OPTIMISATION MODELS AND SOLUTION METHODS FOR LOAD MANAGEMENT

Stig-Inge Gustafsson[a 1], Mikael Rönnqvist[b] and Marcus Claesson[b]
[a]Division of Wood Science and Technology and [b]Division of Optimisation, Linköping Institute of Technology, SE-581 83 Linköping, Sweden

ABSTRACT

The electricity market in Sweden has changed during recent years. Electricity for industrial use can now be purchased from a number of competing electricity suppliers. Hence, the price for each kilowatt-hour is significantly lower than it was just two years ago and interest in electricity conservation measures has declined. However, part of the electricity tariff, i.e. the demand cost expressed in Swedish Kronor (SEK) for each kilowatt, is almost the same as before. Attention has thereby been drawn to load management measures in order to reduce this specific cost. Saving one kWh might lead to a monetary saving of between SEK 0.22 and SEK 914; this paper demonstrates how to eliminate only those kWh that actually save a significant amount of money. A load management system has been installed in a small carpentry factory that can turn off equipment based on a pre-set priority and number of minutes each hour. The question now is what level of the electricity load is optimal in a strictly mathematical sense, i.e. how many kW should be set in the load management computer in order to maximise profitability? In this paper, we develop a mathematical model that can be used as a tool both to find the most profitable subscription level and to control the choices to be made. Numerical results from a case study are presented.

INTRODUCTION

The carpentry industry in Sweden has seldom been the subject of scientific studies. In part this is due to the fact that these factories are situated in rural areas, while the universities are located in larger cities. For small villages in the Swedish countryside, a wood working industry might, however, have significant importance as an employer, and if the company prospers, the industry's influence might even be increased in order to keep the present schools, shops and other industries in the community. The County of Kalmar and the European Community, EC, task 5b have thus financed a number of studies in order to find ways for this industrial branch to improve its profitability. One of the projects deals with optimisation of different processes and, in particular, with load management systems installed at Mörlunda Chair and Furniture Ltd., in Mörlunda, and Bringholz Furniture Ltd., in Ruda. Mörlunda and Ruda are both small villages situated about 350 km south of Stockholm, Sweden. This paper deals primarily with the Mörlunda factory, which was established in 1904. At the time when this study was carried out the factory had about 20 employees manufacturing chairs and other furniture from green lumber of beech, birch and other species. The entire manufacturing process was thus covered. The main

[1]Corresponding author. Fax: int + 46 13 281788, E-mail address: stigu@ikp.liu.se

Tariff element	SET	SD	Total
Fixed fee, SEK	-	8,000	8,000
Subscription fee, year, SEK/kW	-	37	37
Subscription fee, winter, SEK/kW	-	420	420
Reactive power fee, SEK/kVAr	-	205	205
Energy fee			
Jan.-Mar., Mon.-Fri, 06-22, SEK/kWh	0.186	0.052	0.238
Other time, SEK/kWh	0.186	0.029	0.215

Table 1: Electricity tariff for Mörlunda Chair and Furniture Ltd., 1999.

purpose of this paper is to investigate the possibilities of modelling a load management system and, if possible, of developing a decision support system that could be used both to establish an optimal subscription level and to control the system. Even if the fundamentals of the Mörlunda factory have been dealt with in a number of other papers, see e.g. [1] and [2] some basic and important facts must be repeated here in order to show what the mathematical model must deal with.

ELECTRICITY TARIFF

Electricity is currently purchased from or within a deregulated market. Each industrial consumer is obliged to have contracts with two companies, one that sells the electricity (in this case Sydkraft Electricity Trading Ltd., SET) and one that distributes the electricity (in this case Sydkraft Distribution Ltd., SD). The carpentry company can buy electricity from any trading company in Sweden and, in the near future, probably from other countries as well, but it must use the existing power lines from the distribution company. Most of the basic data for the factory have been collected during 1999 and, hence, the 1999 tariffs are shown in Table 1. (One US $ = about 10 SEK.) The company subscribes for 190 kW of active power. If this level is exceeded, the subscription fees in Table 1 are doubled for the excess consumption. The fee for the winter months is applicable to working days from November to March. In the factory, several motors are used for running fans, compressors and other types of machinery. All motors have coils for generating a magnetic field and because of this the sine waves for the current are no longer in perfect synchronisation with the voltage wave. This phase gap results in so-called reactive power, see [3] p. 249 for a more detailed discussion. Reactive power increases the current in the power lines and hence the electricity grid owner seeks to limit its use as far as possible. The company can use reactive power free of charge up to half the level of the active subscription, 95 kVAr in this case. If this level is exceeded, a penalty of 205 SEK/kVAr is imposed. The SD company monitors the electricity consumption with an ordinary electricity meter. Every hour, consumption in kWh is registered, which also shows the demand in kW. The saved registrations, which in turn are used for billing, do not show whether the demand in reality is high during the first 30 minutes and low during the remainder of the hour. Only the average is registered.

