APPENDIX I

OPTIMIZATION OF BIVALENT OIL-BOILER - OUTSIDE AIR HEAT PUMP SYSTEMS WHILE ALSO CONSIDERING ENERGY CONSERVING RETROFITS

In this appendix it is shown how the optimization is elaborated for a bivalent heating system and energy conserving retrofits. The case study, see page 43, in the main thesis is used as an example. The total data file is shown in appendix II, page 123. The optimization procedure is shown here in detail and thus it might be somewhat tedious to read. That is the reason for presenting it as an appendix.

AI.1 DURATION GRAPH

The calculations start with the construction of a duration graph for the existing building, considering the climate for the building site. In the OPERA model the climate is depicted by monthly mean temperatures. These are used for calculating the heat consumption in the building during one year. However, the heating system must also be able to provide enough heat during very cold winter nights and a lowest dimensioning outside temperature is used to ensure that the installed heating equipment power is sufficient. Unfortunately it is not very easy to construct a mathematical expression, suitable for optimization calculations, using these monthly mean values, and thus they are approximated with a straight line function, elaborated with the method of least squares. This function can be shown as:

$$\Delta T = - 0.0025089 \cdot \tau + 22.991 \qquad (\text{ AI } 1 \text{ })$$

where ΔT = the difference between the desireable inside temperature (20 °C) and the monthly mean outside temperature, and

τ = the duration in hours.

Setting Δ T to 0.0 implies that τ = 9164 hours, and setting τ to 0.0 will make Δ T equal to 22.99 °C. The total amount of degree hours during one year will then become:

$$DH = \frac{9164 \cdot 22.99}{2} = 105\ 340$$

This figure should be compared to the "real one", i e 105 241 and, as can be seen, the expression (AI 1) is a very good approximation.

The situation is depicted in figure AI 1, which also can be found in [29].

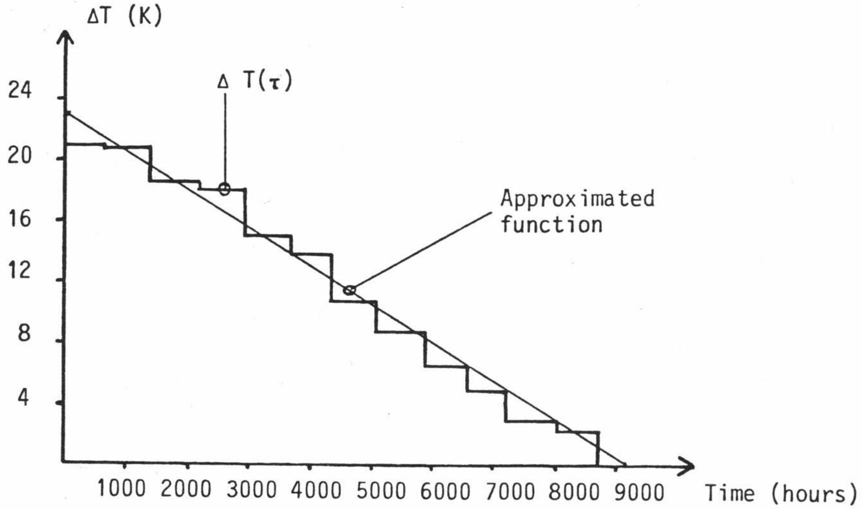

Figure AI 1. Duration graph. Monthly mean temperatures.

In this case study the existing building can thermally be described by an expression showing the thermal losses, i e TRANS + VENT from the equations (AIII 4) and (AIII 5) at page 148. This expression is evaluated to 2.291 kW/K, see table II, page 54, in the main part of the thesis. When multiplying this figure with the maximum temperature difference it is found that the thermal load in the duration graph for energy calculations equals 52.67 kW. The total energy demand during one year will then be:

$$\mathrm{TOD} = \frac{52.67 \cdot 9164}{2} = 241\ 334 \text{ kWh}$$

However, there is also free energy from solar gains and appliances that has to be considered. In table V in the main part, page 59, the figures are presented. In table V the monthly thermal losses are shown as well.

