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19

Insulation and Bivalent Heating System Optimization: Residential Housing Retrofits and Time-of-Use Tariffs for Electricity

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ABSTRACT

Time-of-use tariffs, which reflect the cost of producing one extra unit of electricity, will be more common in the future. In Sweden the electricity unit price will be high during the winter and cheaper during the summer. A bivalent heating system, where an oil-fired boiler takes care of the peak load, when the electricity price is high, and a heat pump the base load, may decrease the cost of space heating substantially. However, insulation retrofits are also likely to reduce the peak space-heating load in a building. This paper shows how a bivalent heating system can be optimized while also considering the insulation measures. The optimization is elaborated by the use of a mixed integer programming model and the result is compared with a derivative optimization method used in the OPERA (optimal energy retrofit advisory) model. Both models use the life-cycle cost (LCC) as a ranking criterion, i.e. when the lowest LCC for the building is achieved, no better retrofit combination exists for the remaining life of the building.

INTRODUCTION

Since 1985 a research project has run, financed by the Swedish Council for Building Research and the municipality of Malmö, Sweden, to develop a method of building retrofit optimization. That is, how should an existing building be retrofitted in order to achieve the best possible combination of the retrofits? This issue is traditionally separated into three different subjects: retrofitting of buildings, life-cycle costing and optimization.

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Very little published work treats the three subjects simultaneously.[1] Nevertheless, at a CIB conference[2] (Conseil International du Batiment pour la Récherche l'Étude et la Documentation) in 1984, one paper was presented about LCC minimization and retrofitting. A single-family house and some plausible retrofits were considered, but the paper treated only building retrofits and not the heating system. No real optimization has thus been achieved.

There are two computer models, CIRA[3] and MSA,[4] that rank different retrofits according to their saving-to-cost ratio. However, they deal only with the thermal envelope and not with the heating system. One author[5] deals with insulation optimization in a similar way as in this paper. However, only insulation is dealt with and not the building as a whole. Kirkpatrick and Winn[6] use traditional optimization methods, but deal with only a few retrofits and only one heating system. The same is true of Bagatin *et al.*[7] In their paper, thermal envelope retrofits are optimized and they find that the existing U value is of no importance to the new optimal one. However, they do not consider the influence of the remaining life of the building asset. Also, only the heating system is dealt with.

As can be found from this brief review, no model has so far been developed that will optimize the total energy system of a building. Therefore this research project resulted in the OPERA model (OPtimal Energy Retrofit Advisory), which enabled the user to study a unique multi-family building with several possible building, ventilation and heating equipment retrofits. The consequent predictions have shown that bivalent systems were often the best solution combined with some cheap envelope and ventilation retrofits, such as attic floor insulation and weatherstripping. The bivalent heating system, in this case an oil-fired boiler combined with a groundwater-coupled heat pump, however, had to be optimized simultaneously with the insulation thickness optimization. The optimal sizes of the oil-fired boiler and the heat pump did depend on the level of insulation in the building, or more accurately on the thermal load imposed on the building.

In Sweden, as in many other countries, the electricity tariff depends on the time of year. During peak conditions for the utility, the unit electricity price is set high, whilst during off-peak conditions the unit price is low. The OPERA optimization, which uses a derivative method in which continuous functions are supposed to emerge, deals with this situation by calculating a normalized price for 1 year. Using this normalized price implies that the utility receives the same income from consumers for identical thermal loads, and so the levels of the rates are identical. However, this approximation might lead to a false optimization.[8] The optimal size of the heat pump differed by approximately 5% when the results of the two methods were compared with each other.

The OPERA model is dealt with in detail elsewhere[1,9,10] and thus only a brief review is presented here in order to show how the model works. A more detailed description of the model and a pertinent case study will be presented elsewhere.[11]

Adding insulation to an attic floor, for example, will decrease the U value and thus also decrease the heat flow through it. The new U value may be expressed as

$$U_{new} = \frac{k_{new} U_{exi}}{k_{new} + U_{exi} t} \tag{1}$$

where U_{new} is the new U value in W/m² K, k_{new} the conductivity for the new insulation in W/m K, U_{exi} the existing U value in W/m² K, and t the thickness of the extra insulation in metres.