ELECTRICITY CONSUMPTION AND COST

Sydkraft monitors the number of kWh the company uses each hour throughout the year. These figures are stored in a database and are also available for the carpentry industry. In 1 and 2 , these values are presented in so-called duration graphs, i.e. the hourly values have been sorted in descending order. Figure

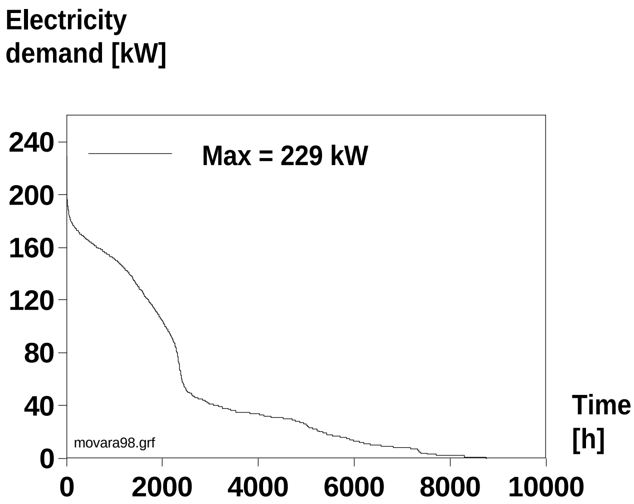

Figure 1: Electricity demand in the form of active power for the Mörlunda factory, 1998.

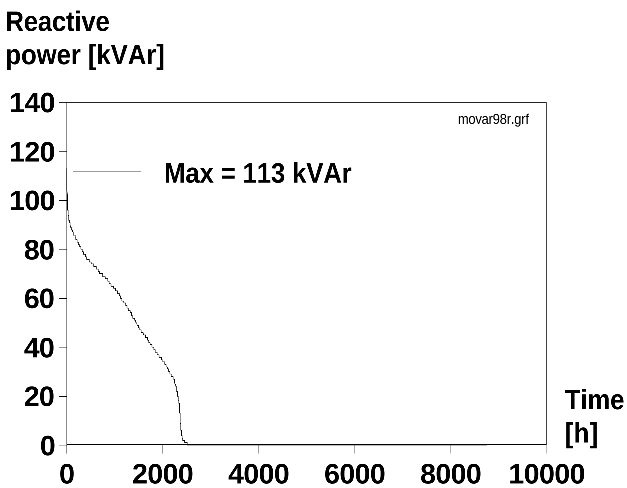

Figure 2: Electricity demand in the form of reactive power for the Mörlunda factory, 1998.

1 shows active power registered during 1998. The maximum demand was 229 kW and, therefore, the subscription level was exceeded by 39 kW, which, in turn, led to a penalty of SEK 35,000. The normal subscription fee was SEK 86,000. Figure 2 shows reactive power and, here again, the factory has exceeded

Fixed fee	8,000
Subscription fee, year	7030
Subscription fee, winter	79,800
Penalty fee, active power	35,646
Penalty fee, reactive power	3,690
Energy cost for 459,611 kWh	102,130
Total cost	236,296

Table 2: Cost elements, and costs i SEK, due to the electricity tariff for Mörlunda Chair and Furniture Ltd.

the subscription level, which was 95 kVAr. The penalty for these extra kVArs was about SEK 4,000. By using a small C computer program which reads the database files, it has been possible to calculate the cost elements in Table . Note, however, that Table shows the costs based on the 1998 load which is presented in Figure 1 , although prices are valid for 1999 as shown in Table . (We used the 1999 prices because the deregulated market was not in practical use before that year.) The cost elements based on the demand fees add up to SEK 122,476, which exceeds the cost for the actually consumed kWh and, hence, the elements are of major interest in seeking to save a significant amount of money. The 190 kW level was exceeded 31 times during 1998. The peak load above 190 kW contained only 209 kWh electric energy which would normally be worth only about SEK 50; see the energy fees in Table 1. Because of the demand fees, which are set according to demand in kW, the cost for this 209 kWh added up to a higher sum of money than all of the other 459,402 kWh which was used during 1998. This serves to exemplify the importance of the demand fees. The load management system can be set to any level and therefore it is not necessary to choose precisely 190 kW. Each kW is worth SEK 454 and hence a still lower level is preferable, but because of the penalty of SEK 908 per kW the factory cannot choose too low a level. However, it must sign a new agreement with Sydkraft Distribution Ltd. in order to change the level and thereby save money.

THE LOAD MANAGEMENT SYSTEM

In earlier studies, [1] and [2], a total of 10 different electricity-consuming components have been identified which could be turned off for a number of minutes each hour. Among the most obvious components are three aero-tempers run on electricity. Each aero-temper is assumed to use 9 kW, according to the rating information, but we have also monitored the consumption with meters scanned each minute by the load management system, called Mintop. All aero-tempers are given the same priority, i.e. 1, which implies that in the event of a hazardous peak the system would turn this equipment off first. Initially, the system has been set so that the aero-tempers must be turned on for one minute in each five-minute interval. This is intended to save 7.2 kWh each hour, or 7.2 kW, with a possible monetary value of SEK 6,580 according to the demand fee. Figure 3 shows the electricity demand, calculated as mean values for one hour, in one of these aero-tempers. From Figure 3 it is also obvious that the maximum electricity demand is about 9 kW, but this maximum value is not

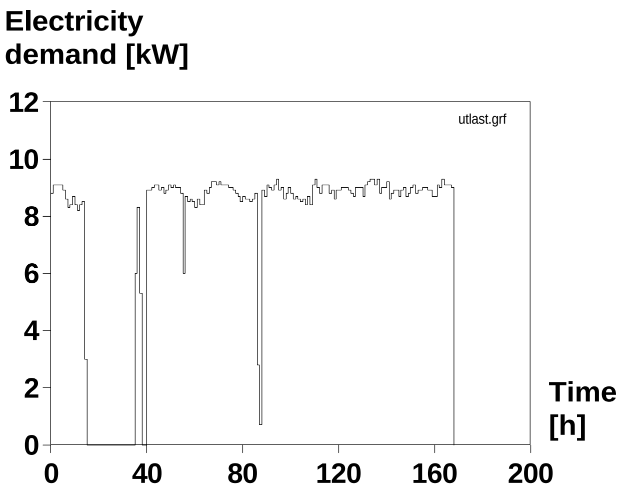

Figure 3: Electricity demand by one of the Mörlunda aero-tempers, March 15 - 21, 1999.