During the summer, i e when the heating equipment is turned off, this valuable free energy equals:

$$55.2 + 29.7 + 17.2 + 20.3 + 38.8 = 161.2 \text{ MJ}$$

or 44 711 kWh. This "summer energy" must be depicted in the duration diagram and the duration is calculated as:

$$\tau_1 = \frac{44\ 711 \cdot 2}{P_{som}} \quad , \quad \tau_1 = \frac{9\ 164 \cdot P_{som}}{52.67} \quad ,$$

and thus $\tau_1 = 3\ 944$ hours and $P_{som} = 22.67$ kW.

The free energy during the rest of the year is also calculated by use of table V. The heat delivered from the heating system is 573.7 MJ. Excluding the domestic hot water part, will result in 321.7 MJ. The total energy loss in the building is 868.1 MJ and the free energy during the heating season can be calculated as:

$$868.1 - 321.7 - 161.2 = 385.2 \text{ MJ},$$

or 107 257 kWh. This free heat must answer to some thermal load in the duration graph and therefore it is spread out during the heating season:

$$P_{fhs} = \frac{107\ 257}{(9\ 164 - 3\ 944)} = 20.54\ \text{kW}.$$

Of course this is an approximation but it makes the calculation much easier than calculating on monthly mean values. The situation is depicted in figure AI 2.

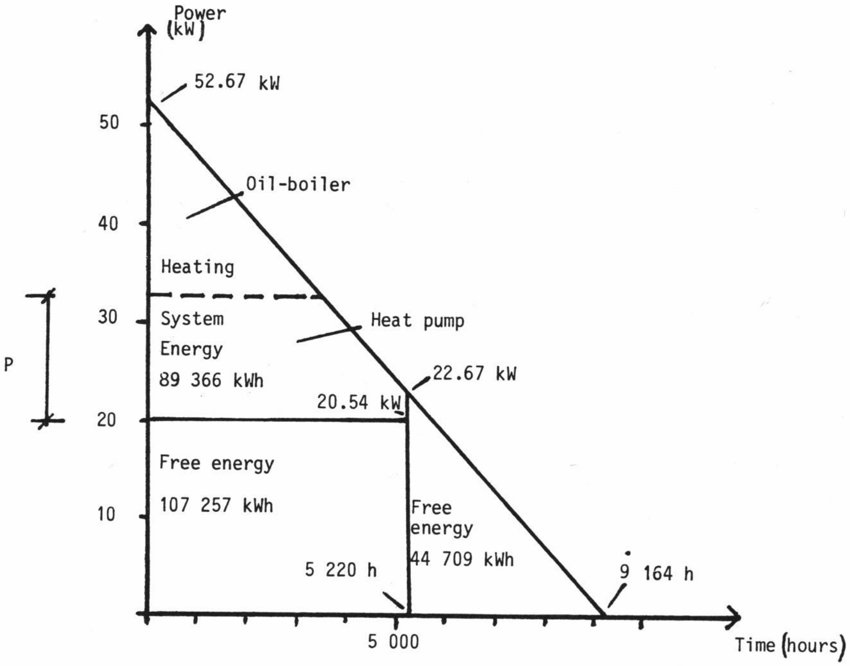

Figure AI 2. Approximated duration graph

This approximated duration graph is used for the optimization calculations. It is obvious that much of the heat consumed in the building comes from appliances etc and these values may be too high. However, in Sweden it is common to turn off the heating system during the four summer months and from the energy balance calculations in table V, page 59, the free heat could provide the building with the desireable climate during five months. This implies that the magnitude is approximately right.

Later in this appendix there are also calculations where the free gains have been reduced in order to show the influence on the optimization, see page 115.

