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OPTIMIZATION OF BUILDING RETROFITS IN A COMBINED HEAT AND POWER NETWORK

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Abstract—We describe a mathematical model of a linear program for optimization of the following: the use of purchased and produced electricity (for power and for heat), the fuel mix in a district-heating plant, and implementation of energy-conservation measures in the building stock of the Malmö municipality. We find that energy retrofits are not profitable when compared to the cost for producing or purchasing more electric power and heat. The reasons for this result are, the low cost of electricity purchased from the national grid even under peak conditions and the exploitation of waste heat generated at the district-heating plant.

INTRODUCTION

When electric power is generated in an ordinary condensing power plant, steam is produced by burning fuel in the boiler. To generate the maximum amount of electricity generated, the pressure difference between the steam inlet and the condensate outlet must be kept as high as possible. This is often done by cooling the condenser with cold seawater. A disadvantage of this method is that about 60% of the energy of the fuel will be wasted in the form of lukewarm water and only 40% converted to useful electric power. By utilizing a district-heating network as a cooling device for the condenser, waste heat can be exploited to heat buildings. The low temperature of the wastewater, around 10°C, prevents direct use. The temperature could be increased to 100°C or more and distributed over the district-heating network, but this approach would diminish the electric power generated by about 10–15%. Economic theory indicates that energy should be priced according to its short-term marginal cost, which implies that all plants should be used optimally. This cost is defined as the money saved when one less unit is produced or as the cost of producing one additional unit. The cost of scarcity must also be taken into account. The district-heat consumer should therefore pay only for the power potential lost when the temperature is raised in the condenser of the combined heat and power plant, CHP. Cogeneration provides a certain amount of heat for the district-heating system. However, this amount is often not sufficient, and it is necessary to burn fuel in the district-heating plant also. The cost of so doing will depend on the fuel mix but will always increase the short-term marginal cost. When the electric power generation cost exceeds the market price, generation should, of course, be terminated.[1] The purpose of the study discussed in this paper is to determine the best ways of operating the equipment and how to change the energy system to achieve the optimal solution.

In recent years, there has been increased interest in using linear programming techniques to optimize energy systems of various sizes.[2] Some journals have published articles dealing with energy and economic mathematical modelling in an overall perspective, but there is, to the best of my knowledge, no article describing an optimal building-energy system in detail, although a suitable mathematical method has been presented.[3] Two authors have described a model used to minimize the cost of the national grid in former West Germany, but buildings and building retrofits were not given special attention in these studies.[4–6] Another published model deals with building retrofits and bivalent heating systems[7] but no CHP system was included. In the model[8] forming the basis of our study, a CHP plant is described and analyzed through linear programming but without including retrofitting in the discussion.

In a linear program, there is always an objective function which must be minimized or maximized.[9] In the study at hand, this function contains the life-cycle cost LCC of the energy system, and this cost is then the subject for minimization. The LCC is defined as the sum of all building, maintenance and operating costs of the system. The objective function is constrained by a number of equations, e.g., electricity need, heat need, or the upper and lower size constraints of a power plant. An item of special interest is the influence of steps on the cost function, e.g., demolition of old boilers or other equipment, before evaluation of the linear part of the cost function. Such problems are solved by using binary variables whose value can only be 0 or 1. The same technique is applied when nonlinear functions are to be approximated by piecewise linear parts, e.g., when estimating the influence of additional insulation on outside walls. The mathematical problem may then be solved by using an appropriate program for solving a mixed-integer problem.[10] We have frequently used the LAMPS and ZOOM programs.[11,12] The LAMPS system has been implemented on a DEC 2065 machine while ZOOM can be run on various computers.

CASE STUDY

To exemplify the linear programming method, a case study is presented concerning the Swedish municipality *Malmö*. The electricity load is presented in Table 1. The load, monitored in 1988, is split into several time segments because of the time-of-use TOU tariff structure of the selling company *Sydkraft*. High- and low-price periods are represented; the cost of electricity is shown in Table 2. Furthermore, there is a demand charge of 270 SEK/kW during the high-price period stretching from November to March (1 US$ = 6 SEK.) The district-heating load was not monitored for the same time segments, i.e., it was not split in TOU during the day and, therefore, a gigantic fictitious building has been projected, one which would have a climatic load of the same magnitude as the district-heating load monitored during one year. The reason for this is that there must be a consistent influence between the retrofit

Table 1. Electric load in Malmö.