Device	Max. demand [kW]	Priority
Aerotemper, loading ramp	9	1
Aerotemper, assembly	12	1
Aerotemper, upholstery	9	1
Electric radiators, showrooms	2	1
Kiln dryer no 1	8	2
Kiln dryer no 2	20	3
Motor to fan no 1	10	4
Motor to fan no 2	10	5
Motors to fan no 3 etc.	25	6
Wood chipper	10	7

Table 3: Electricity consuming devices taking part in the load management system.

always present. Sometimes the aero-temper was turned off for other than load management reasons. Reality is, however, even more complicated. The graph in Figure 3 is built up from average values over one-hour intervals. In Figure 4, two such intervals are shown in closer detail and it is obvious that demand in the aero-temper varies significantly. The problem is accentuated even more in Figure 5, where demand is shown for a kiln dryer used for drying wood and operating on electricity. This dryer can also be heated by using steam from two boilers, one fired with wood chips and the other with oil, [2]. The kiln is also included in the load management system but has a higher priority than the aero-tempers. From March 1999, ten different electricity-consuming devices were coupled to the system, see Table 3. The wood chipper is not actually coupled to the system because of hazardous health effects but, instead, a yellow lamp flashes when the machine must not be used. Most systems are turned on automatically by Mintop, but this is not the case for fan no. 3. In fact,

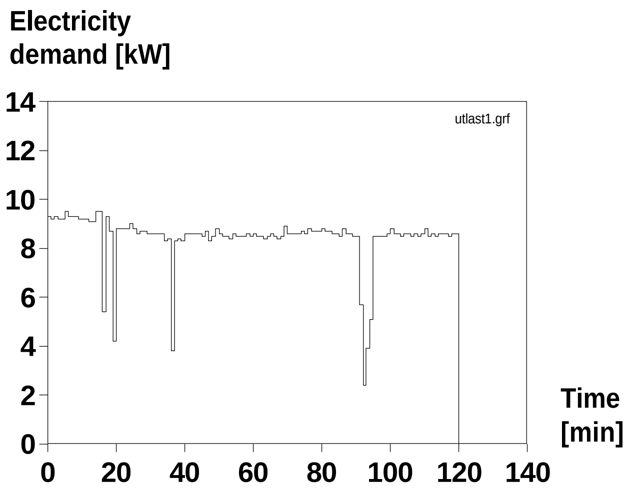

Figure 4: Electricity demand by an aero-temper 6.00 - 8.00 February 15, 1999.

Figure 5: Electricity demand for two hours in one of the kiln dryers, starting 23.00 11 May, 1998, [1].

this item contains three motors, running two fans and one wood chip screw. It was not easy to modify the original design of the automation equipment, so, when Mintop has turned it off, it has to be restarted by hand. There is also a last resort. If Mintop has turned off all equipment in Table 3, a yellow lamp flashes in the foremans cabin. He then has to go to the factory and manually turn some equipment off. All these devices and tariff structure elements must now be introduced in a mathematical model which in turn will also be used for optimisation.

OPTIMISATION BASICS

The mathematical method which we have used in this paper employs the technique of Mixed Integer Linear Programming, MILP. The first thing to do is to construct a so-called objective function, which is a mathematical expression showing the cost for a system. In our case this function therefore shows the operating cost for the carpentry factory. Hence, the electricity tariffs and all other costs must be included in this objective function. The expression also contains a number of variables, showing for instance the electricity demand in kiln dryer no. 1, see Table 3. From Figure 5 it is obvious that the model must be designed with very short time steps, i.e. of one minute length, which also was the basis for our measurements. Assume now that the electricity demand in the kiln dryer is P_{kd} kW. For one hour the cost of electrical energy will become:

$$\frac{P_{kd1} + P_{kd2} + P_{kd3} + P_{kd4} + \ldots + P_{kd59} + P_{kd60}}{60} \times 0.238 + \ldots$$

assuming P_{kd1} is used for the first minute and P_{kd2} for the second minute of the specific hour, and 0.238 shows the cost for one kWh, See Table 1. The objective function must now be minimised, i.e. we want to find the lowest possible cost for operating the carpentry factory. The optimisation might now result in some of these variables being equal to zero. If all are equal to zero we would have a minimum point for the objective function but then nothing is dried in the dryer. A number of constraints are therefore introduced which ascertains that at least some of the P_{kd} variables must have a positive value which in turn results in a warm dryer. Such a constraint can look like this, if, for example, 25 kWh must be used:

$$\frac{P_{kd1} + P_{kd2} + P_{kd3} + P_{kd4} + \ldots + P_{kd59} + P_{kd60}}{60} \geq 25.0$$

Another strategy can also be used, i.e. to use binary variables that equal one if the dryer operates and zero if it is turned off. By adding such decision variables it is easy to force the model to use the dryer for say, 20 minutes each hour. Other constraints and variables ascertain that, for example, the motor to fan no 2 must be used every now and then or else all wood chips will be transported only to the filter and not to the chip bin. As shown above this leads to a very large number of variables, 60 for only one hour and one component in the factory, for which optimal sizes must be calculated. In fact, the number of variables became so large that ordinary procedures for solving the problem did not work. Even modern computers choked when they started to calculate our models with thousends of binary and other variables. One of the authors to this paper therefore needed help from the other two in order to solve such vast optimisation problems. The basis for the MILP technique cannot be presented in detail here and the reader should therefore consult references [4], [5] or [6] for further reading. The three references show text books on optimisation that are commonly used at universities and at least one of them should be easy to locate. MILP has also been used in numerous studies and the method is becoming increasingly interesting because of faster computers and more powerful algorithms. Models that were impossible to solve just a few years ago can now be dealt with in just a few minutes. Examples of recent studies in the field of energy science and MILP can be found in [7], [8], [9], while a study on load management can be found in [10].

