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Energy Conservation and Optimal Retrofits in Multifamily Buildings

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Abstract *This article deals with minimizing the future Life-Cycle-Cost (LCC) of multifamily buildings. Building retrofits, installation retrofits, such as exhaust-air heat pumps, and heating system retrofits are dealt with simultaneously. It is shown that considerable energy savings, about 75%, can be achieved with no major increase of the optimal LCC. This is made possible through a mathematical function that shows the LCC for the retrofitted building. It is very flat in the region where the minimum point is located, and by adding more than optimal insulation, it is possible to achieve considerable energy savings at little cost. Uncertainties in the input data will often have a greater impact on the LCC than will increased costs for further insulation of the building envelope. A case study is presented, showing that it was optimal to replace the existing heating system with a natural gas boiler, combined with attic floor insulation. A sensitivity analysis shows that this solution was very reliable even in the face of rising prices for natural gas. If the price exceeds the defined upper limit, a district heating system will become the optimal solution. The building retrofit strategy will however not change very much; attic floor insulation is still a viable component in the optimal solution.*

Keywords retrofitting, buildings, optimization, life-cycle cost, energy conservation

Introduction

In 1985, natural gas was introduced to southern Sweden as a fuel alternative. The supply is piped from Danish reservoirs in the North Sea to a location near Malmö, Sweden's third largest city. Since Sweden is committed to a future phase-out of nuclear power, a supply of natural gas would have potential significance for the Swedish energy system.

At the Institute of Technology of Linköping University, Sweden, a mathematical model has been developed to determine the best retrofit strategy for any multifamily building. Called the OPERA model, OPtimal Energy Retrofit Advisory, it is used to minimize the remaining Life-Cycle-Cost (LCC) by viewing the building as an energy system. The LCC includes building costs, maintenance costs, and running costs for the building. Because these costs do not emerge at the same time, the present worth calculations are used to transfer them to a base year where they can be added together. A number of building retrofit options are simulated and the OPERA model selects the option with the lowest LCC. LCC is addressed in more detail in Ruegg and Petersen (1987).

The OPERA Model

The model and its associated computer program have been described in an earlier paper (Gustafsson and Karlsson 1989) and will be briefly dealt with here. Its most important features are

- The retrofit strategy is optimized, i.e., no better solution can be found.
- The model determines the magnitude of various measures.
- The interaction between different measures is taken into account.

As shown in Gustafsson and Karlsson (1989), no other model or computer program deals with all these items simultaneously. The OPERA model has other advantages as well; some are described in closer detail later. Figure 1 is a schematic view of how the model works. The procedure starts by assessing all the input data, around two hundred parameters that describe:

- Geometry of the building
- Remaining life of the building parts
- Properties of material
- Heating equipment cost
- Real discount rate
- Outdoor climate
- Thermal status
- Present heating system
- Building costs
- Project life
- Ventilation retrofit cost
- Etc.

The complete list is presented in Gustafsson (1988). The program proceeds by calculating the LCC of the building prior to retrofitting. This includes the inevitable (unavoidable) retrofits, but obsolete equipment and fixtures will be replaced by new items having the same thermal status.

Retrofits may be inevitable for reasons apart from energy conservation; for instance, it may be necessary to replace windows damaged by rot. Note that it is important to include inevitable retrofits in the calculations because their time horizons may be altered by other, energy-related retrofits.

The next step is to simulate implementation of a retrofit, such as attic floor insulation, and calculate a new LCC. If this new LCC is lower than the preceding one, the retrofit is considered to be a candidate component for an optimal solution, otherwise it is rejected. A number of retrofits are tested in this way, and the appropriate ones selected. Note that the OPERA model also determines the optimal amount of additional insulation. The method is described more closely in Gustafsson and Karlsson (1989). However the LCC for this insulation thickness should be examined more closely. Experience has shown that it is often best to leave the existing building part the way it is. Extra insulation, even if it is of optimal thickness, might increase the LCC instead of decreasing it.