Month	High (GWh)	Low (GWh)	Month	High (GWh)	Low (GWh)
January	117.9	103.5	July	68.1	56.7
February	122.1	94.9	August	96.7	70.9
March	131.0	98.5	September	107.2	81.0
April	105.7	94.1	October	111.5	99.5
May	87.9	69.6	November	129.9	98.4
June	88.6	65.1	December	135.6	111.2

Table 2. Electricity price, Sydkraft 1990.

Month	Energy price [SEK/kWh]	
	High price	Low price
November - March	0.235	0.142
April, September, October	0.126	0.0997
May - August	0.068	0.057

Table 3. *U*-values and areas for different parts of the fictional building.

Building part	Area (Mm^2)	U-value (W/m^2K)	UxA (MW/K)
Attic floor	3.1	0.5	1.55
External wall	9.7	0.7	6.79
Floor	3.1	0.5	1.55
Windows,1.2 Mpc · 1.5 m^2	1.8	2.5	4.50

Table 4. District-heating load in Malmö in GWh.

Month	Load	Month	Load	Month	Load
January	340.5	May	173.9	September	134.3
February	323.1	June	113.2	October	204.3
March	312.9	July	84.2	November	254.8
April	239.3	August	91.4	December	304.2

activities on a building and the decrease of the thermal load. U-values for various areas of the building are shown in Table 3. The thermal loss due to ventilation is set equal to 5.07 MW/°C and the heat expended for domestic hot water is calculated to be 350 GWh annually. It is also assumed that the indoor temperature is 21°C, while the monthly outdoor temperature is the mean values for a 30 yr period, as monitored by the Swedish Meteorological and Hydrological Institute. These values result in the district-heating load shown in Table 4.

Electricity prices are shown in Table 2, but electricity can also be generated at the CHP plant where the boiler is fuelled by natural gas (NG). The model also contains a gas-turbine operating on NG. The CHP plant already exists, the equipment cost is a sunk cost. The gas-turbine does not exist in the real-world plant but is included in the model to examine at what point new equipment will enter the optimal solution and at what price. District-heat can be produced in a number of ways. Sources include garbage incineration, industrial waste heat, heat pumps in water or sewage treatment systems, and boilers fired by coal, oil or NG. The cost of operating these different facilities, their sizes, efficiencies, and other relevant data are presented in Table 5.

Other costs emerge when the building is retrofitted. These are shown in Table 6. The costs are expressed as linear functions: the first constant shows the cost of raising scaffolds and other preparatory work and is here set equal to zero. The next constants concern insulation, the first showing the general insulation effect, the second showing the insulation start-up costs, and the third the cost per meter of insulation. The same method was used in the design of the so-called OPERA model.[13,14] The cost of new windows is related to size. There are, of course, other retrofit options, but these two are implemented in the model because they were selected through an OPERA optimization.

Thermal power and electric power are examples of variables that appear in the model. If these items are known, all other aspects of the energy system can be calculated. The cost of operating the energy system depends, for instance, on the rate at which electricity is purchased from the market (see Table 2). The rate is split into several segments according to time-of-year and time-of-day uses. Both the climatic and the electricity loads are related to the monthly

Table 5. Equipment in the district-heating plant, efficiency, and prices.

Equipment Type	Fuel price (SEK/MWh)	Efficiency (SEK/MWh)	Taxation (SEK/MWh)	Heat price (SEK/MWh)	Size (MW)
Garbage	54	1.0	-	54	65
Ind. waste	100	1.0	-	100	30
Coal	42	0.8	55	107.5	125
Heat pump	198	3.0	50	116	40
Natural gas	85	0.85	29	129	120
Oil	57	0.8	89	160.3	240
Gas-turbine	85	0.25	-	340	New
CHP plant	85	0.85	-	100	120

Table 6. Building retrofit costs.