The cost for the new insulation is expressed as

$$C_{ins} = A + Bt \tag{2}$$

where C_{ins} is the cost for extra insulation in SEK/m², A the initial cost in SEK/m², and B the direct insulation cost in SEK/m² m. (1 US$ ≡ 6 SEK.)

The cost for new heating equipment is expressed in the same way, but with P as a variable showing the thermal power of the equipment. These expressions, however, must be evaluated as present values, i.e. costs for future changes of the equipment are to be transferred to a base year. This is also the situation for the operating cost. Adding more insulation to the attic floor will decrease the need for heat in the building and subsequently decrease the cost for both heating and heating equipment acquisition. The decrease will occur in the future and thus present value calculations are necessary. The methods for doing this are presented in detail by Ruegg and Petersen.[12] Adding all these costs provides the operator with an expression which shows the LCC for the building and its possible retrofit measures. Gustafsson and Karlsson[9] show that the cost may be expressed as

$$\mathrm{LCC} = C_1 + \frac{C_2}{C_3 + C_4 t} + C_5 P_{hp} + \frac{C_6 P_{hp}^2}{C_7 + C_8 t} + \frac{C_9 P_{hp}^2 t}{C_7 + C_8 t} + C_{10} t \tag{3}$$

where P_{hp} is the thermal power for the heat pump in kilowatts, and $C_{1,2}$ are the different constants indicating insulation costs, heating equipment costs, energy costs, etc.

The optimal conditions are achieved when the derivatives with respect to P_{hp} and t equal zero simultaneously. However, this problem is not easily solved in a strictly analytical way and thus OPERA is provided with a numerical optimization process, which examines the derivatives for different values of P_{hp} and t. When the derivatives are close enough to zero, the process is terminated.

The derivative method works well as long as the LCC is made up of continuous functions. When a time-of-use tariff for electricity is introduced, this is no longer the situation because the tariff is designed in discrete steps. Another method has to be chosen if a bivalent system is to be optimized. Here linear programming is considered. The procedure starts with an objective function which is to be minimized. This function expresses the total LCC for the building. The mathematical model, or the linear program, also contains a set of constraints which show the ranges within which the values of the variables are to be located. However, the model must be linear, which is a major disadvantage with the method. All non-linear functions in the program must thus be approximated by means of linear pieces in order to solve the problem. It is not appropriate in a paper of this kind to show how to solve linear programming problems; the methods are described in detail elsewhere.[13] Instead this paper will emphasize how the mathematical model is designed, using a case study from Malmö, Sweden.

CASE STUDY

The building under consideration is located in the block Ansgarius in Malmö, Sweden. The building envelope is in a poor condition and renovation is necessary. The building has thus been the subject of an extensive analysis by means of the OPERA model, which is now used by the municipality. In this paper, however, only a part of the OPERA calculations are shown and they are also simplified to highlight the use of the two different optimization methods shown here. The OPERA model showed that the best retrofit strategy was to change the original oil-fired heating equipment to a bivalent oil-fired boiler–heat pump system and to combine this with attic floor insulation. The calculations are now presented.

Heating equipment costs

Information from contractors in Malmö showed that the oil-fired boiler cost, C_{ob}, in SEK could be expressed as

$$C_{ob} = 55\,000 + 60P_{oil}$$

where P_{oil} is the thermal power of the oil-fired boiler in kilowatts.