The retrofits simulated at this stage are

- Attic floor insulation
- External wall insulation
- Exhaust-air heat pump
- Weatherstripping

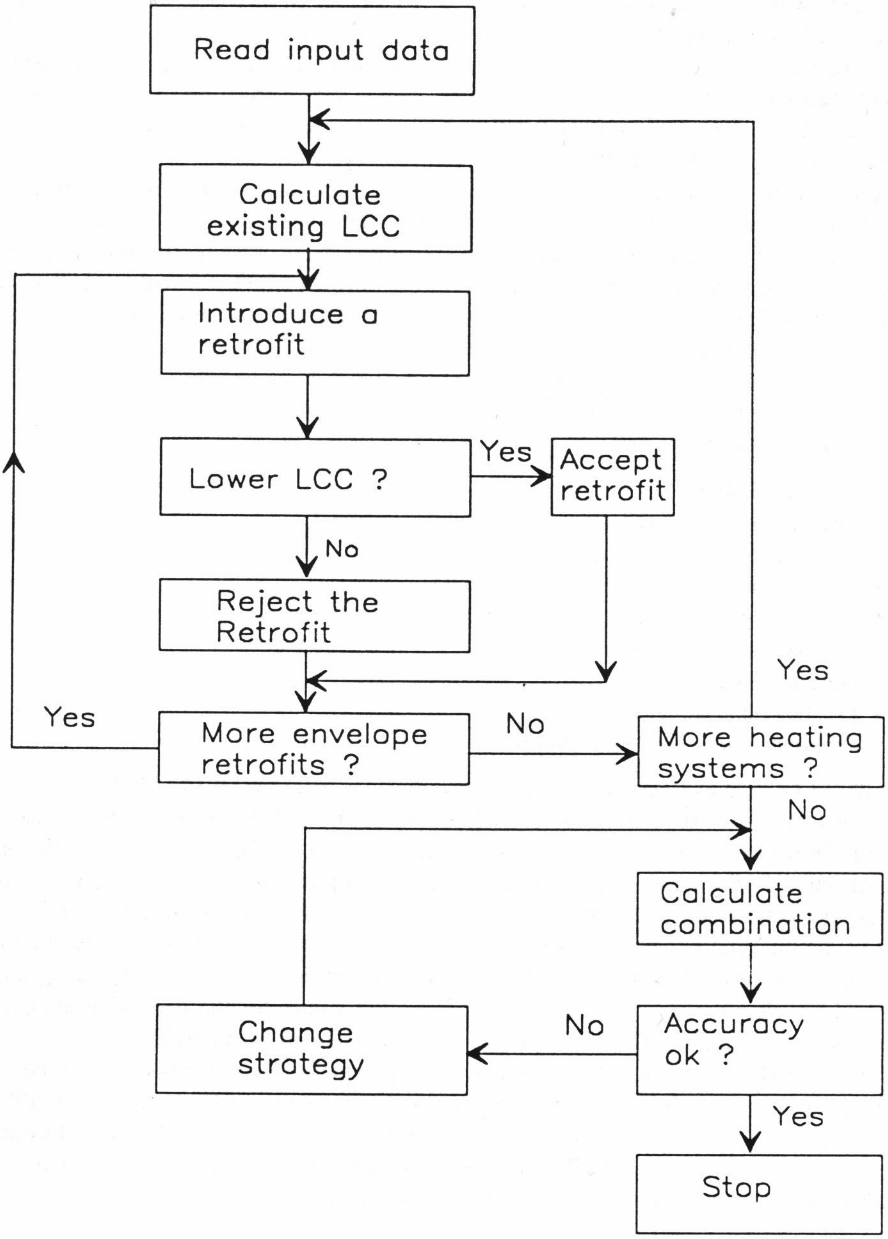

Figure 1. Principle flow chart of the OPERA model.

- Floor insulation
- Three fenestration retrofits

Certain types of retrofits are not dealt with, e.g., thermostatic valves and water heater blankets. These do not influence the building as an energy system. Thermostatic valves only provide the proper indoor temperature, which is set as input data in the program, and water heater blankets will only conserve energy as long as the heater is located outside the building. Heat flowing from the water heater will yield more incidental heat gains, and this value is submitted to the model as input data. Improving the insulation of the hot water tank will actually save energy during the seasons space heating is not required. In Sweden, however, heating cost for off-season heat is very low, so this retrofit is not included in the model.