Building asset	Total cost (SEK/m^2)
Attic floor insulation	0 + 260 + 530 x t
New double glazed windows	0 + 1 100 x A_f
Triple glazed windows	0 + 1 300 x A_f

pattern. Moreover, one rate element shows the cost of maximum electricity demand in SEK/kW. To model the cost of the energy system, it is therefore necessary to implement 29 time segments, one for high-price conditions and another for low-price conditions for each of the 12 months and another for the maximum electricity load for the 5 months from November to March. Because of the large number of equations, the model is presented only for the month of January. The optimal solution presented in this study, represents the complete model.

Generation of electric power

The municipality of Malmö can produce its electric power by burning NG in a steam boiler. In Sweden, generation of electric power is not taxed at the source, but the NG heat delivered to the district-heating network is taxed at the rate of 29 SEK/MWh. The price of NG is 85 SEK/MWh and it is used with an efficiency of 0.85. *The first part of the objective function, which is to be minimized shows the cost for electricity production by use of a steam turbine as follows*:

$$(EDH1 \times 336 \times 100.0 + EDL1 \times 408 \times 100.0 + HEH1 \times 336 \times 129.0 + HEL1 \times 408 \times 129.0) \times 18.26 \times 10^{-6} \quad (1)$$

where EDH = power production under high-price conditions (336 h) in MW_e, EDL = power production under low-price conditions (408 h) in MW_e, HEH = heat from the condenser under high-price conditions in MW, HEL = heat from the condenser during low-price conditions in MW, 1 = the month of year which in this case is January, 18.26 = the present worth factor for a 5% real discount rate and a 50-yr project life. The model must also include an expression showing the market-power need and the gas-turbine production. Table 1 shows the monthly power need. By determining variables for the market demand and for the gas-turbine power generation, we get

$$(EDH1 + GTH1 + REH1) \times 336 \geqslant 117.9 \times 10^3, \quad (2)$$

$$(EDL1 + GTL1 + REL1) \times 408 \geqslant 103.5 \times 10^3, \quad (3)$$

where REH = electric power purchased under high-price conditions in MW_e, REL = electric power purchased under low-price conditions in MW_e, GTH = electric power generated by the gas-turbine when high-price conditions prevail, and GTL = electric power generated by the gas-turbine when low-price conditions prevail. Expressions (2) and (3) are two constraints on the model, i.e., the need for electric power must always be satisfied. However, the cost of obtaining the power, whether generated by a gas-turbine or purchased on the market, must be reflected in the expression (1). Table 2 shows the cost for each kilowatt hour. It is necessary to add to expression (1)

$$(REH1 \times 336 \times 235 + REL1 \times 408 \times 142) \times 18.26 \times 10^{-6}. \quad (4)$$

The gas-turbine utility does not exist; this equipment must be bought and installed. The cost for installment and operation is assumed to be reflected in the following cost estimate which is added to the objective function:

$$3.0 \times GTMF + 85.0 \times 18.26 \times 10^{-6} \times (GTH1 \times 336 + GTL1 \times 408)/0.25, \quad (5)$$

where 3.0 = cost of a gas-turbine in $MSEK/MW_e$, GTMF = maximum fuel demand in MW for the gas-turbine in any time segment, 0.25 = efficiency of the gas-turbine, 85.0 = price of NG. It is also necessary to ascertain that GTMF is the largest value in MW_e used during any time segment. This is accomplished by use of the following constraints:

$$(GTH1/0.25) - GTMF \leqslant 0.0, \quad (6)$$

$$(GTL1/0.25) - GTMF \leqslant 0.0. \quad (7)$$

The model will contain 12 equations of type (6) and 12 of type (7), two for each month, and, since all must be valid at the same time, GTMF must be set equal to the largest value for any month. The same technique is used to ascertain the maximum demand for the electrical charge. Here, only 5 months are significant, November–March, and subsequently the following constraint for each month must be added to the model:

$$\text{EDH1} + \text{PMAX} + \text{GTH1} \geqslant 443.1, \tag{8}$$

where PMAX = maximum purchase during any of the 5 months in MW_e, GTH1 = gas-turbine production in January (high-price conditions) in MW_e, and 443.1 = maximum monitored need in January. The demand charge 270 SEK/kW must be added to the objective function. In MSEK, the expression becomes