The economic life of the boiler has been assumed to be 15 years. There is also a cost for installation, here assumed to be $200P_{oil}$ SEK, which has an economic life of 50 years. For a project life of 50 years and a real discount rate of 5%, the LCC in SEK for the oil-fired boiler equipment is

$$\mathrm{LCC}_{ob} = 97\,000 + 305{\cdot}93P_{oil} \tag{4}$$

The heat pump has an acquisition and installation cost in SEK of

$$C_{hp} = 60\,000 + 5\,000P_{hp}$$

and the cost for heat pump retrofits etc. was assumed to be $1500P_{hp}$ SEK. The acquisition cost occurs only once during 50 years, while the retrofit cost emerges each 10 years. The LCC in SEK for the heat pump will thus be

$$LCC_{hp} = 60\,000 + 8546{\cdot}34P_{hp} \qquad (5)$$

The two expressions (4) and (5) are calculated by use of the net present value method.[12]

Operating costs

The total transmission loss has been calculated to be 4·780 kW/K, including losses from the ventilation system. The peak load in the building is 167 kW, according to the Swedish building code.

Climatic conditions
In the OPERA model, the climate is described by monthly mean temperatures for different sites in Sweden. This is also suitable for the other optimization method and the need for heat is shown in Table 1.

The electricity tariff
The time-of-use electricity tariff introduced in Malmö is designed as shown in Table 2. The prices include taxation at the rate of 0·072 SEK/kW h.

TABLE 1
Climatic Conditions in Malmö, Sweden for the Ansgarius Building

Month	*Peak load (kW)*	*Monthly heat loss (kW h)*
January	102·8	76 460
February	103·7	70 326
March	93·7	69 704
April	71·7	51 624
May	47·8	35 563
June	28·7	20 650
July	18·2	13 514
August	20·6	15 292
September	35·9	25 812
October	57·8	43 031
November	77·0	55 409
December	90·8	67 570

TABLE 2
The Malmö Time-of-Use Electricity Tariff

Fixed fee (SEK)	5 000
Subscription fee and power fee (SEK/kW)	230
Energy fee (SEK/kW h)	
November to March	
Monday to Friday, 06.00–22.00	0·392
Other times	0·252
April, September and October	
Monday to Friday, 06.00–22.00	0·252
Other times	0·222
May–August	
Monday to Friday, 06.00–22.00	0·222
Other times	0·187

However, in the models the tariff must correspond to the energy need in Table 1 and thus the energy fee is recalculated as

Energy fee (SEK/kW h)	
November to March	0·314
April, September and October	0·236
May to August	0·204

In the OPERA calculations, these values are normalized to a unit price valid for all the year. The price must result in the same income to the utility and thus the total energy cost, for 1 year is calculated using the values in Table 1 and the tariff above. Dividing this cost by the total amount of energy consumed during 1 year yields the normalized unit price of 0·28 SEK/kW h.

Optimization with the OPERA model

We have shown elsewhere[1,9] how OPERA evaluates the energy cost for the bivalent system and simultaneously considers the influence of attic floor insulation. The method is not repeated here, but the LCC functions have been evaluated as

$$LCC_{ob} = 143\,383 + \frac{196{\cdot}33}{0{\cdot}04 + 0{\cdot}8t} - 305{\cdot}93P_{hp} \tag{6}$$

$$LCC_{hp} = 60\,000 + 8546{\cdot}34P_{hp} \tag{7}$$

$$LCC_{fee} = 1399{\cdot}9P_{hp} \tag{8}$$

$$LCC_{fix} = 91\,300\,\text{SEK} \tag{9}$$

$$\mathrm{LCC}_{\mathrm{ehp}} = 16\,265 P_{\mathrm{hp}} - \frac{13{\cdot}55 P_{\mathrm{hp}}^2 + 271{\cdot}19 P_{\mathrm{hp}}^2 t}{0{\cdot}1912 + 3{\cdot}457 t} \tag{10}$$

$$\mathrm{LCC}_{\mathrm{eob}} = 2\,653\,797 + \frac{11\,262}{0{\cdot}04 + 0{\cdot}8t} - 51\,194 P_{\mathrm{hp}} + \frac{42{\cdot}66 P_{\mathrm{hp}}^2 + 853{\cdot}6 P_{\mathrm{hp}}^2 t}{0{\cdot}1912 + 3{\cdot}457 t} \tag{11}$$

$$\mathrm{LCC}_{\mathrm{ins}} = 71\,625 + 171\,900 t \tag{12}$$

The indices are explained below.