Solar panels are seldom profitable in the Swedish climate, so this item is excluded from the model, as are exhaust-air heat exchangers. This latter type of equipment is very expensive due to difficulties in distributing warm air in old multifamily buildings. Almost all buildings that might be subjected to retrofitting are equipped with natural ventilation systems that use ducts in the chimneys for the air flow. Use of an exhaust-air heat exchanger would mean connecting it to all ducts and redistributing the heated fresh air to all apartments. This would require many new insulated ducts and considerable work.

When all the building retrofits have been tested, a number of heating system alternatives are simulated; these are

- New oil boiler
- District heating, fixed rate
- Ground-connected heat pump
- District heating, time-of-use rate (TOU)
- Bivalent, ground-connected heat pump
- Electric boiler, fixed rate
- Natural gas heating
- Electric heating, TOU rate
- Bivalent, outside-air heat pump

Each of the heating systems will have its own optimal building retrofit strategy, and the system with the lowest resulting LCC will be selected by OPERA as the most feasible solution. However, in some cases, retrofits might interact, affecting the plausible savings, so the OPERA model recalculates the LCC, implementing the candidates of the optimal retrofit strategy, and once again selects the solution with the lowest LCC. If some of the building or ventilation retrofits have a very low profitability, the interaction might be of interest. Thus it is possible to examine if the option of excluding them yields a lower LCC than the option to keep them. Different values for the insulation thickness might also be examined. The true optimal solution thus can be determined.

The OPERA model has been extensively applied in Malmö where a number of buildings are being evaluated for renovation, see Gustafsson and Karlsson (1989; 1990) for examples. A group of major Swedish contractors, the so-called Seven Builders Group, has also used the OPERA model in several case studies. One, applied to a building located in the Ansgarius block in Malmö, is discussed here.

Case Study

The building contains 34 apartments and has a total floor area of 2,047 m^2. It is in rather poor thermal condition. The building was erected in 1902 and refurbished in part in 1937. The existing thermal transmittance, or U-values, are

• attic floor, area 573 m^2,	0.8 W/m^2, K
• external wall, area 1,611 m^2,	1.2 W/m^2, K
• floorage 830 m^2,	0.5 W/m^2, K
• windows, total area 352 m^2,	2.5 W/m^2, K

Table 1
Input Data for Insulation Measures, Ansgarius, Malmö.
(US$ 1 = Skr 6).

Retrofit	A [Skr/m^2]	B [Skr/m^2]	C [Skr/m^2, m]
Attic floor	0	125	300
Floor	250	195	250
Ext. wall, out.	425	275	395
Ext. wall, in.	100	175	555

The design outdoor temperature in Malmö is defined in the Swedish building code as −14 °C (7 °F), and the parameter for the indoor temperature is set to +21 °C; this results in a total energy consumption of 367,000 kWh/year, including 100,000 kWh/year for hot water production. Incidental heat gain from occupants, appliances, etc. is set to 14,000 kWh/month. These values and the ventilation losses, 0.6 renewals/hour, result in a transmission factor of 4,780 W/K, which leads to a design thermal load of 167 kW. It is beyond the scope of this article to present complete details about the building and its retrofit costs, but we can briefly discuss some pertinent input data and some results of the OPERA operation.

The costs for additional insulation of the building assets, C_{ins} are submitted to OPERA by an equation:

$$C_{\text{ins}} = A + B + C \cdot t \tag{1}$$

where A = fixed cost for scaffolds, demolition, etc., B = fixed cost for the insulation, C = variable insulation cost, and t = insulation thickness.

OPERA must also be provided with values showing the remaining life of the actual building assets to be replaced, the new life span of each retrofitted part, and the thermal properties of each retrofit. Table 1 presents the costs for the insulation measures under consideration.

Table 2 shows the remaining life, the new life, and the thermal properties of the insulation assets.

The inner insulation of the external wall, abbreviated "in." in Tables 1 and 2, will

Table 2
Input Data for Insulation Measures, Ansgarius, Malmö.