THE MODEL

The model shown below is presented partly from a mathematician's point of view, and also from a general aspect, but we will elucidate the conditions with a number of examples for the less familiar reader. To describe the mathematical model we need to introduce certain definitions, decision variables and parameters. We start by defining the dimensions of four important aspects.

- nd Number of devices used in the load management system
- nt Number of time periods. We use $t \in T = \{1, \ldots, nt\}$
- np Number of reading time periods. We use $q \in Q = \{q_1, \ldots, q_{np}\} \subset T$
- $na(i)$ Number of turn-off alternatives for device i.

The decision variables used in the optimisation must reflect the possible alternatives and be able to measure certain important values, e.g. if the kiln dryer is operating or not. We have introduced the following decision variables. The variable Z_q is given a positive value if the subscription level is exceeded; otherwise it will be given a value of 0. Other such variables are:

X_{ijt} 1, if device i is turned off according to alternative j in time period t, otherwise it will equal 0.

Z_q Excess energy during reading period q. ($q \in Q$)

Y_t Controllable amont of energy during time period t.

The Y_t variable is introduced because there is also other equipment in the factory which is not affected by the load management system. Only a small part of the total energy demand can be dealt with. Further, a number of parameters and information are needed in order to formulate the mathematical model. Certain data are related to the variables chosen and other data are independent of the model. The energy demand is made up of two parts: firstly, the part that can be changed, see above, and secondly, a part that cannot be controlled since it derives from machines that are outside the load management system. Hence, we introduced the following independent parameters.

W_t The energy amount that is not controllable during time period t.

L Subscription level of energy per reading period, i.e. hour.

P Penalty in SEK per kW for exceeding the subscription level.

An obvious solution is to turn off all devices that are controllable during the entire day. This is of course impossible, since they must be operating during some period. In order to model this, we have set restrictions on the energy consumption during a number of fixed time periods. One example could be that kiln dryer no. 1 must use, say, 30kWh for each time period of four hours (maximum is 8×4=32 kWh). Data relating to this aspect and information relating to the turn-off alternatives for the devices are:

$nr(i)$ Number of sets of time periods where device i must meet the energy restrictions.

g_{ip} Lower level of energy consumption for device i during time set p

G_{ip} Set defining time set p by using lower and upper bounds on time. $G_{ip} = \{t : gl_{ip} \leq t \leq gu_{ip}\}$.

8 Energy consumption of device i if it is not turned off during time period t.

a_{ij} Number of time periods that define alternative j for device i.

b_{ijq} Energy reduction in device i during time period t according to alternative j. $q = 1, \ldots, a_{ij}$.

c_{ij} Cost of turning off device i according to alternative j.

To illustrate the notation, we study for example Kiln dryer no. 2. This corresponds to device 6, and we may assume that we consider alternative 1. In this alternative, the dryer is turned off for 3 minutes, after which it must be on for 5 minutes. The total time for the alternative is therefore 8 minutes (where the dryer cannot be turned off for the last 5 minutes) i.e. $a_{61} = 8$.. The energy consumption (using a time period of 1 minute) if the device is not turned off is 166 Wh, i.e. $e_{6t} = 8$. In the same way the energy reduction during the 8 minutes is 166 for the first three minutes and then 0 for the remaining 5 minutes. To represent this we have $b_{611} = 166, b_{612} = 166, b_{613} = 166, b_{614} = 0, b_{615} = 0, b_{616} = 0, b_{617} = 0, b_{618} = 0$. The cost corresponds to its priority, which gives $c_{61} = 3$. This alternative is illustrated in Figure 6. We can now formulate the

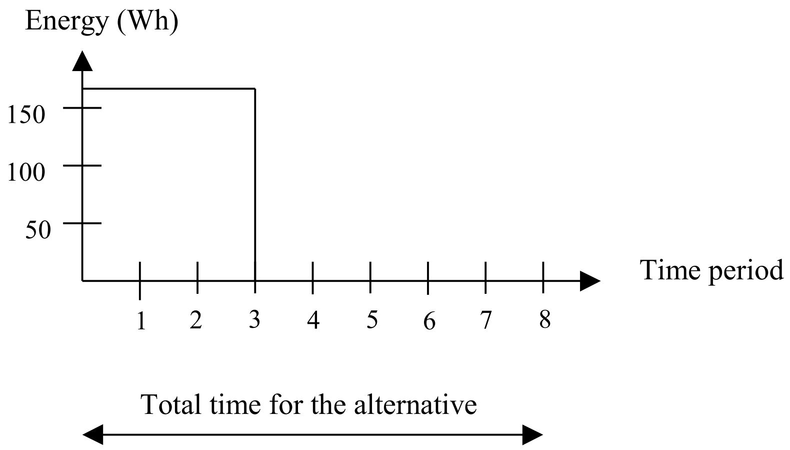

Figure 6: An illustration of a possible turn off alternative for the device Kiln dryer no 2.

mathematical model as follows.

