building. This method is called the OPERA- model, OPtimal Energy Retrofit Advisory, which is described in detail in Ref. [2].

In the project a number of buildings have been the subjects for retrofit considerations and in this paper one of the buildings, sited in the block Ansgarius, is dealt with. The building is a rather small multi-family building with 34 apartments in a rather poor thermal status. The total transmission loss in the building, including ventilation losses is 4.780 kW/K. The peak load in the building according to the Swedish building code is 167 kW. Running the OPERA model for this building implies that a bivalent system shall be combined with attic floor insulation in order to reach optimal conditions, see Ref. [3]. Ten different heating systems and eight different envelope retrofits have then been considered. In this paper however, only the bivalent heating system is optimized, the building envelope retrofits are not considered.

## Climate conditions

In order to show the optimization process it is necessary to start with the climate conditions in Malmö. In Figure 1 the monthly mean duration curve is shown.

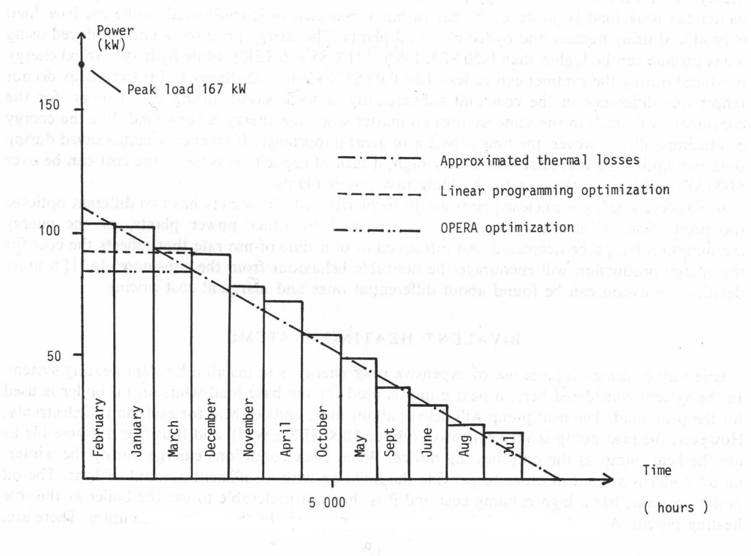


Figure 1: Duration graph for the building Ansgarius in Malmö, Sweden.

The highest load is found for February, 103.7 kW. Note that this is monthly mean values, the "real" peak load is still 167 kW. The number of hours in February is set to 678 which implies that the energy used is 70 326 kWh. The conditions are also shown in Table 1.

| Month | Load [ kW ] | Heat loss [ kWh ] | Month | Load [ kW ] | Heat loss [ kWh ] |
|---|---|---|---|---|---|
| Jan | 102.8 | 76 460 | July | 18.2 | 13 514 |
| Feb | 103.7 | 70 326 | Aug | 20.6 | 15 292 |
| March | 93.7 | 69 704 | Sept | 35.9 | 25 812 |
| April | 71.7 | 51 624 | Oct | 57.8 | 43 031 |
| May | 47.8 | 35 563 | Nov | 77.0 | 55 409 |
| June | 28.7 | 20 650 | Dec | 90.8 | 67 570 |

Table 1: Climate conditions in Malmö Sweden considering the building Ansgarius.

The influence of free energy from persons and applications is neglected here in order to show the linear programming method. In the OPERA model this can be studied because of the extensive use of energy balance calculations. This is necessary due to both heating system, and building envelope optimization.

## The electricity tariff

In Malmö a time-of-use rate is introduced as follows:

- Fixed fee = 5 000 SEK
- Subscription fee = 60 SEK/kW
- Power fee = 170 SEK/kW
- Energy fee
  - Nov - March, Mon - Fri, 06 - 22, = 0.392 SEK/kWh
  - otherwise, = 0.252 SEK/kWh
  - April, Sept, Oct Mon - Fri, 06 - 22, = 0.252 SEK/kWh
  - otherwise, = 0.222 SEK/kWh
  - May - Aug, Mon - Fri, 06 - 22, = 0.222 SEK/kWh
  - otherwise, = 0.187 SEK/kWh

The prices above include taxation of 0.072 SEK/kWh. It is now possible to calculate the applicable price for each month using the number of high and low price hours. The calculations result in the energy fee:

- Energy fee
  - Nov- March 0.314 SEK/kWh
  - April, Sept and Oct 0.236 SEK/kwh
  - May - August 0.204 SEK/kwh

**Normalization**

In the OPERA model only continuous functions can be dealt with, i.e. when bivalent systems are to be optimized, and thus these energy prices must be normalized to a fixed price. The normalization means that the utility will achieve the same income for identical thermal loads no matter how the tariff is designed. See Ref. [1] for a more thorough discussion about normalization. The procedure is shown below:

$$0.314 \times 76460 + 0.314 \times 70326 + \ldots + 0.314 \times 67570 = 152366$$

The subscription fee is calculated to 10 039 SEK and the power fee to 28 446 SEK. The total cost for the energy during one year is thus 190 851 SEK. The annual energy loss is 544 915 kWh and thus the normalized price will become 0.35 SEK/kWh.