Retrofit	Remaining Life [Years]	New Life [Years]	Thermal Conductivity [W/m, K]
Attic floor	0	50	0.04
Floor	40	50	0.05
Ext. wall, out.	15	50	0.05
Ext. wall, in.	10	15	0.05

Table 3
Fenestration Input Data, Ansgarius, Malmö.

Retrofit	D [Skr]	E [Skr/m²]	Remaining Life [Years]	New Life [Years]	New U-value [W/m², K]
Double-glazed	1,890	560	0	50	2.5
Triple-glazed	2,350	790	—	50	2.0
Energy-glazed	2,580	1,020	—	50	1.2

decrease the habitable area of the apartments with a subsequent loss of rent from the tenants, in this case, Skr 400/m^2 per year. Fenestration retrofits are dealt with in a similar way but the costs, C_w, are presented as:

$$C_w = D + E \cdot A_w \tag{2}$$

where D = initial retrofit cost, E = window-size dependent cost, and A_w = window area. The window input data used in this case study is shown in Table 3.

Weatherstripping is expected to decrease the ventilation flow through the building by 0.1 renewals/hour and the cost is estimated as Skr 200 for each item to be caulked. The life of the weatherstripping is set to 10 years.

Exhaust-air heat pumps might be a profitable retrofit and the cost for the equipment, C_{ehp}, is expressed in the equation:

$$C_{\text{ehp}} = F + G \cdot P \tag{3}$$

where F = initial cost, G = direct thermal power cost, and P = thermal power of the exhaust air heat pump.

There is also a cost for connecting the heat pump to the ventilation and hot water systems. This cost has been estimated to be influenced by the number of apartments in the building and has a longer life than the heat pump itself. The values are presented in Table 4.

The temperature of the ventilation air flow is assumed to be +20 °C upon arrival at the heat pump and +5 °C after passing through the heat pump; the drop in air temperature is thus 15 degrees. The pump is assumed to have a coefficient of performance, COP, of 2.0.

In the OPERA model, heating equipment costs are expressed as:

Table 4
Exhaust-Air Heat Pump Input Data, Ansgarius, Malmö.

Retrofit	F [Skr]	G [Skr/kW]	Life [Years]	Piping Cost [Skr/apa.]	Life [Years]
Heat pump	10,000	4,500	15		
Pipes				5,000	30

Table 5
Heating Equipment Data, Ansgarius, Malmö.

Heating Device	H [Skr]	I [Skr/kW]	Life [Years]	J [Skr/kW]	Life [Years]	COP [1]
Oil boiler	55,000	60	15	200	50	0.75
Electr.	20,000	100	25	0	—	1.0
Dis. heat.	40,000	60	25	300	50	0.95
Heat p. gw	60,000	5,000	50	1,500	10	3.0
Natur. gas	55,000	60	20	200	50	0.9
Heat p. oa	40,000	6,000	15	200	40	*

$$C_{he} = H + I \cdot P_{he} + J \cdot P_{he} \tag{4}$$

where H = initial cost, I = cost influenced by the thermal power, P_{he} = thermal power of the equipment, and J = piping cost.

The reason for separating *I*-cost and *J*-cost is their different life-cycles. The program is also supplied with COP data for different types of boilers. The situation is presented in Table 5.

The COP for the outdoors air, oa, heat pump, see "*" in Table 5, varies according to the outdoor temperature. In our example, this variation is expressed as:

$$\text{COP} = (-\text{T} + 66.43)/20.53 \tag{5}$$

where T = the temperature difference between the desired inside and outside temperature (Gustafsson 1988).

When the heat pump for outdoors air is considered, it is possible to provide OPERA with a calculation of the maintenance LCC calculation. In the case at hand, it is assumed to be 10% of the acquisition cost every 7 years. The LCC for the maintenance of other types of heating equipment must be included in the *H, I* and *J* parameters in Equation 4.

Using as input data to the OPERA model these values and others not described here, we obtained the results shown in the tables, i.e., optimal strategies for (1) several optimization periods, ranging between 10 and 50 years, (2) several real discount rates between 3 and 11%, and (3) energy prices escalating at between 0 and 3% annually. Table 6 shows part of the output of an OPERA run using the parameters 5% discount rate, 50-year optimization time, and 0% energy price escalation.