$$min \sum_{q\in Q} PZ_q + \sum_{i=1}^{nd}\sum_{j=1}^{na(i)}\sum_{t=1}^{nt} c_{ij}x_{ijt}$$

$$subject\ to$$

$$\sum_{j=1}^{na(i)}\sum r = t - a_{ij} - 1x_{ijr} \leq 1 \forall i,t \qquad (A)$$

$$y_t = \sum_{i=1}^{nd}(e_{it} - \sum_{j=1}^{na(i)}\sum_{r=t-aij+1}^{t}(b_{ij,r-t} + a_{ij}xijr))\forall t \qquad (B)$$

$$\sum_{t\in G(i,p)} e_{it} - \sum_{j=1}^{na(i)}\sum_{r=t-a_{ij}+1}^{t}(b_{ij,r-t} + a_{ij}x_{ijr}) \geq g_{ip}\forall i,p \qquad (C)$$

$$\sum_{t\in T(q)}(W_t + y_t) - z_q \leq L\forall q \in Q \qquad (D)$$

$$x_{ijt} \in \{0,1\}\forall i,j,t, y_t \geq 0\forall t, z_q \geq 0\forall q \qquad (E)$$

The objective function, i.e. the one that must be minimised, consists of two terms. The first is related to the penalty associated with exceeding the subscription level. The second is related to choosing a device according to a priority list. This means that we in fact have a multi-objective function, but we place them together with a given weighting. Generally, finding the correct weighting can be a complex task. For example, it is difficult to compare a direct cost expressed in SEK with an environmental impact. In our case, such impact is not present. The penalty is the dominating factor and the priority list will therefore have less influence. This means that the solution will tend to have a zero penalty, if possible. If this is so the priority list will also be accomplished. We have chosen the weights in such a way that a cost coefficient of 1000 corresponds to one SEK. There are four different types of constraints. Constraint set (A) states that at most one turn-off alternative for each device can be active for any time period.

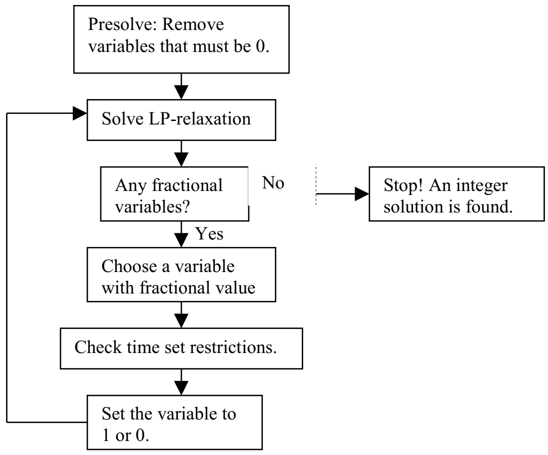

Figure 7: Illustration of the solution process based on a sequential LP approach.

Constraint set (B) is used to compute the controllable energy during time period t. Constraint set (C) restricts the energy consumption for device i during time set p. This means, for example, that Kiln dryer no. 2 must be operating for at least, say, 80% of the total time for any two-hour interval. Constraint set (D) is used to compute the cumulative energy consumption between two reading time periods and to measure the excess energy at each reading point. Set (E) corresponds to the restrictions on binary variables and non-negative continuous variables. It should be mentioned that the constraints are used either to model a practical limitation or to compute values that can be used to simplify other expressions. For example, the constraint set (B) can be removed but then a much trickier solution must be used in set (D). This is a standard procedure in many models.

SOLUTION METHOD

The mathematical model is an integer programming (IP) model of considerable size. In practice, it is not possible to solve the model directly with a commercial solver. Therefore, it is necessary to develop a new method that not only solves the problem but also does this within a reasonable time. The approach we have chosen is to use a sequential linear programming method. Here, we solve the linear programming (LP) relaxation of the IP problem i.e. we relax the restrictions that the variables should be an integer (i.e. 0 or 1). This means that the restriction $x \in \{0, 1\}$ is replaced by $0 \leq x \leq 1$. Once the LP relaxation is solved, we choose a variable with fractional value and set that variable to 1 or 0. Then we resolve the modified problem and repeat the process. This is repeated until no integer variables have fractional values. This process can be viewed as a limited branch and bound procedure where no backtracking in the search tree is performed. We, therefore, do not solve the integer programming model to theoretical optimality which also means that the branch and bound tree is not too large. The approach is summarised in Figure 7. In the step of choosing a fractional variable, there are several possibilities. We use a method where we find the largest value that is based on a combination of fractional value and the time period with which the variable is associated. In the step of checking restrictions, we compute whether the variable must be 0 with respect to the energy consumption for individual devices during the time steps. There is no guarantee of finding an optimal solution with this process. However, tests have indicated that the quality is very high. The optimisation model and the solution approach is implemented in the AMPL modelling language, [11]. Four major components are needed in order to solve our problem using AMPL. These are mathematical model, model data, solution approach, and optimisation system. The generic model is described using a syntax that is very similar to the original mathematical model given earlier. The model does not contain any data itself. Instead, this is provided through a number of input files, which makes it easy to run different cases on the same model. The solution approach is implemented in an AMPL script code, which is similar in syntax to MATLAB. To solve the underlying LP-problems, for example, a suitable optimisation package library must be used. AMPL is a general modelling language and, hence, it is possible to use essentially any package. We have used the package from CPLEX 6.5, which is a commercial program for solving MILP problems. There is no traditional source which could be used as a reference but the reader finds information on the program on the WWW-page, see [12].