$$\text{PMAX} \times 270 \times 10^{-3}. \tag{9}$$

The existing steam boiler has a capacity ceiling of 120 MW_e. A capacity floor is also defined in terms of efficiency loss; the plant will be turned off when the power load drops below 40% of the maximum power capacity. The model must therefore contain expressions stating that profitable electric power production lies between 48 and 120 MW, and any other value will cause the plant to be turned off. Such problems are solved by using binary variables whose value can only be 0 or 1. The expressions for January will be

$$\text{EDH1} - \text{INTH1} \times 120 \leqslant 0, \tag{10}$$

$$\text{EDL1} - \text{INTL1} \times 120 \leqslant 0, \tag{11}$$

$$\text{EDH1} - \text{INTH1} \times 48 \geqslant 0, \tag{12}$$

$$\text{EDL1} - \text{INTL1} \times 48 \geqslant 0, \tag{13}$$

where INTH1 = binary variable for high-price conditions and INTL1 = binary variable for low-price conditions. The expressions (10)–(13) state the EDH1 must be ⩽120 MW if INTH1 equals 1 and also >48 MW. If INTH1 = 0, EDH1 will implicitly equal 0.

District-heat production

The district-heat is produced in part from waste heat obtained through the generation of electric power, but mostly by burning various fuels in the boilers of the utility. In the waste-heat case, three units of heat are assumed to be produced for each unit of electric power. This fact must be incorporated in the model. By using the variables from expression (1), a set of equations may be represented as follows:

$$3.0 \times \text{EDH1} - \text{HEH1} = 0, \tag{14}$$

$$3.0 \times \text{EDL1} - \text{HEL1} = 0. \tag{15}$$

If the waste heat of electricity generation does not suffice to supply the thermal load, the district-heating plant will have to be fuelled. Table 5 shows a number of possible fuels and their costs; these costs must be implemented in the objective function. Expression (1) must therefore include;

$$(\text{HG1} \times 54 + \text{HW1} \times 100 + \text{HC1} \times 107.5 + \text{HHP1} \times 116 + \text{HGAS} \times 129) \times 18.26 \times 744 \times 10^{-6}, \tag{16}$$

where HG1 = thermal power derived from garbage incineration, HW1 = thermal power from industrial waste heat, HC1 = thermal power from the coal boiler, HHP1 = thermal power from the heat pumps in the sewage water treatment system, and HGAS = thermal power from the NG boiler. The energy cost for the heat pump, i.e., 116 SEK/MWh, is an approximation of the real cost which ultimately depends on the cost of electricity, generated either from the steam- or gas-turbine or purchased from the energy market. The real cost of electric power is ultimately determined by the mix of the electricity supply sources. It has not proved possible to

Table 7. Optimal solution when no energy conservation retrofits are present, values in MW.

Month	CHP		Purchase		District-heating				
	High	Low	High	Low	Garbage	Waste	Coal	Heat p.	Nat. gas
January	120.0	48.0	230.9	205.7	65.0	30.0	120.8	-	-
February	120.0	48.0	243.4	215.7	65.0	30.0	120.7	-	-
March	120.0	-	236.0	262.0	65.0	30.0	125.0	22.3	-
April	51.6	-	263.0	245.1	65.0	30.0	125.0	40.0	-
May	-	-	249.6	177.6	65.0	30.0	125.0	13.7	-
June	-	-	251.6	176.8	65.0	30.0	62.2	-	-
July	-	-	202.8	139.1	65.0	30.0	-	-	-
August	-	-	262.9	188.5	65.0	30.0	27.9	-	-
September	-	-	304.6	220.1	65.0	30.0	91.5	-	-
October	-	-	331.8	244.0	65.0	30.0	125.0	40.0	14.7
November	120.0	-	248.8	267.4	65.0	30.0	82.6	-	-
December	120.0	-	264.9	283.6	65.0	30.0	125.0	18.5	-

model this dependence in terms of a linear or mixed-integer program. Therefore an estimate from the energy authorities of Malmö has been used for the energy price of the heat pump.