## The OPERA optimization

The costs for the oil-boiler and the heat pump as well as the energy cost from the two devices now has to be calculated. In the OPERA model, and the same is valid for the linear programming system, the optimal solution is found when the total life-cycle cost for the building is as low as possible. The oil-boiler cost in this case study is assumed to be 55 000 + 60 $\times P_{oil}$ where $P_{oil}$ shows the thermal power of the oil-boiler. The economic life of the boiler is set to 15 years. Furthermore there is another cost for installation i.e. 200 $\times P_{oil}$ which has a longer economic life, 50 years, than the boiler itself. Using the present value method the boiler life-cycle cost will become:

$$55000 + 60 \times P_{oil} \times (1 + 1.05^{-15} + 1.05^{-30} + 1.05^{-45} - \frac{2}{3} \times 1.05^{-50}) +$$
$$+200 \times P_{oil} \times 1 = 55000 + 305.93 \times P_{oil} \tag{1}$$

The same expression for the heat pump has been evaluated as:

$$60000 + 8546.34 \times P_{hp} \tag{2}$$

The real discount rate is set to 5 % and the project life to 50 years.

In Refs. [2] and [4] it is shown that the energy cost for the oil-boiler can be calculated as:

$$(548263 - 9562 \times P_{hp} + 41.7 \times P_{hp}^2) \times \frac{18.26 \times 0.22}{0.75} \tag{3}$$

and the heat pump energy cost as:

$$(9562 \times P_{hp} - 41.7 \times P_{hp}^2) \times \frac{18.26 \times 0.35}{3.0} \tag{4}$$

The value 548 263 is the total energy need during one year, using the approximation in figure 1 due to the method of least squares. The present value factor 18.26 emerge from annual recurring costs for 50 years and 5 % real discount rate. The energy prices for the oil-boiler and the heat pump are 0.22 and 0.35 respectively and 0.75 and 3.0 are the efficiency and the COP for the heating systems. Adding the expressions 1 to 4 together, and noting that $P_{oil} = 167 - P_{hp}$, result in the total life-cycle cost for the building heating system. The expression is minimized by setting the derivative to 0 and the minimum is reached for a heat pump equaling 84 kW.

## The linear programming optimization

Another means to optimize the problem above is to use a linear programming method. In this paper it is not possible to make a review of how the method works, and thus only references are made to Refs. [5] and [6].

The problem to minimize must be expressed in an objective function and in this case the function is:

$$Cost_{hp} + Cost_{ob} + Cost_{energy\ hp} + Cost_{energy\ ob}$$

The first two parts of the objective function can be found in expressions 1 and 2 above, while the energy cost for the heat pump and the oil-boiler must be shown for each month and further using the applicable energy price. The first and last part of the objective function will become:

$$8546.34 \times P_{hp} + 305.93 \times P_{ob} + E_{Jan} \times \frac{18.26 \times 0.314}{3.0} + O_{Jan} \times \frac{18.26 \times 0.22}{0.75} +$$
$$+E_{Feb} \times \frac{18.26 \times 0.314}{3.0} + O_{Feb} \times \frac{18.26 \times 0.22}{0.75} + \ldots +$$
$$+ \ldots + E_{Dec} \times \frac{18.26 \times 0.314}{3.0} + O_{Dec} \times \frac{18.26 \times 0.22}{0.75}$$

In the expression $E_{Jan}$ equals the heat demand from the electrical heat pump during January, and $O_{Jan}$ the need for oil during the same month. The constant parts of the objective function are not necessary to encounter because they do not influence the size of the heat pump. There are also some constraints that must be satisfied. First the energy need for each month must be provided. This is achieved by setting:

$$E_{Jan} + O_{Jan} > 76460$$

$$E_{Feb} + O_{Feb} > 70326$$

$$E_{March} + O_{March} > 69704$$

and so on for each month during the year, see Table 1. Another constraint that has to be satisfied is that the heat pump power P must equal the heat pump energy devided by the number of hours for each month i.e.:

$$P_{hp} - E_{Jan} \times \tau_{Jan}^{-1} > 0$$

$$P_{ob} - O_{Jan} \times \tau_{Jan}^{-1} > 0$$

The last constraint is due to the total need for power in the building. The sum of $P_{hp}$ and $P_{ob}$ must exceed 167 kW. The objective function with the constraints above result in a linear program with 26 variables which has been solved using the LAMPS computer program, see Ref. [7]. The solution found optimal, implies that $P_{hp}$ shall equal 91 kW. $P_{ob}$ shall thus equal 76 kW. The heat provided by the heat pump is then 524 996 kWh and the oil-boiler energy equals 19 960 kWh each year, see Figure 1.

### Derivative versus linear programming optimization

Using linear programming offers a more straightforward method to find the optimal solution in this case study. The problem can be solved without the approximations which are necessary in the derivative method. The difficulty in linear programming is instead to elaborate the problem itself in such a way that it is possible to solve it with commercial computer programs. The case discussed above was rather small but introducing also envelope measures and ventilation retrofits will increase the number of variables very much.

In this case the two methods of optimization did result in a difference in the heat pump size of 7 kW or about 6 % so it will not cause any severe misoptimization if the derivative method is used instead of linear programming which in this case seems to be te best method to use. It must also be remembered that using monthly mean temperatures is an approximation of the real climate conditions. Using diurnal mean values makes the problem more like a continuous function which implies better performance from the derivative method.

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