CASE STUDIES

We have set up a test case with 11 electricity consuming devices, which is used throughout the tests. Aero-tempers, kiln dryers, motor fans etc. are included in the case. There is no need to control these devices during times when there are limited operations in others and, hence, no peak will occur. Load management is useless during off-peak conditions. We have therefore chosen to control consumption during the normal working day which is essentially 10 hours. We use time periods with a length of one minute, which means that the number of time periods is 600. Each reading period is one hour. To model the requirement that devices must be operating during sensitive periods, we have introduced so-called time sets. These time intervals differ, depending on the device, and each interval has minimum electricity demand. This means that we can force a heater in a dryer, for example, to operate for at least, say, 75% any two-hour period, or otherwise too low a temperature in the dryer might be hazardous for the timber. The number of such time set periods varies between 2 and 5 depending on the type of device. Certain basic data relating to the case are given in Table 4.

The Cost column gives minimum and maximum costs for the turn-off alternatives. A wood-chipper is by this more costly to turn off compared to an aero-temper. This under the assumption that they are turned off for the same number of minutes. In the same way, the Time column gives the corresponding times for the alternatives. The Required energy column shows the electricity demand necessary in each time set.

Device	Electricity demand. (Wh)	No. of alternatives	Cost	Time	No. of time sets	Required energy (kWh)
1	150	4	3-4	2-5	2	25
2	200	4	2-3	2-5	3	18-24
3	150	4	2-4	2-5	2	25
4	33	6	1-8	6-35	3	2-4
5	167	5	3-5	6-10	2	25
6	167	5	5-6	6-10	4	10-15
7	167	5	5-8	36-40	2	20-30
8	167	5	1-20	11-30	5	10
9	167	5	2-25	11-30	1	50
10	167	6	3-60	21-50	2	25
11	16	9	1-8	2-61	1	0

Table 4: Electricity demand and other input data for the case study.

COMPUTATIONAL RESULT

We have set up four different scenarios in order to investigate the model, solution approach and practical performance. The first (A) is designed to solve the model with different subscription levels. The second (B) is for solving the model with a fixed subscription level but with a different number of alternatives. The third (C) is intended to solve the model with a different number of alternatives and with the subscription level as a decision variable. The fourth (D) is provided to simulate one days operation where the estimated and actual demand are different. Some of the data used are estimates of the real situation. This includes the setting of the time sets and the lower level of energy demand for the devices.

SCENARIO A

In the first scenario we have five different subscription levels ranging from 160 to 120 kWh/h or kW. Note that the utility monitors the actually used kWh during each hour. The purpose of using such low levels was to make it a harder to optimise the situation. In reality the company could use 190 kW but such a level only occured a few times during the studied year. We, however, wanted to create a difficult problem so we could analyse performance in a difficult planning situation. The number of constraints in the model is 7,237, the number of 0/1 variables 35,391 and the number of continuous variables 1,209. Data and results are given in Table 5. The column No. of decisions shows the number of times any device was turned off according to an alternative. The column Objective Penalty gives the objective function value relating to exceeding the subscription level. In the same way, the Objective Control column gives the objective value relating to turn-off costs. From Table 5, it is obvious that it is possible to control the electricity consumption with zero penalty by turning off devices. Figures 8 and 9 illustrate how the electricity consumption increases for each of the ten hours, which in turn builds up the demand in kW.

The cases shown are those with subscription levels of 160 and 120 kW, respectively. A closer study of the data sets shows that nothing needed to be done for the first 3 hours in Figure 8. The electricity demand was lower than

Case	Subscription level (kWh)	No. of decisions	Objective Penalty	Objective Control
A1	160	22	0	88
A2	150	63	0	250
A3	140	152	0	629
A4	130	278	0	1212
A5	120	369	0	1969

Table 5: Results with different subscription levels.

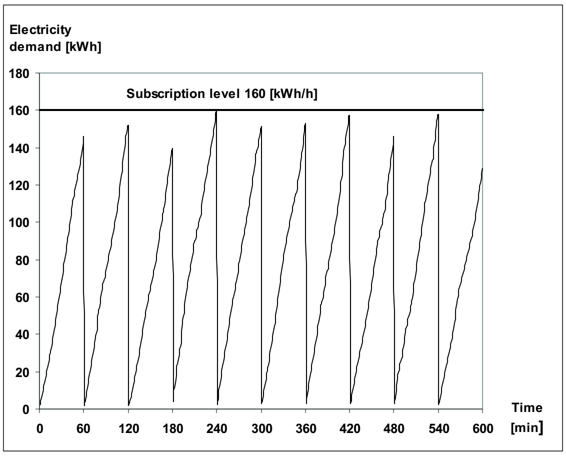

Figure 8: Electricity demand for ten consecutive hours with demand side management for a subscription level of 160 kWh/h.

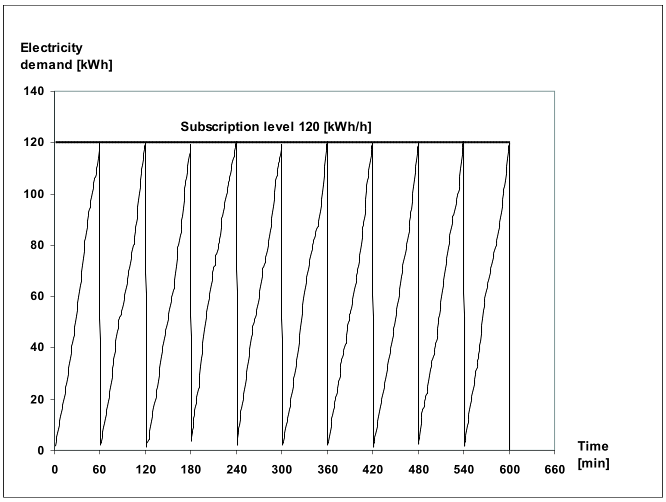

Figure 9: Electricity demand for ten consecutive hours with demand side management for a subscription level of 120 kWh/h.