AI.2 THE HEATING EQUIPMENT COST

Some of the heat, produced by the bivalent heating system, comes from the oil-boiler and the rest from the heat pump. In this case the oil-boiler has to provide the total thermal load during very cold winter periods, i e 77.9 kW.

In the input data chapter, page 48, it is found that oil-boilers cost 20 000 + 350 · P where P is the power for the heating system concerned. Outside air heat pumps cost 40 000 + 6 000 · P. The oil-boiler as well as the heat pump have a life span of 15 years. The heat pump must also be renovated to a cost of 10 % of the initial cost each 7.5 years. It is also assumed here that other costs when installing the oil-boiler, such as piping costs etc, cost 150 · P with a life span of 30 years. Such costs for the heat pump are assumed to be 200 P with a life span of 40 years. See appendix II for the total input file.

The total cost for the oil-boiler can be calculated as:

$$(20\,000 + 350 \cdot 77.9) \cdot (1 + 1.05^{-15} + 1.05^{-30} + 1.05^{-45} -$$

$$- \frac{2}{3}\, 1.05^{-50}) + 150 \cdot 77.9 \cdot (1 + 1.05^{-30} - \frac{1}{3} \cdot 1.05^{-50}) =$$

$$= 97\,498 \text{ SEK.}$$

The total cost for the heat pump with the power P is calculated in the same way i e:

$$(40\,000 + 6\,000 \cdot P) \cdot (1 + 0.1 \cdot 1.05^{-7.5} + 1.05^{-15} +$$

$$+ 0.1 \cdot 1.05^{-22.5} + 1.05^{-30} + \ldots\ldots = 75\,440 + 11\,575 \cdot P$$

The costs above show the present value for the heating equipment, formula (A III 2) in appendix III, page 144.

AI.3 THE ENERGY COST

The energy cost is a little more complicated to elaborate. The first thing to do is to find an expression for the oil-boiler duration time, τ_2. From equation (AI 1) it is found that:

$$\tau_2 = \frac{-\Delta T + 22.991}{0.0025089} = -398.58 \cdot \Delta T + 9\,164,$$

However, $P_1 = \Delta T \cdot (\mathrm{TRANS} + \mathrm{VENT}) = 2.291 \cdot \Delta T$ and thus

$$\tau_2 = -398.58 \cdot \frac{P_1}{2.291} + 9\,164 = -173.97 \cdot P_1 + 9\,164,$$

$$P_1 = P + 20.54 \text{ and thus } \tau_2 = -173.97 \cdot P + 5\,590,$$

The situation is depicted in figure AI 3.

Now it is suitable to evaluate the heat pump energy:

$$E_{hp} = 5\,220 \cdot P - \frac{(5\,220 - \tau_2) \cdot (P - (22.67 - 20.54))}{2} =$$

$$= 2\,610 \cdot P + 5\,559 + \frac{P}{2} \cdot \tau_2 - 1.065 \cdot \tau_2,$$

Inserting the expression above for τ_2 gives:

$$E_{hp} = -86.98 \cdot P^2 + 5\,590 \cdot P - 394,$$

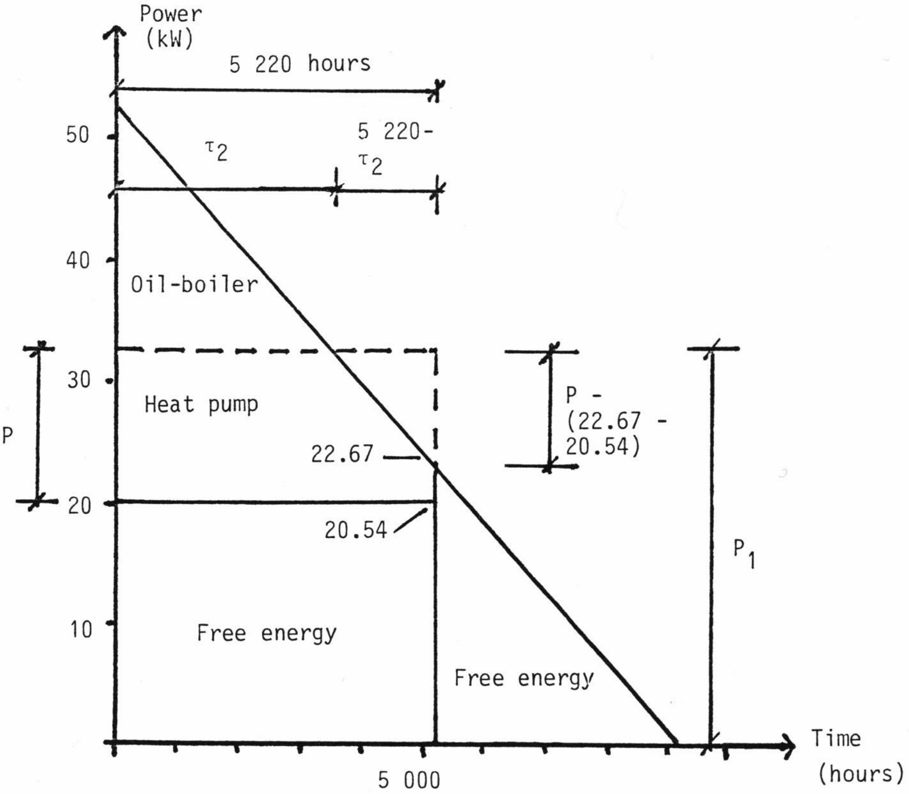

Figure AI 3. Energy evaluation graph.

The oil-boiler energy is calculated as:

$$E_{ob} = \frac{(\ 52.67 - P - 20.54\ ) \cdot \tau_2}{2} = -\ 0.5 \cdot \tau_2 \cdot P + 16.07 \cdot \tau_2,$$

and thus:

$$E_{ob} = 86.98 \cdot P^2 - 5\ 590 \cdot P + 89\ 803,$$

Now the energy cost must be calculated. The heat pump has a varying COP and accordingly a mean value during the heating season is used. Equation (4), page 29 in the main part shows the situation. For

$\Delta T = 22.991$ the COP equals 2.12 and for $\Delta T = 0$ it will become 3.24. These two values answer to 0 and 9 164 hours in the duration graph. A new COP function can be calculated i e :

$$COP = 2.11 + 0.000122 \cdot \tau$$

Implementing $\tau = 5\,220$ hours i e the end of the heating season, see figure AI 2, evaluates the COP to 2.75. The mean value will be:

$$COP_{mv} = \frac{2.11 + 2.75}{2} = 2.43$$

The electricity price is assumed to be 0.089 SEK/MJ, 0.32 SEK/kWh, and the net present value factor for annually recurring costs can be evaluated to 18.26, see expression (AIII 3) in appendix III page 144. The heat pump energy cost thus can be calculated as:

$$EC_{hp} = (-86.98 \cdot P^2 + 5\,590 \cdot P - 394) \cdot \frac{0.32 \cdot 18.26}{2.43} =$$

$$= - 209.15 \cdot P^2 + 13\,441 \cdot P - 947 \text{ SEK.}$$

The oil-boiler energy cost will be:

$$EC_{ob} = (86.98 \cdot P^2 - 5\,590 \cdot P + 89\,803) \cdot \frac{0.18 \cdot 18.26}{0.8} =$$

$$= 357.36 \cdot P^2 - 22\,966 \cdot P + 368\,956 \text{ SEK.}$$

AI.4 TOTAL COST AND OPTIMIZATION

Adding the cost functions, i e the oil-boiler cost, the heat pump cost and the energy cost, result in :

$$LCC = 148.21 \cdot P^2 + 2\,050 \cdot P + 540\,947 \text{ SEK}$$

The minimum point for this function will emerge when:

$$P = \frac{-2\,050}{2 \cdot 148.21} = -\ 6.92 \text{ kW}$$

which of course is very disappointing as negative heat pumps do not exist. However this result means that the best thing to do is to reject the outside air heat pump. It is better to heat the building using only an oil-boiler. OPERA thus sets the heat pump power to 0 kW.