The first column of Table 6 contains the LCC for the building using the existing district heating system, i.e., MSkr 2.67. If the attic floor is subjected to an optimal thickness of floor insulation, MSkr 0.04 can be saved during the project life, here 50 years. External wall insulation and other tested retrofits were found not to be cost-effective and are indicated by "—." Certain retrofits that have no bearing on the cost of any of the heating systems are not listed. The expected new LCC, for the existing heating system, is therefore MSkr 2.63. Some retrofits might interact and affect one another's saving values. Therefore, the LCC is recalculated, simulating the implementation of these candidates for optimal retrofittings. In the first instance, where only one retrofit is selected, the two values are of course identical. The situation is quite different

Table 6

LCC and Savings in MSkr, Ansgarius, Malmö.

	Dist. Heat	Electr. Boiler	Natural Gas	Bivalent Heating	Oil Boiler
LCC with no building retrofits	2.67	3.94	2.37	2.74	3.08
Savings:					
Attic insulation	0.04	0.16	0.01	0.04	0.07
Ext. wall ins.	—	0.22	—	—	—
Weatherstripping	—	0.04	—	—	—
Exh.-air heat p.	—	0.10	—	—	—
Expected new LCC	2.63	3.42	2.36	2.70	3.01
Recalc. new LCC	2.63	3.50	2.36	2.70	3.01

when electrical heating is selected as the heating system, since it has a much higher energy cost, Skr 0.40 compared to Skr 0.21/kWh. Other building and installation retrofits are also candidate components of an optimal solution, which means that interaction effects may arise. The bottom line of Table 6 depicts this situation where the retrofit combination implies that the LCC will be MSkr 3.50 instead of MSkr 3.42 as originally anticipated. However, none of these heating systems can compete against the best solution, i.e., natural gas and attic floor insulation, since natural gas is currently available in Sweden for Skr 0.16/kWh. The optimum thickness of attic floor insulation is calculated to 0.15 m of mineral wool. The OPERA model can also be used to evaluate an LCC scenario when greater or lesser thicknesses of insulation are applied. Table 7 describes this situation.

In Table 7, the various LCC components are presented, together with the thermal conditions, for varying thicknesses of insulation. If no additional insulation whatsoever

Table 7

Thermal Performance and Present Values Due to Attic Floor Insulation Thickness. All Costs in kSkr.

Insul. Thick. [m]	Therm. Load [kw]	Energy Use [MWh]	Inevit. Cost	Boiler Cost	Energy Cost	Conn. Fee	Insul. Cost	Total Cost
0.00	167.3	366.6	1,007	129.4	1,212	20.1	0	2,369
0.05	159.3	345.7	1,007	127.1	1,143	19.1	80	2,377
0.10	156.6	338.8	1,007	126.3	1,120	18.8	89	2,361
0.15	155.2	335.1	1,007	125.9	1,108	18.6	98	2,358
0.20	154.5	333.2	1,007	125.7	1,102	18.5	106	2,359
0.30	153.6	330.8	1,007	125.5	1,094	18.4	123	2,368
0.40	153.1	329.6	1,007	125.3	1,090	18.4	140	2,381
0.50	152.7	328.7	1,007	125.2	1,087	18.3	158	2,395

is applied, the thermal load will be 167 kW and the energy use 366.6 MWh, during a one-year interval. The inevitable retrofit cost is kSkr 1,007 during the optimization period. In Table 7 this cost includes the salvage value of the existing boiler. The total LCC with no new insulation on the attic floor is kSkr 2,369.