The model must also contain expressions about the need for heat in the different time segments. This need is shown in Table 4. The resulting expression indicates that the sum of all heat produced must exceed the need, i.e.

$$(\mathrm{HG1} + \mathrm{HW1} + \mathrm{HC1} + \mathrm{HHP1} + \mathrm{HGAS}) \times 744 + \mathrm{HEH1} \times 336 + \mathrm{HEL1} \times 408 \geqslant 340.5 \times 10^3. \tag{17}$$

The different types of equipment have limited sizes in megawatts (see Table 5) which yield the following constraints:

$$\mathrm{HG1} \leqslant 65, \qquad \mathrm{HW1} \leqslant 30, \qquad \mathrm{HC1} \leqslant 125, \qquad \mathrm{HHP1} \leqslant 40, \qquad \mathrm{HGAS} \leqslant 120. \tag{18}$$

The above equations complete the electricity and heat-generation parts of the model. Only January is presented. The model contains around 150 variables and even more constraint equations. Before the energy-conservation part of the model is presented, this preceding part will be optimized and discussed in some detail. Because of the many variables and cumbersome way of presenting the mathematical model in MPS format to the optimizing computer program, a short FORTRAN program was developed to write the input data file. The program and the MPS input data file are not presented in this paper but will appear in a separate report.[15]

Table 7 presents the optimal solution, including the electric and thermal loads for the various types of equipment. Generation of electricity at the CHP plant is found to be optimal under high-price conditions from November to April, but it is optimal only during January and February under low-price conditions. The plant should operate at its maximum load except during April and at its lowest capacity during the low-price hours. At all other times, the plant should be idle. Electricity should be purchased during all of the time segments, but the NG

Table 8. Number of hours in the different time segments.

Month	High price hours	Low price hours
January	336	408
February	336	360
March	368	376
April	336	384
May	352	392
June	352	368
July	336	408
August	368	376
September	352	368
October	336	408
November	352	368
December	352	392

turbine had no optimal use at all. In the district-heating plant, the garbage-incineration plant and the waste heat should be utilized at full capacity throughout the year, but coal should not be used during July. The optimum use of the heat pump will entail its maximum use in April and October, but it will be completely turned off for 7 months. The NG boiler is to be used only in October and then only 14.7 MW are necessary. The needs for both electricity and heat are adequately covered. The January electricity demand is approximately 351 MW during the high-price period, i.e., $117.9 \times 10^3/336$ as seen in Tables 1 and 8. The CHP production and purchase in Table 7 add up to 350.9 MW. The total production of CHP electricity is $120 \times 336 + 48 \times 408$ equalling 59,904 MWh, implying that three times more heat or 179,712 MWh are delivered to the district-heating network. Table 4 shows that 340.5 GWh heat must be delivered. Using 65×744 of the garbage heat, 30×744 of the waste heat and 120.8×744 MWh of coal heat leaves 179.9 GWh or nearly the estimated contribution of the CHP plant. The values in Table 7 could be looked at more closely to find their underlying determinants but this has not been done in this paper. Instead, the model will be completed by introducing the important measures for energy conservation.

Energy-conservation measures

One possible energy-conservation measure, e.g., insulation of the attic floor, might reduce both electricity and heating needs. In this study, the heat load is calculated for a gigantic fictitious building (see Tables 3 and 4). If the attic floor in this building is given additional insulation, the need for heating will be reduced. It has been shown that the new U-value for the attic floor could be calculated as[16]

$$U_n = k_n \times U_e/(k_n + U_e \times t) \tag{19}$$

where U_n = new U-value in W/m^2 K, k_n = thickness of extra insulation in metres. Equation (19) cannot be implemented directly in the model because it is not a linear statement. However, it is possible to calculate a new expression for which parts of the nonlinear function have been linearized. The method is described in Ref. 8 and is only briefly presented here. The underlying idea is to change Eq. (19) so that it will no longer be a function of t but of some binary variables (A) instead, which correspond to different values for t. First the objective function must be changed. Tables 3 and 6 show the area of the attic floor to be insulated and the cost of doing so. The insulation cost is a linear function of t but Eq. (19) is not and, therefore, both expressions must be changed. Calculating the cost of additional insulation for some consecutive values of t will yield a new equation. We assume that the optimal insulation thickness lies in the range of 0–0.3 m. We also assume that an approximation in steps of 0.05 m is sufficient. The cost for 0.05 m of additional insulation is

$$3.1 \times 10^6 \times (260 + 530 \times 0.05) = 888.15 \text{ MSEK}. \tag{20}$$