In the case above only space heating has been considered. Adding also the hot water production to the heating load, which has a duration all over the year will enhance the profitability for the heat pump. In [29] it is shown that it is optimal to let the heat pump produce all the hot water due to its 100 % duration.

OPERA calculates the total LCC for this new system, setting the heat pump thermal load equal to the hot water production thermal load. The operator however, will be informed that this situation has emerged in order to avoid mistakes.

AI.5 CHANGING THE AMOUNT OF FREE ENERGY

In the example presented above the optimization leads to an impossible result, a negative heat pump. This is because of the low thermal demand and a high heating equipment cost. Above was also mentioned that the free energy from appliances etc might be overestimated. It could be interesting to show what will happen if the free energy is decreased. This has been done below , estimating that the monthly free gains is 0 kWh. The only free energy in the building comes from the solar radiation. A new duration graph has to be presented, figure AI 4.

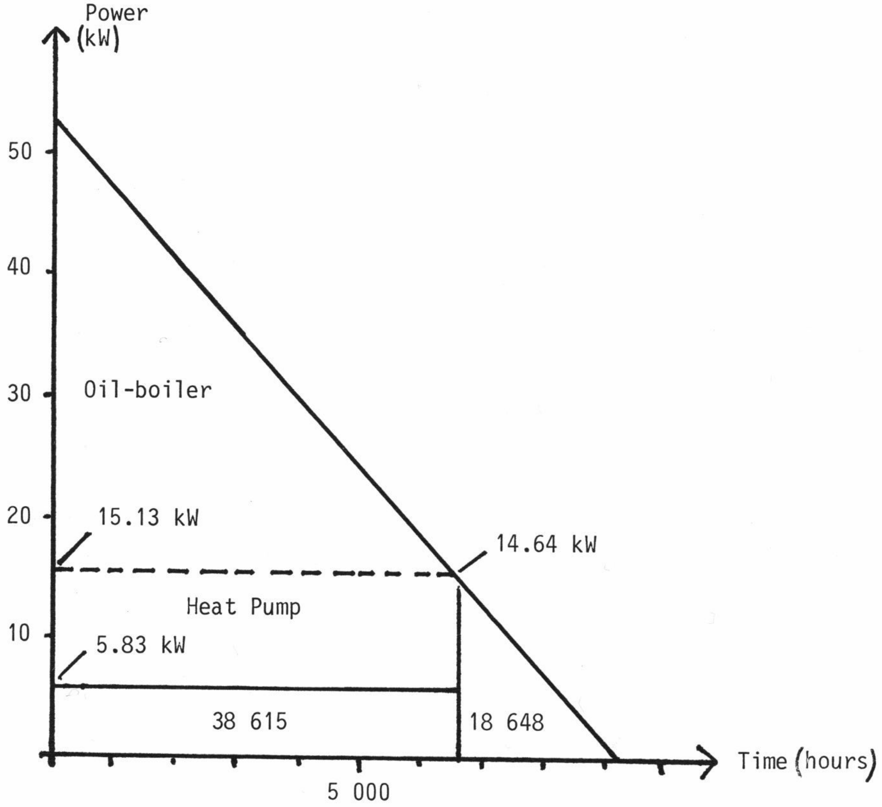

Figure AI 4. Duration graph. Free energy equals solar gains.

This new situation leads to a more encouraging result. The heat pump size ought to be 9.3 kW if only space heating is considered and 17 kW if the hot water thermal load is added. It shall be emphasized here that the COP for the hot water production is calculated for the 100 % duration.