If 0.05 m of insulation is applied, the thermal load will drop, as will energy consumption in the building. The energy cost, boiler cost, and connection fee, in this case Skr 120/kW, all decrease. The increased insulation cost, however, cancels out the savings, leading to a higher LCC, i.e., kSkr 2,377 instead of kSkr 2,369. This solution thus is not acceptable. Applying still more insulation reduces energy costs even more, but the cost of additional insulation is less than when the calculation starts from scratch. The LCC will continue to drop until it reaches a point where the insulation cost increases faster than energy and other costs decrease. The breakeven point is about 0.15 m of additional insulation. It is interesting to note that the LCC is approximately the same for, on the one hand, an additional 0.3 m of insulation and on the other, for no insulation at all. In this case, where the price of natural gas is only Skr 0.16/kWh and might conceivably rise, it could be advisable to insulate even more than even the optimal solution suggests. This is because the LCC increases more slowly for additional insulation but faster when insulation is not sufficient.

Table 7 shows the decrease in energy consumption. The result might seem rather scanty, since the decrease is only about 10% compared to the original situation. However, it must be remembered that attic floor insulation is the only retrofit that was found profitable and this sole retrofit cannot influence the total situation very much. If the study is concentrated only to the attic floor, the situation becomes clearer. The original U-value is 0.8 W/m^2,K and the attic floor area is 573 m^2. Using 60,000 degree hours, which is the approximate situation according to the energy balance defined by the OPERA model, will lead to an energy flow of 27,500 kWh annually. Optimal insulation will decrease the U-value to 0.2 W/m^2,K and subsequently the energy flow will become 6,700 kWh. The energy flow is thus decreased by 75%. An extensive insulation, say 0.3 m will decrease the energy flow through the attic floor by about 85%, although the LCC will remain lower than in the original situation (see Table 7).

The OPERA model can also be made to show the effects of one or several additional retrofits. As seen previously, it was not an ontimal solution to install external wall insulation. However, local building codes, and so forth might make additional insulation mandatory for building subsidy eligibility. Table 8 shows the situation if the external walls must be retrofitted and how the LCC varies due to the insulation thickness. We see that the inevitable retrofit cost will increase immediately when the wall is retrofitted. No renovation was necessary for another 15 years, but when it is nonetheless done during the base year, cost will be considerable, i.e., kSkr 373. This cost has its origin in the value *A* in Equation 1. Table 7 also shows that the insulation cost increases from kSkr 98 to kSkr 542. This is due to the cost *B* in Equation 1, which shows the starting cost when insulation is applied. If the building requires retrofitted walls, the LCC immediately increases by kSkr 915.

However, the LCC may be decreased by applying more insulation. For 0.15 m insulation, the cost is the lowest possible, or kSkr 2,797. It should be noted that the value for optimal insulation thickness of the wall is not identical to the attic floor insulation; the two values just happened to coincide. From Table 8 we see that a radical change has occurred in the overall energy loss of the building. It has dropped from 366.6 MWh to 198.8 MWh where the optimal insulation thickness is used and will drop even more if the LCC is allowed a slight increase. The energy flow has, in other words, decreased by

Table 8

Thermal Performance and Present Values Due to External Wall Insulation Thickness. Costs in kSkr.

Insul. Thick [m]	Thermal Load [kw]	Energy Use [MWh]	Inevit. Cost	Boiler Cost	Energy Cost	Conn. Fee	Insul. Cost	Total Cost
0.00	155.2	335.1	1,380	125.9	1,108	18.6	542	3,161
0.05	118.3	239.3	1,380	115.2	791	14.2	573	2,874
0.10	107.4	211.2	1,380	112.1	698	12.9	605	2,809
0.15	102.3	198.8	1,380	110.6	657	12.3	637	2,797
0.20	99.2	191.8	1,380	109.8	634	11.9	669	2,805
0.30	95.7	184.7	1,380	108.8	611	11.5	732	2,844
0.40	93.9	180.1	1,380	108.2	598	11.3	796	2,894
0.50	92.7	178.6	1,380	107.9	590	11.1	859	2,949

about 50%. It also must be emphasized that if the refurbishment of the external wall is postponed 15 years, the measure will be an inevitable cost. The total inevitable cost increases then to kSkr 1,365, but there is no way to stop this. The insulation of the external wall might thus be found to be a profitable future activity. If optimal retrofits were always implemented when other renovation measures are necessary, the building stock will be continually improved, even from an energy-efficiency standpoint.