For 0.1 m, the cost will be 970.3 MSEK and so forth. Multiplying these values by binary variables leads to

$$888.2 \times A_1 + 970.3 \times A_2 - 1052.5 \times A_3 + 1134.6 \times A_4 + 1216.8 \times A_5 + 1298.9 \times A_6. \tag{21}$$

As mentioned before, the binary variables can only have the value 1 or 0, thereby setting the constraint

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \leq 1, \tag{22}$$

which will imply that only one or none of the values A_1–A_6 can be chosen. The additional insulation leads to a lower thermal need as shown in Eq. (19). By calculating the difference between the old and new U-values, this decrease in heat flow can be implemented in the model. Assuming that the existing U-value is 0.5 W/m^2 K (see Table 3) and that the new conductivity for the additional insulation is 0.04 W/m K, the use of Eq. (19) will result in values for an insulation thickness of 0.05, 0.1, 0.15 m, etc. as shown in Table 9. Before these can be

Table 9. New and reductions of U-values in W/m^2K for a retrofitted attic floor.

Variable	New U-value	Reduction
t = 0.05	0.308	0.192
t = 0.10	0.222	0.278
t = 0.15	0.174	0.326
t = 0.20	0.143	0.357
t = 0.25	0.121	0.379
t = 0.3	0.105	0.395

implemented in the model, the values of Table 9 must be multiplied by the area of the attic floor (i.e., $3.1 \times 10^6 m^2$) and also by the number of degree-hours assumed to apply for the Malmö area each month. In Malmö, the outside mean temperature is assumed to be −0.5°C, the indoor temperature is assumed to be 21°C, and the number of hours for the same month is 744, which means that the number of degree-hours in January is assumed to be 15,996. The decrease in the heat flow for an additional 0.05 of attic insulation will then be $0.192 \times 15{,}996 \times 3.1 \times 10^6$ or 9.52 GWh, which must correspond to the A_1 value above. The right-hand side of Eq. (17) must therefore be (in GWh)

$$340.5 - 9.5 \times A_1 - 13.8 \times A_2 - 16.2 \times A_3 - 17.7 \times A_4 - 18.8 \times A_5 - 19.6 \times A_6. \tag{23}$$

The function (23) is valid only for January and the model must subsequently contain 11 more equations of the same type. Equation (22) requires that only one or none of the A variables is chosen and, if one is chosen, the appropriate value is added to the objective function. At the same time, the thermal need decreases according to the additional insulation. Another way to decrease the heat flow is to replace the existing double-paned windows with triple-paned glass. The cost of replacing the windows is found in Table 6. It is assumed that the replacement must be done immediately. The cost of replacing them with new double-paned windows is, as shown in Tables 3 and 6,

$$(0 + 1100 \times 1.5) \times 1.2 \times 10^6 = 1980\,\text{MSEK}. \tag{24}$$

The cost of replacing the windows with triple-paned ones is calculated in the same way to be 2340 MSEK. The difference in cost is 360 MSEK. It is further assumed that the new windows have a life span of 30 yr. In our study, a project life of 50 yr is assumed and later differences in cost must be transferred to the base year by using the present-worth method. The discount rate is 5%, and using a binary variable B for window replacement, the cost in the objective function will be

$$360 \times [1 + (1 + 0.05)^{-30} - (10/30) \times (1 + 0.05)^{-50}]B = 432.8\,B. \tag{25}$$

The window replacement will influence the thermal needs, reducing the U-value from 2.5 to $2.0\,W/m^2K$ and the right-hand side of Eq. (17) must therefore also include

$$-15{,}996 \times 1.5 \times 1.2 \times 10^6 \times (2.5 - 2.0) \times 10^{-9} \times B = -14.4 \times B\,\text{GWh}. \tag{26}$$