AI.6 ADDING INSULATION TO THE ENVELOPE

If extra insulation is implemented on e g the attic floor, it may be possible to get a lower LCC due to a lower energy bill or cheaper heating equipment. The case with a low amount of free energy led to a successful optimization and that example is used for showing how the optimization is elaborated, considering also an insulation measure i e the attic floor. The basic ideas are identical to the ones used in [29], but here the complexity is larger due to energy balance calculations and free energy considerations. As earlier, the procedure starts with the heating equipment costs.

Adding more insulation to the attic floor will decrease the thermal load for the building and subsequently a smaller oil-boiler can be used. In [3] it is shown that the new U - value can be expressed as:

$$U_{new} = \frac{k_{new} \cdot U_{ex}}{k_{new} + U_{ex} \cdot t}$$

where U_{new} = the new U - value
k_{new} = the conductivity of the new insulation
U_{ex} = the existing U - value
t = the thickness of the extra insulation

The new thermal loss for the building now has to be recalculated and this is done by subtracting the loss through the original attic floor and adding the new loss using the expression above. Mentioned above in the input data chapter, page 44, are the original U - value, i e 0.8 $W/m^2 \cdot K$ and the attic floor area equalling 396 m^2. The total TRANS + VENT factor in the building has been calculated to 2.291 kW/K and the new situation will result in:

$$2\ 291 - 0.8 \cdot 396 + \frac{0.04 \cdot 0.8 \cdot 396}{0.04 + 0.8 \cdot t} = 1\ 974 + \frac{12.67}{0.04 + 0.8 \cdot t}$$

The oil-boiler cost will be:

$$(20\ 000 + 350 \cdot (1.974 \cdot 34 + \frac{0.01267 \cdot 34}{0.04 + 0.8 \cdot t})) \cdot 1.7655 +$$

$$+ 150 \cdot (1.974 \cdot 34 + \frac{0.01267 \cdot 34}{0.04 + 0.8 \cdot t}) \cdot 1.201 =$$

$$= 88\ 873 + \frac{343.8}{0.04 + 0.8 \cdot t} \quad \text{SEK}$$

The heat pump cost will be the same as before or:

$$75\ 440 + 11\ 575 \cdot P$$

In order to evaluate the heat pump energy the duration for the oil-boiler must be expressed. Above it is shown, page 112, that:

$$P_1 = \Delta\, T \cdot (\ \mathrm{TRANS} + \mathrm{VENT}\ )$$

In this case (TRANS + VENT) is not a constant but a function of t, i e the insulation thickness. The expression is shown above and thus:

$$P_1 = \Delta\ T \cdot \left(\ 1.974 + \frac{0.01267}{0.04 + 0.8 \cdot t}\ \right) \text{ and}$$

$$\tau_2 = -398.58 \cdot \frac{P_1}{\left(\ 1.974 + \dfrac{0.01267}{0.04 + 0.8 \cdot t}\right)} + 9\ 164 \text{ hours}$$

In this case where the insulation optimization is emphasized $P_1 = P$, see page 18, and this expression can be simplified to:

$$\tau_2 = - \frac{15.94 \cdot P - 839.69 + 318.86 \cdot P \cdot t - 14\ 471 \cdot t}{(\ 0.09163 + 1.5792 \cdot t\ )}$$

In the first case studied above, page 109, the heating season period could actually be calculated by the energy balance subroutine. Here however the heating season is a function of t. Fortunately the heating season will not change very much for a single retrofit why an approximation can be used. In [14] it is shown that rather thick insulations, i e about 0.2 meter, often can be found optimal, and such an insulation will result in a very low thermal flow through the attic floor. OPERA thus calculates the heating season for the building as if no heat at all was transferred through this asset. In this case the heating season is calculated to 6 616 hours. The energy produced by the heat pump now can be approximated as:

$$E_{hp} = 6\ 616 \cdot P - \frac{(\ 6\ 616 - \tau_2\ ) \cdot (\ P - 12.61\ )}{2} =$$

$$= 3\ 308 \cdot P + 41\ 713 + 0.5 \cdot P \cdot \tau_2 - 6.31 \cdot \tau_2$$

Implementing the expression for τ_2 gives:

$$E_{hp} = 3\,308 \cdot P + 41\,713 - $$

$$- \frac{7.97 \cdot P^2 + 159.43 \cdot P^2 \cdot t - 9\,245 \cdot P \cdot t - 520.34 \cdot P + 91\,239 \cdot t + 5\,294.5}{0.09163 + 1.5792 \cdot t}$$

The oil-boiler energy will become:

$$E_{ob} = \frac{(\ 22.991 \cdot (\ 1.974 + \frac{0.01267}{0.04 + 0.8 \cdot t}\ ) \cdot 9\,164}{2} -$$

$$- 16\,068 - E_{hp} = 191\,882 + \frac{1\,334}{0.04 + 0.8 \cdot t} - E_{hp}$$

The constant 16 068 shows the amount of free energy during the summer if no heat at all is transferred through the attic floor. This is an approximation because the real value is a function depending on the thickness of the extra insulation.

The same approximations must be made for the COP of the heat pump, the real COP is approximated with the one calculated for the building with no thermal transport through the attic floor. The situation leads to the following expression:

$$\text{Cost}_{ob} + \text{Cost}_{hp} + \text{Energy cost}_{ob} + \text{Energy cost}_{hp} + \text{Cost}_{ins} = \text{Total cost}$$

From the expressions above it is achieved:

$$\text{Total cost} = 878\,958 + 5\,731 \cdot P + \frac{5\,824.5}{0.04 + 0.8 \cdot t} +$$

$$+ \frac{14.08P^2 + 281.7P^2t - 16\,335Pt - 919.4P + 161\,210t + 9\,354.8}{0.09163 + 1.5792 \cdot t} +$$

$$+ 49\,500 + 118\,800 \cdot t$$

Now this cost function must be derivated in order to find the minimum point.

$$f^1_P(\ t,\ P\ ) = 5\ 730 + \frac{563.4Pt + 28.16P - 919.5 - 16\ 336t}{(\ 0.09163 + 1.5792 \cdot t\ )}$$

$$f^1_t(\ t,\ P\ ) = - \frac{4\ 659.6}{(\ 0.04 + 0.8 \cdot t\ )^2} + \frac{3.57P^2 - 43.7P - 3}{(\ 0.09163 + 1.5792 \cdot t\ )^2} + 118\ 800$$

These two derivatives shall equal 0 simultaneously for the minimum point. However it is not very easy to actually calculate this point. OPERA has thus been provided with an iterative process that tests the result for a number of alternatives for t and P. This process results in a heat pump power equalling 13.26 kW and an extra insulation of 0.197 meter. The LCC field is shown in figure AI 5. See also figure 5, at page 27, for a graphic presentation. Note however, that figure 5 is elaborated from a slightly different mathematical expression.