Sensitivity Analysis

The OPERA model is equipped with special routines for examining the influence of changes in input data. If the optimal solution found for the base case will change very much due to small variations in, for example, the real discount rate, there will be a severe risk of misoptimization. In this case study, a natural gas heating system is the best solution for optimization periods from 10 to 50 years, real discount rates from 3 to 11%, and annual escalating energy prices from 0 to 3%.

Note, however, that only one variable at a time has been changed. The solution with natural gas heating, therefore, seems quite reliable. However, the OPERA model will reject the combination with attic floor insulation if the optimization period is shorter than 40 years or the real discount rate is higher than 7%.

If energy prices escalate faster than 2% annually, weatherstripping will be a viable component of the optimal solution. Remember that the attic floor insulation varies according to the previous input parameters. Optimal insulation is thus 0.21 m when annual energy price escalation is 3%.

It might be interesting to examine at what point the natural gas boiler loses its viability. Price and connection fee are two of the parameters affecting the natural gas solution. In the base case, which reflects the real situation in Malmö, the price is Skr 0.163/kWh and the fee equals Skr 120/kW. In Figure 2, which shows results of a bivariate sensitivity analysis, the optimal heating system, the LCC, and the thickness of insulation are presented for a range of values. The insulation thickness in meters is shown above the mark and the LCC in MSkr below it.

When energy price has a low value, i.e., Skr 0.14/kWh, the alternative for attic floor insulation is rejected. The natural gas heating system is best when connection fees fall between Skr 80 and 160/kW. When energy price is higher, between Skr 0.16 and 0.20/kWh, the natural gas heating system should be combined with attic floor insulation. The thickness of insulation then varies between 0.15 and 0.18 m. It is important to note that the connection fee will have a minor influence on the thickness of attic floor insulation. When natural gas prices are Skr 0.22/kWh or higher, the natural gas heating system is rejected. The district heating system is then the best one in combination with 0.18 m of additional attic floor insulation. When this point is reached, the impact of the natural gas price and connection fee is no longer significant. Higher values do not affect the LCC at all.

Some extra OPERA runs have been carried out in order to find when the connection fee will start to influence the heating system solution. If the natural gas price is set to Skr 0.20/kWh, the fee must be of the magnitude Skr 300/kW, or the district heating system will not be the optimal choice.

It is also interesting to study the influence of the real discount rate and the optimization time in more detail. This is done in Figure 3.

From Figure 3, it is obvious that the attic floor insulation is unprofitable for high real discount rates and short project lives. In the case studied here, the project life must be longer than 30 years, when a 3% real discount rate is encountered, if the insulation is to be profitable. When a 5% rate is used, the project life must be longer than 40 years in order to achieve profitable insulation. The thickness of insulation must be greater than 0.15 m to be cost-effective.

If insulation is applied in thinner layers, the LCC will be higher than the alternative