The sum of the functions described by equations (6) to (12) is now calculated and the derivatives with respect to P_{hp} and t are calculated and set to 0. The solution from the minimization is:

Thermal power of the heat pump	77 kW
Thermal power of the oil-fired boiler	78 kW
Heat from the oil-fired boiler	38 000 kW h/year
Heat from the heat pump	468 900 kW h/year
Oil-fired boiler cost, present value, $\mathrm{LCC}_{\mathrm{ob}}$	120 900 SEK
Heat pump cost, present value, $\mathrm{LCC}_{\mathrm{hp}}$	718 100 SEK
Power fee cost, present value, $\mathrm{LCC}_{\mathrm{fee}}$	107 800 SEK
Fixed fee cost, present value, $\mathrm{LCC}_{\mathrm{fix}}$	91 300 SEK
Energy cost, heat pump, present value, $\mathrm{LCC}_{\mathrm{ehp}}$	797 900 SEK
Energy cost, oil-fired boiler, present value, $\mathrm{LCC}_{\mathrm{eob}}$	203 800 SEK
Insulation cost, 0·18 m mineral wool, $\mathrm{LCC}_{\mathrm{ins}}$	102 600 SEK

Adding the above costs together results in a total LCC of 2 142 400 SEK.

Mixed-integer programming optimization

When the linear programming method is used, it is necessary to describe the energy cost, as well as the other costs, in the objective function. This is done by calculating the energy cost month by month and adding the costs together. In Fig. 1, the monthly thermal losses are shown if no attic floor insulation is implemented in the building.

The total energy cost for January will thus be

$$\mathrm{EC}_{\mathrm{Jan}} = (P_{\mathrm{hp}} T_{\mathrm{Jan}} El_{\mathrm{Jan}})/Eff_{\mathrm{hp}} + (P_{\mathrm{ob}} T_{\mathrm{Jan}} Oil_{\mathrm{Jan}})/Eff_{\mathrm{oil}} \tag{13}$$

where $\mathrm{EC}_{\mathrm{Jan}}$ is the energy cost for January, T_{Jan} the number of hours in January, El_{Jan} the electricity price in January, Oil_{Jan} the oil price in January, Eff_{hp} the coefficient of performance for the heat pump, and Eff_{oil} the efficiency for the oil-fired boiler.