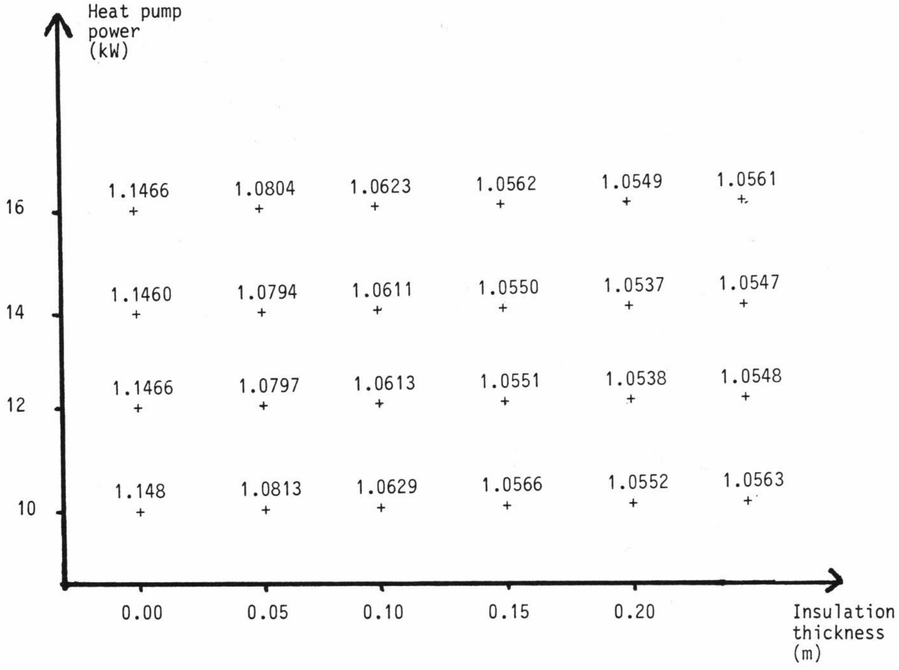

Figure AI 5. LCC field for insulation and heating system optimization.

It must be observed that it is not correct to implement the values for P and t above and calculate the total LCC. This is due to the free energy consideration see page 118. Thus the insulation thickness value is implemented in the (TRANS + VENT) equation and the optimization starts once again now for the building including its insulated attic floor. The process is shown in the beginning of this appendix and is not repeated here. However it results in a heat pump power equalling 7.63 kW if the hot water load is excluded.

AI.7 EXHAUST AIR HEAT PUMPS

An exhaust air heat pump can be used to take care of the heat in the ventilation air. In this case when there already is a heat pump in the heating system it is most important to consider the duration graph when calculating the new LCC, see figure AI 6.

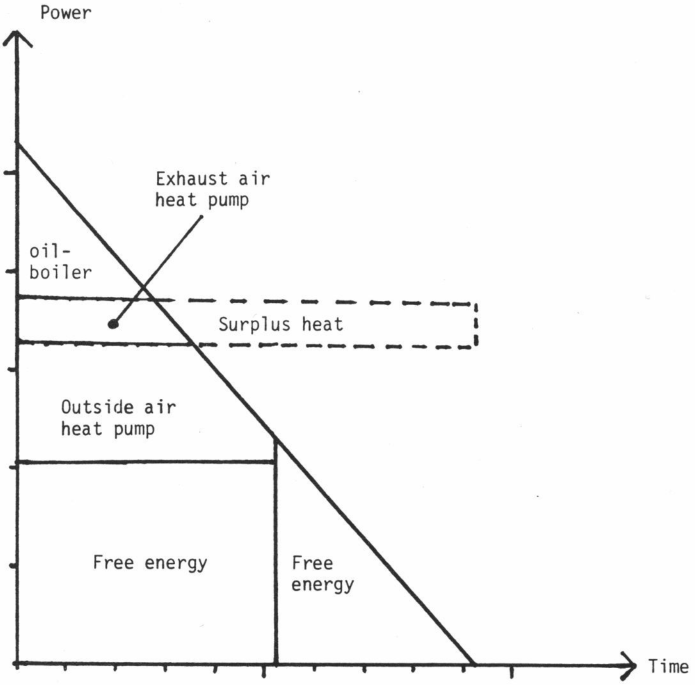

Figure AI 6. Exhaust air heat pump and heating system duration graph.

In OPERA the space heating is default and the exhaust air heat pump delivers heat for the hot water production only when there is a heat surplus from the space heating. However, the hot water is produced with the ordinary heating system and thus the profitability of an exhaust air heat pump will mostly vanish. OPERA tests if the device is profitable or not using the process described in the beginning of this appendix and thus only the duration graph will be presented here.