The district-heating load in MW is also influenced by the building retrofits. The model does not contain any expression for this cost because suitable cost data are lacking. The existing equipment can meet all heating needs and no new boilers or other equipment is necessary. The model must also contain expressions showing the cost and the consequences of conserving electricity. Unfortunately, it is not clear how an individual retrofit will affect the electricity load. It was thus necessary to design another gigantic fictitious building which could be insulated and otherwise modified in the same way as described previously. Data received from energy authorities in Malmö indicate that this building would have a transmission coefficient of about 1.975 MW/K. This value was used to calculate the use of electric power for space heating; the result is shown in Table 10. It is further assumed that the performance of the two buildings is the same, whether heated by electricity or from the district-heating network. It is supposed that about 35% of the electric space-heating load is ventilation-dependent, implying

Table 10. Assumed electricity used in GWh for space heating in Malmö.

Month	High price	Low price	Month	High price	Low price
January	14.27	17.33	July	2.52	3.06
February	14.03	15.03	August	3.13	3.19
March	14.25	14.56	September	5.21	5.45
April	9.95	11.38	October	8.03	9.75
May	6.95	7.74	November	11.19	11.70
June	4.17	4.36	December	13.21	14.71

that 1.284 MW/K is a result of heat transmission through walls. To investigate if attic floor insulation will be optimal, the same reasoning is applied. It is assumed that about 11% of the heat flow or 0.138 MW/K depends on this feature. If the existing attic has the same U-value of $0.5\ \mathrm{W/m^2\,K}$, the implied attic floor area is $276{,}000\ \mathrm{m^2}$. We now have enough information to design amendments to the objective and other functions and constraints. To start with, the expression (1) must be completed [see the design of expression (21)] as follows:

$$79.25 \times D_1 + 86.58 \times D_2 + 93.91 \times D_3 + 101.24 \times D_4 + 108.57 \times D_5 + 115.90 \times D_6, \qquad (27)$$

where the variables D_1–D_6 are binary variables. The influence on the electric load is calculated as before [see Eq. (23)]. The results are

$$117.9 - 0.384 \times D_1 - 0.56 \times D_2 - 0.65 \times D_3 - 0.71 \times D_4 - 0.76 \times D_5 - 0.79 \times D_6, \qquad (28)$$

$$103.5 - 0.47 \times D_1 - 0.67 \times D_2 - 0.79 \times D_3 - 0.87 \times D_4 - 0.92 \times D_5 - 0.96 \times D_6, \qquad (29)$$

$$D_1 + D_2 + D_3 + D_4 + D_5 + D_6 \leqslant 1. \qquad (30)$$

The expression (28) must be added to (2) and (29) to (3). Expression (28) deals with the high-price period and expression (29) with the low-price period. The additional attic floor insulation will also decrease the electricity demand in megawatts. This is significant during the 5 months when the cost is related to the TOU tariffs. The design outdoor temperature is assumed to be −14°C and the desired indoor temperature is 21°C. The right-hand side of Eq. (8) must therefore be completed by

$$443.1 - 1.86 \times D_1 - 2.69 \times D_2 - 3.16 \times D_3 - 3.46 \times D_4 - 3.67 \times D_5 - 3.82 \times D_6. \qquad (31)$$

Finally, the model contains heat storage for low-price periods. The storage is assumed to involve the use of a water tank, which stores heat at about 4.18 kJ/kg K or about $1.16\ \mathrm{kWh/m^3\,K}$. It is also assumed that the temperature range of the water is about 40 K, which means that about 46 kWh can be stored in $1\ \mathrm{m^3}$. The total storage cost is assumed to be $7000\ \mathrm{SEK/m^3}$ or about 150 SEK/kWh, according to a limited study described in Ref. 15. The storage tank is used to retain heat from 10 p.m. to 6 a.m. Eight hours are indicated for each workday, while Saturdays and Sundays are viewed as one uninterrupted low-price period. The workdays will thus comprise the period that determines the size of the storage tank. January has 21 workdays, i.e., 168 h are available for storing heat while 336 h could be used for the discharge. The model must take this fact into consideration, i.e.,