Reading period	60	120	180	240	300	360	420	480	540	600
Slack [kWh/h]	0.23	0.56	0.51	0.21	0.78	0.40	0.68	0.70	0.60	0.77

Table 6: Slack energy levels for the ten consecutive hours in case A5.

Device	B1	B2	B3	B4
1	4	2	3	5
2	4	2	3	5
3	4	2	3	5
4	6	4	5	7
5	5	3	4	6
6	5	3	4	6
7	5	3	4	6
8	5	3	4	6
9	5	3	4	6
10	6	4	5	7
11	9	7	8	10

Table 7: Number of turn off alternatives for each combination of device and case.

160 kW without any actions by the load management system. For hour no. 4, however, device no. 2 had to be turned off for minutes nos. 192, 203, 211 and 216 respectively. Load management was of no benefit during the following 4 hours, but at minute 481 device no. 2 was turned off again. Further devices turned off for several minutes were 5, 7 and 8. From Figures 8 and 9, it is obvious that it is more difficult, i.e. more decisions are required, to solve the problem with a low subscription level. Further, from Figure 8 it is clear that there is considerable slack at virtually all reading points, indicating that this case is easy to control. It is difficult to see the energy slack (subscription level - actual energy consumption) values for case A5, see Figure 9, but Table 6 gives a more detailed description of the case.

SCENARIO B

When investigating the impact of the number of alternatives, we have generated four cases with the same subscription level (130 kWh/h). The number of alternatives varies, see Table 7. The number of constraints is 7,237 for all cases and the number of continuous variables 1,209. The number of binary variables differs between cases. Basic data and results for the cases are shown in Table 8. From the results, we can see that the solution approach is not optimal since problem B3 has a higher objective value compared to B2. However, there is a clear general trend in which the objective value is better (lower) with an increased number of alternatives. Also, with more alternatives it is easier to choose the best possible turn-off alternative in different situations. This is clear from the fact that the number of decisions decreases with the number of alternatives.

Case	No. of variables [0 1]	Subscription level [kWh/h]	No of decisions	Objective penalty	Objective control
B1	22,191	130	319	0	1,420
B2	28,791	130	302	0	1,201
B3	35,391	130	278	0	1,212
B4	41,991	130	211	0	1,047

Table 8: Results with a constant subscription level but a different number of alternatives.

Case	Subscription level [kWh/h]	No. of decisions	Objective penalty	Objective energy	Objective control
C1	124.32	465	0	56,814,200	1,820
C2	118.11	409	0	53,978,600	2,178
C3	118.11	392	0	53,978,600	2,280
C4	116.22	341	0	53,112,100	2,181

Table 9: Results with subscription level as a decision variable.

SCENARIO C

Instead of using a fixed subscription level, we introduce the level, L, in the model as a new decision variable. Another modification in the model is that the objective function (below) includes a cost for the actual level so that it is possible to balance the actual cost against the penalty. In our case, we have C_L = 420 + 37 = SEK 457 per kW, see Table 1.

$$min \sum_{q \in Q} PZ_q + \sum_{i=1}^{nd} \sum_{j=1}^{na(i)} \sum_{t=1}^{nt} c_{ij} x_{ijt} + c_L L$$

We use the same data as in scenario B, where we have a different number of alternatives for the devices. The magnitudes of the problems in terms of constraints and variables are the same as for scenario B, except that the number of continuous variables is increased by one to 1,210. Basic data and results for the cases are given in Table 9. The new objective Energy column gives the part of the objective function associated with the energy costs below the subscription level. (Note that the cost is based on Wh here and, hence, the large value). It is clear that the dominating part of the objective function is associated with the selected electricity subscription level. The penalty part related to exceeding the subscription level is in all cases zero, indicating that it is possible to control the energy demand. Figure 10 illustrates the situation for case C1, and we can clearly see that the subscription level is almost reached in every reading period. Figures 11 and Table 10 provide a more detailed description of the results obtained for case C1.

SCENARIO D

In an operative environment where decisions have to be made in real time, it is not possible to know the actual electricity demand in each time period.

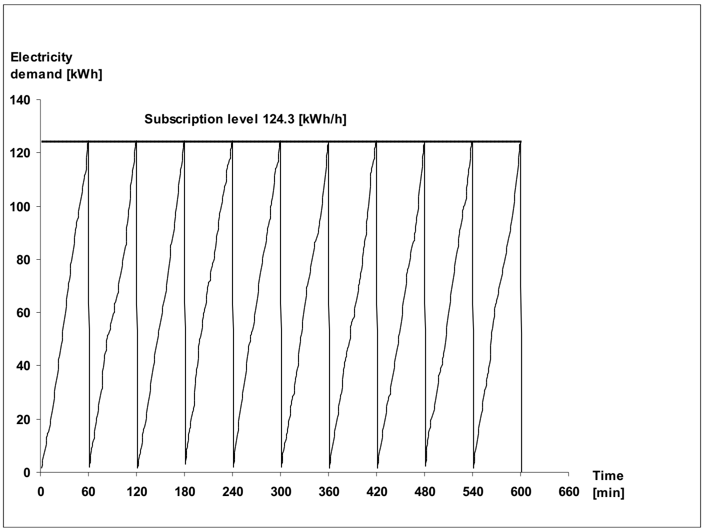

Figure 10: Electricity demand when the subscription level is used as a variable in case C1.