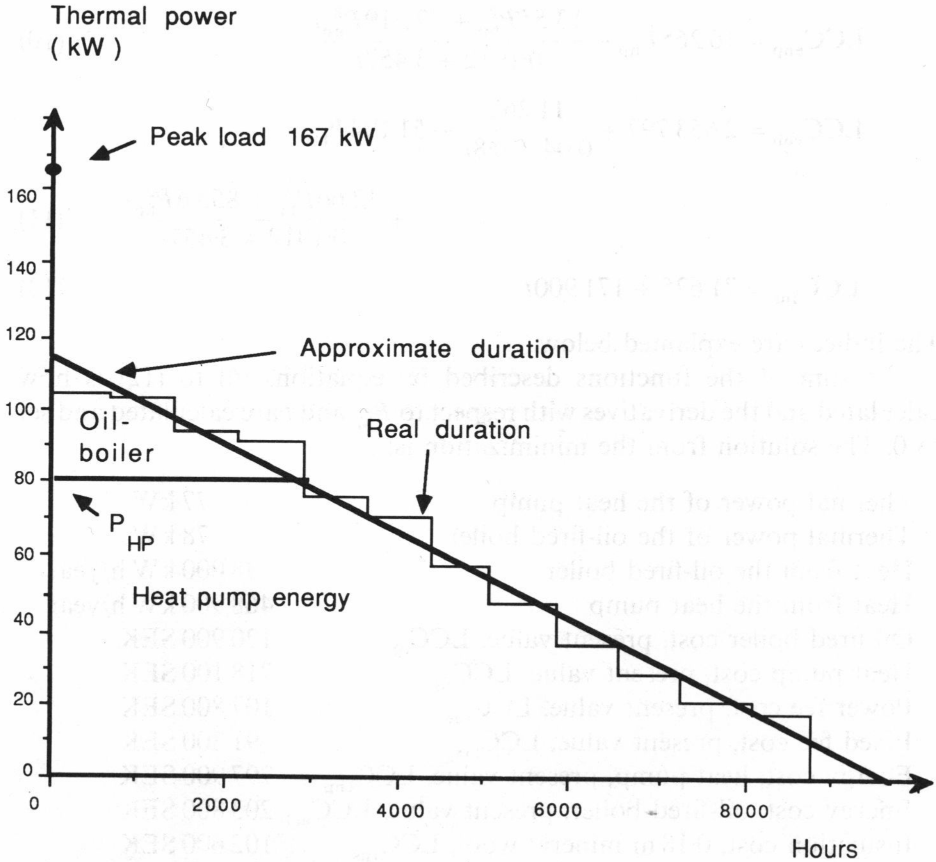

Fig. 1. Duration of imposed thermal load according to climatic conditions in Malmö, Sweden.

The total annual energy cost is calculated by adding the energy costs for each month together, i.e.

$$EC_{tot} = EC_{Jan} + EC_{Feb} + EC_{Mar} + \cdots + EC_{Dec} \tag{14}$$

The energy LCC is now easy to calculate: the total energy cost has to be multiplied by the present value factor to account for annual recurring costs.

Electricity power fee cost

The power fee and subscription fee for electricity are to be paid annually. In this case, the total fee, F_{el}, is

$$F_{el} = 230 P_{hp}/Eff_{hp} \tag{15}$$

Also this cost has to be multiplied by the applicable present value factor to achieve the total fee LCC.

Insulation cost
The cost for thermal insulation is represented in the model by equation (2). In this case study, the area of the attic floor is 573 m^2, the initial cost for insulation is 125 SEK/m^2 and the direct insulation cost is 300 SEK/m^2 m. The insulation cost occurs only once and thus no present value is required. The cost in SEK will emerge as

$$C_{\text{ins}} = 71\,625 + 171\,900t \tag{16}$$

Expression (16) is used with a discrete function which shows the insulation cost for discrete amounts of insulation. The reason for this is that the decrease in energy cost is not a linear function of the insulation thickness.

Piecewise linearization
In Fig. 2, the U value is shown as a function of the attic floor insulation thickness. The energy cost, and hence the total LCC are not linear functions