$$336 \times \mathrm{HSH}_1 - 168 \times \mathrm{HSL}_1 = 0, \qquad (32)$$

where HSH_1 = heat flow in MW for January high-price periods and HSL_1 = heat flow in megawatts during the January low-price periods. It is assumed that the stored heat will decrease the electric load, and Eqs. (2) and (3) must be completed on the left-hand side with the following expressions, respectively,

$$+\mathrm{HSH}_1 \times 336, \qquad (33)$$

$$-\mathrm{HSL}_1 \times 168. \qquad (34)$$

The model must also include a statement showing the maximum energy storage for any month, i.e.,

$$-\mathrm{HSL}_1 \times 168 + \mathrm{HSMAX} \geqslant 0, \qquad (35)$$

where HSMAX = maximum energy storage in megawatt hours for any month. Equation (8) is influenced and the HSH_1 variable must be added to the left-hand side of this statement. The objective function must include the cost for the storage, i.e., HSMAX $\times$ 150,000 $\times 10^{-6}$ divided by the number of days in operation, all expressed in MSEK. The expression completes the model, which now contains 220 variables and 211 constraints.

Optimizing the model shows that not a single retrofit action intended to conserve electricity or heat proves to be profitable. Nor is it profitable to generate more electricity from a new gas turbine or to store heat. The outcome of the optimization is the strategy shown in Table 7. However, the model can be slightly changed to force it to choose such times as the attic floor insulation retrofit. This can be done by deleting the $\leqslant$ sign in the expression (22) which means that one of the A variables must be set equal to 1. Optimizing this new situation shows that the value of the objective function is increased by about 747 MSEK (from 11,877 to 12,625) and the smallest amount of additional insulation is used (0.05 m). Reference 15 describes a closer study of the different costs for the insulation, and it is shown that the insulation will not be profitable enough to be included in an optimal solution until the cost is decreased by about 80%. The originally assumed U-value was rather low, 0.5 W/m^{2}K, meaning that profitable additional insulation is very hard to achieve. A higher U-value in the first fictitious building, say 1.0 W/m^{2}K, probably would have changed the result. This is also the reason for not replacing the windows since the U-value change is only from 2.5 to 2.0 W/m^{2}K and the savings from this change cannot compete with the replacement cost. It might be possible to change the heating system in an existing building to get a lower LCC. In this case, however, the building is heated via the district-heating network using waste heat derived from the generation of electric power, incinerated garbage and industrial waste heat. In other words, the heat has a very low cost compared to the competing heating systems which are available. Savings related to electricity show a much better profit. Deleting the $\leqslant$ sign in Eq. (30) forces the model to choose extra insulation; the optimal situation is to set $D_2 = 1$, which means that 0.1 m of additional insulation should be installed. The value of the objective function will now increase by 49 MSEK, which gives a much lower over-all figure than when additional insulation is used to reduce the needed supply from the district-heating network. If the demand charges in the electricity rate are increased, the gas turbine, heat storage, and conservation retrofits will all become more profitable. The conservation measures will not emerge as optimal until the cost for the gas turbine exceeds 4200 SEK/kW. Some experiments with the model show that the heat storage will be optimal if the cost for it is decreased by about 50%. A higher electricity tariff will lead to increased production in the CHP plant and further use of the gas turbine, before optimal heat-storage implementation is achieved.

CONCLUSIONS

This paper shows that it is possible to build a mathematical model of a municipal energy system which includes both new and existing production units, CHP plants, district-heating equipment, and gas turbines, as well as factors such as attic-floor insulation, window replacement, and heat-storage facilities. The model is designed as a mixed-integer program, which includes linear expressions containing both ordinary and binary variables. The binary variables are used to solve expressions which were originally nonlinear. When the model is optimized, we find that, on the basis of currently valid Swedish prices for heat, electricity, operating equipment, and energy-conservation measures, none of the energy-conservation measures tested in the model was found to be included in an optimal solution. Instead, it is better to produce more heat or more electricity in the plants already in operation. When energy needs could not be covered by more production, purchase from the energy market was optimal.

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