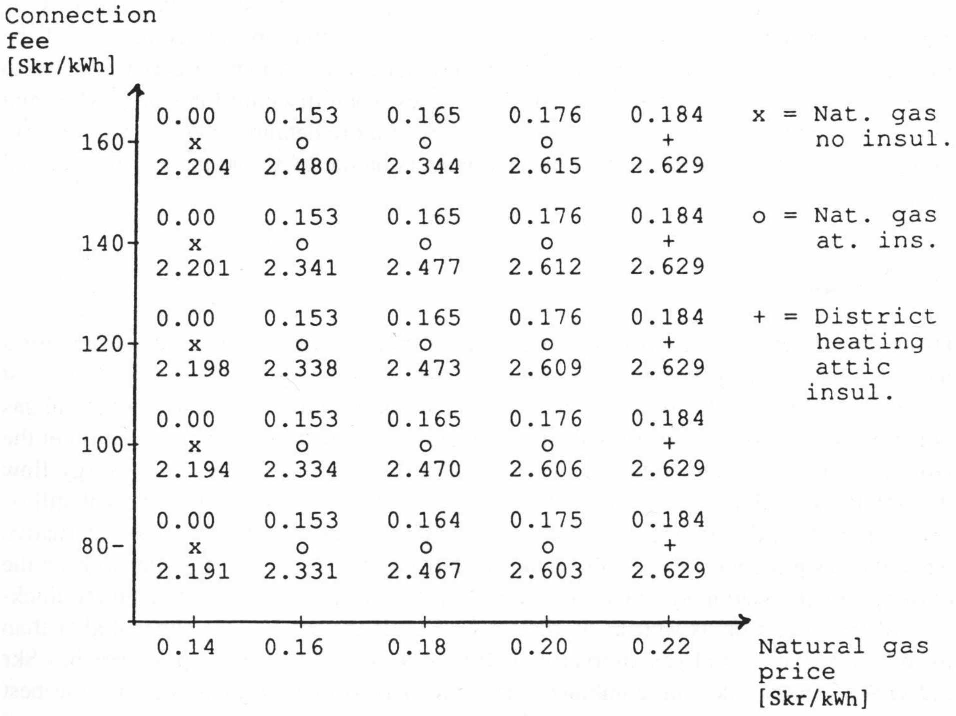

Figure 2. Bivariate sensitivity analysis, Ansgarius, Malmö.

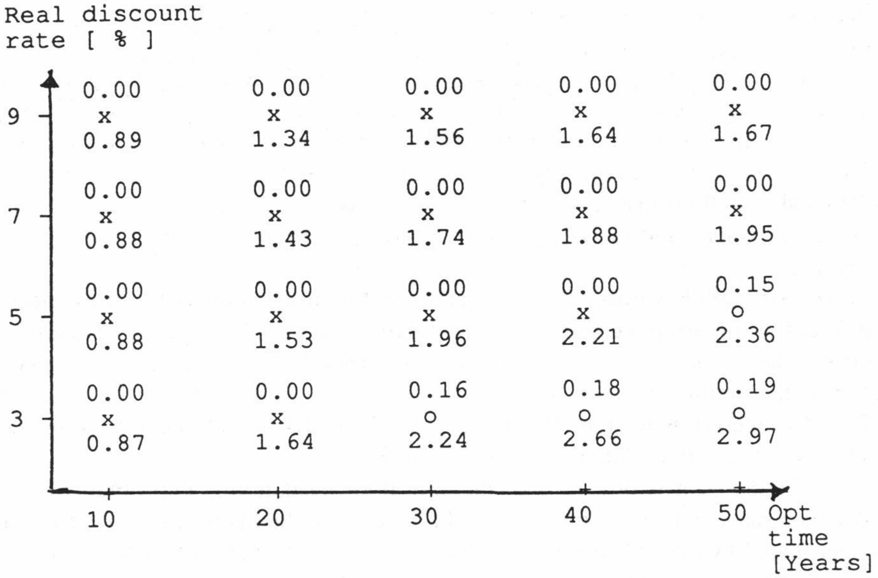

Figure 3. Bivariate sensitivity analysis, Ansgarius, Malmö.

with no additional insulation at all. Important to note is that the natural gas heating system is never rejected for the values studied. Figure 3 also shows that the LCC is highly influenced by the real discount rate. However it must be remembered that the LCC is just one criterion of rank; it is not a good idea to use a high discount rate just to reduce the LCC. Note also that the LCC increases when discount rates are higher and the project life is very short. In Gustafsson (1988), a more detailed study is elaborated in order to illuminate the impact of various parameters on the LCC and the ensuing optimal strategy.

Conclusions

Using the OPERA model enables the operator to study the optimal retrofit solution for a particular multifamily building. In the case study presented in this article, the best solution was found to be to replace the existing district heating system with a natural gas boiler and combine this retrofit with approximately 0.15 m of additional insulation of the attic floor. It is also shown that if this insulation thickness is doubled, the energy flow through the attic floor will decrease by more than 75%, while the total LCC will still be lower than the existing one. The natural gas boiler is rejected as a viable alternative when the gas price increases to about Skr 0.22/kWh; in this event it is better to keep the existing district heating system. The connection fee has a minor influence on the thickness of the insulation as well as on the total LCC. If the fee is three times higher than today's rate, the natural gas alternative will be rejected when the gas price reaches Skr 0.20/kWh. District heating combined with attic floor insulation then becomes the best solution. The thickness of insulation will, however, be about the same as when a natural gas system is used.

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