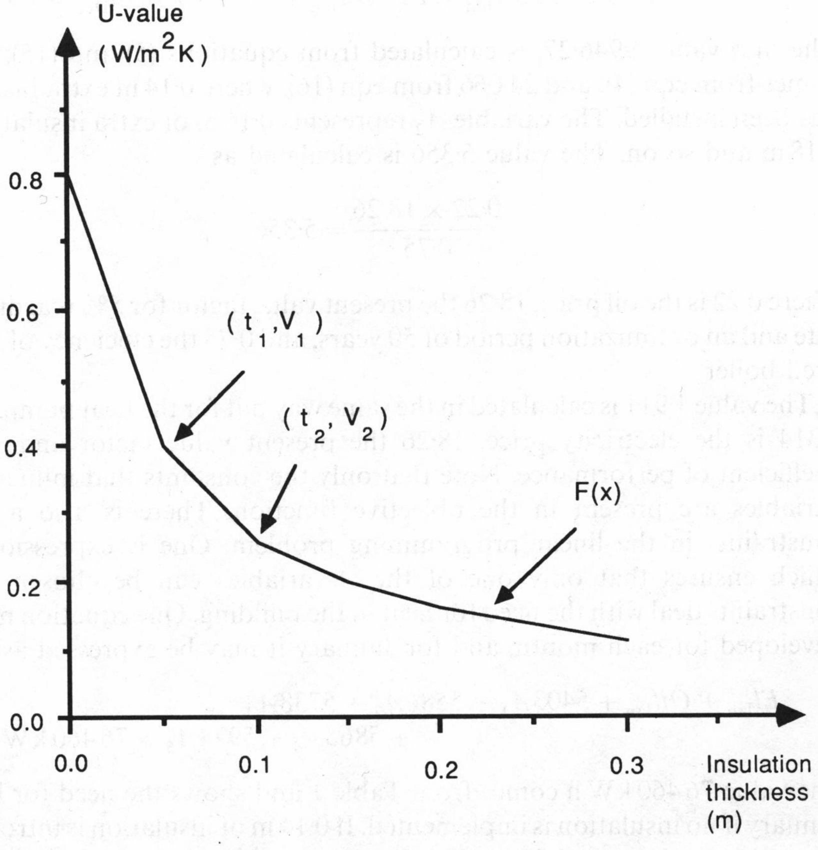

Fig. 2. *U* value as a function of insulation thickness.

of t. In this case, a method from Foulds[13] is used in which the non-linear expression is substituted by a linear one of the following type

$$A_1V_1 + A_2V_2 + A_3V_3 + \cdots = F(x) \tag{17}$$

where $A_{1,2,3...}$ are binary integers, 0 or 1, and $V_{1,2...}$ are the discrete values calculated from $F(x)$ for different values of t. The integer values must correspond to the expression

$$A_1 + A_2 + \cdots = 1 \tag{18}$$

One of the values $V_{1,2,3}$ thus has to be selected by the model. The linear expression is now a function of $A_{1,2,3}$ and not a function of t.

This also implies that the linear insulation cost function should be transferred to a function of $A_{1,2,3}\ldots$ instead of t. The beginning and end of the objective function in this case will thus be

$$9946{\cdot}27P_{\text{hp}} + 305{\cdot}93P_{\text{ob}} + 24\,066A_1 + 27\,504A_2 + 30\,942A_3 + \cdots$$
$$+ 5{\cdot}356Oil_{\text{Jan}} + 1{\cdot}911El_{\text{Jan}} + \cdots + 5{\cdot}356Oil_{\text{Dec}} + 1{\cdot}911El_{\text{Dec}}$$

The first value, 9946·27, is calculated from equations (5) and (15); 305·93 comes from eqn (4); and 24 066 from eqn (16), where 0·14 m extra insulation has been installed. The variable A_2 represents 0·16 m of extra insulation, A_3 0·18 m and so on. The value 5·356 is calculated as

$$\frac{0{\cdot}22 \times 18{\cdot}26}{0{\cdot}75} = 5{\cdot}356$$

where 0·22 is the oil price, 18·26 the present value factor for 5% real discount rate and an optimization period of 50 years, and 0·75 the efficiency of the oil-fired boiler.

The value 1·911 is calculated in the same way but for the heat pump, where 0·314 is the electricity price, 18·26 the present value factor and 3·0 the coefficient of performance. Note that only the constants that influence the variables are present in the objective function. There is also a set of constraints in the linear programming problem. One is expression (17), which ensures that only one of the A-variables can be chosen. Other constraints deal with the need for heat in the building. One equation must be developed for each month, and for January it may be expressed as

$$El_{\text{Jan}} + Oil_{\text{Jan}} + 5403A_1 + 5586A_2 + 5738A_3$$
$$+ 5865A_4 + 5974A_5 > 76\,460\ \text{kW h}$$