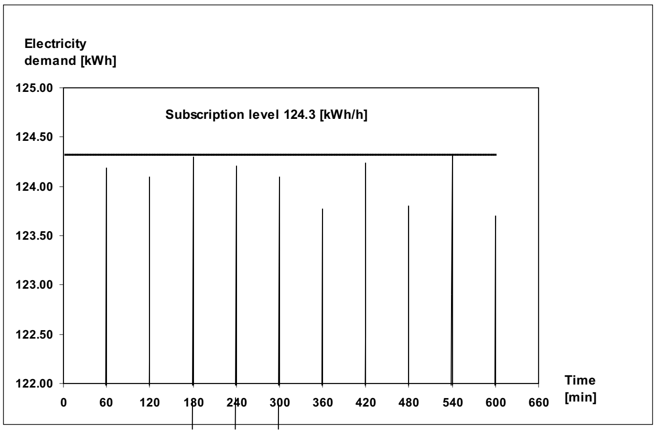

Figure 11: Slack energy levels for reading periods in case C1.

Reading period	60	120	180	240	300	360	420	480	540	600
Slack [kWh/h]	0.13	0.59	0.40	0.21	0.14	0.28	0.29	0.41	0.0	1.21

Table 10: Slack energy levels for reading periods connected to case C1.

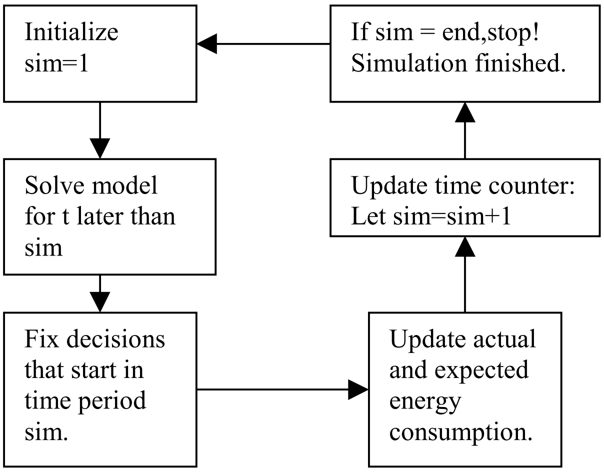

Figure 12: Description of the simulation process.

Case	Subscription level [kWh/h]	No. of decisions	Objective penalty	Objective control
D1	185	51	0	216
D2	175	118	0	540
D3	165	215	54,840	1117

Table 11: Results with simulation and different electricity consumption.

Instead, the actual demand can be viewed as a stochastic variable. To test this situation, we have developed a simulation process where the demand of one day and control are simulated. The basic simulation process used is described in 12. In each time period, we solve an entire model describing the situation from a given simulation time until the end of the day. This model uses data which we have expected to apply for the remainder of the day. Given this solution, we fix decisions made for the next time step. We then update with the actual electricity demand and formulate another optimisation problem, which in turn is resolved. This process is continued until we have a simulation time equal to the end of the day. The index *sim* gives the simulation time (in time periods) and the notation *end* gives the last time period, which in our case is 600 (minutes). Figure 13 shows the estimated and monitored electricity demand throughout the day. We have deliberately made the fluctuations throughout the day more difficult in order to test the performance of the models and methods. The number of constraints is 7,223, the number of binary variables 35,391 and the number of continuous variables 1,209. The result is given in Table 11. In problem D3, it was not possible to find a control strategy that would enable a demand below the subscription level. This was due to a lower estimated consumption than the measured level. The excess amount was, however, very small. Due to the scaling of the problem, the actual cost was only SEK 54.84.

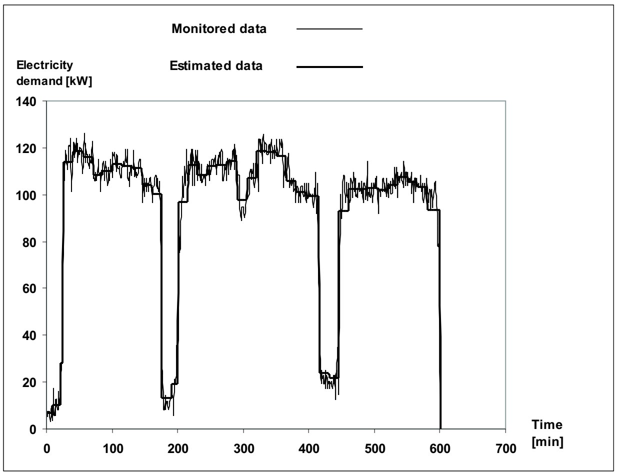

Figure 13: Estimated and measured electricity consumption in the simulation tests.

CONCLUDING REMARKS

We have shown that it is possible to construct mathematical models in the form of Mixed Integer Linear Programs, MILP, which in turn have been used for optimisation of a load management system installed in a carpentry factory, Mörlunda Chair and Furniture Ltd. The models are based on monitored data collected from electricity meters coupled to various equipment in the factory as well as the meter used by the utility for the electricity bill. The meters are scanned each minute and this also sets the resolution in time. Since some of the data needed for the model were not known in detail, estimates have been used. We have also tested against a lower subscription level because the current level was so high that too few decisions were needed by the optimisation in order to really test the performance of the model. The model developed is of considerable size in terms of constraints and variables. Generally, integer programming problems are very difficult to solve. However, with the proposed methods it is possible to obtain solutions within a reasonable time. If the model is to be used in a real time system, it is necessary also to improve its solution times. On the other hand, in such a case the model only need to consider one hour and not ten hours. This will obviously make the process much faster.

The model can accurately be used to determine a subscription level which is almost optimal. By running the model on daily data representing the worst days for, say, the previous month, it would be possible to suggest a suitable subscription level. The introduction of more turn-off alternatives leads to better results. In our tests, we have generated these alternatives manually. An interesting area for further research is to generate these dynamically in the solution process.

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