The value 76 460 kW h comes from Table 1 and shows the need for heat in January if no insulation is implemented. If 0·14 m of insulation is introduced, this value is decreased by 5403 kW h, i.e. A_1 equals unity. The value is calculated by the use of eqn (1), where k_{new} equals 0·04 W/m K, $U_{exi} = 0{\cdot}8\ \mathrm{W/m^2\ K}$ and $t = 0{\cdot}14$ m. Thus U_{new} will equal 0·2105 W/m² K. The original U value = 0·8 W/m² K and the decrease in U value is 0·5895 W/m² K. The area of the attic floor is 573 m² and the number of degree hours in January equals 76 460/4·78, i.e. 15 996. The decrease is thus

$$0{\cdot}5895 \times 15\,996 \times 573 = 5403\ \mathrm{kW\,h}$$

There is also one set of constraints that must be satisfied when choosing a heat pump and an oil-fired boiler with sizes large enough to deliver the required amount of heat in the building. Two expressions for each month must thus be satisfied, namely

$$P_{hp} - (El_{Jan}/T_{Jan}) > 0$$
$$P_{ob} - (Oil_{Jan}/T_{Jan}) > 0, \text{ etc.}$$

The heating equipment must be able to provide the thermal peak load of the building. This will yield the last constraint

$$P_{hp} + P_{ob} + 11{\cdot}82A_1 + 12{\cdot}22A_2 + 12{\cdot}56A_3 + 12{\cdot}84A_4 + 13{\cdot}07A_5 > 167\ \mathrm{kW}$$

The LAMPS program has been used to find the solution to the model, which is accomplished after 28 iterations. The solution is

Heat pump power	84 kW
Oil-fired boiler power	70 kW
Heat from the oil-fired boiler	18 500 kW h
Heat from the heat pump	485 500 kW h
Oil-fired boiler cost, present value	118 700 SEK
Heat pump cost, present value	777 900 SEK
Power fee, present value	117 600 SEK
Fixed fee, present value	91 300 SEK
Electricity cost, present value	822 900 SEK
Oil cost, present value	98 000 SEK
Cost of introducing 0·18 m extra insulation	102 600 SEK

The total LCC, using the linear programming optimization, will thus be 2 129 000 SEK.

OPERA versus mixed-integer programming optimization

From the above discussion, it is apparent that it is possible to use both methods to optimize a bivalent heating system and at the same time consider insulation retrofits. The mixed programming method will solve the problem with a high degree of accuracy but no severe penalties occur if the derivative method is used instead. The predicted insulation thickness will be exactly the same with the two methods while the heat pump size will be slightly smaller, by approximately 8%, if the derivative method is used. The total LCC is a little higher (by about 0·6%) if the OPERA model is used, and this difference can almost always be neglected. The optimal oil-fired boiler size is somewhat larger when the OPERA model is used: this also implies that the oil-fired boiler energy cost will be higher than the true optimal solution. This may be due partly to the approximation of the climate condition by the method of least squares. The 'real' need for heat in the building, without any insulation measures, is calculated to be 545 000 kW h while the OPERA model gives 548 200 kW h. The maximum load, when the energy need is considered, is 115 kW using the derivative method, but 103 kW is the 'real' value. This results of course in a larger oil-fired boiler when the OPERA model is used. The OPERA model, however, has some major advantages. When the problem is more complex than the one studied above, the number of variables increases significantly. The OPERA model deals with ten different heating systems and ten different building and ventilation retrofits. A linear program that solves such a big problem will be very tedious to design and it might not be possible to solve at all with small computers like IBM AT and others. When the mixed-integer problem was designed, the base for it was elaborated by an OPERA operation. It was thus possible to emphasize the work on a much smaller problem than was originally the case. One more drawback with the mixed programming method is that one has to start with a very strict mathematical problem which has to be implemented using a commercial computer program when the problem is to be solved. Furthermore it is not very easy to design the problem and afterwards interpret the solution in a language understood by a non-mathematically skilled building designer. The conclusion from this paper is therefore that the OPERA model works well for the bivalent heating system optimization when time-of-use tariffs for electricity are implemented. If very accurate predictions are desirable, the solution from OPERA must be scrutinized with a mixed-integer programming method.

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