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Mathematical Modelling of District-Heating and Electricity Loads

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ABSTRACT

In recent years it has been more common to use linear or mixed-integer programming methods for finding optimal solutions to the complicated operating options in modern Combined Heat and Power (CHP) networks. Electricity may be bought from the national grid or it may be produced in ordinary condenser or CHP plants owned by the utility. In the same manner, district heat can be produced by the use of waste heat from industries or from a CHP plant. Other options are burning garbage in an incineration plant, using heat pumps in a sewage water plant or just burning fuels in an ordinary boiler. Combining these options and including the possibility of using conservation measures in industry or in the housing stock will result in a very complex situation if one tries to find the optimal solution characterized by the lowest Life-Cycle Cost (LCC). Load management equipment, such as hot-water accumulators, will aggravate the problem even further. By the use of modern computers, complicated problems can be solved within a reasonable period of time. The bases for the mathematical models are the thermal and electrical loads. Splitting these loads into finer and finer segments will yield a model that will depict reality more closely. Two methods have been used frequently, one where the high and low unit price hours in each month have been lumped together, resulting in 24 segments plus one segment showing the influence of the maximum electricity demand. The other method tries to model the loads by lumping the energy demand in six electricity-tariff segments, but also using about 15 elements for a more versatile picture of the district-heating load. This paper describes the two methods using monitored data for 1990–1991 from Kalmar in the south of Sweden. It also discusses which of the methods is preferable or whether a combination must be elaborated upon in order to model reality closely enough for practical use.

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INTRODUCTION

There has been increased interest in recent years in mathematical modelling by the use of linear mixed-integer programming methods. This is mostly the result of the more common use of computers with high calculation capacity. Previously, solving optimization problems with the linear programming technique was very tedious, and neither the speed nor the memory capacity was usually sufficient in small desktop machines. These problems, at least in part, have been solved by the introduction of 386 and 486 processors running in protected mode. This paper, however, does not deal with how to find the optimal solution for a linear program; see Refs 1 or 2 for details.

One of the major drawbacks with this technique is that the entire mathematical problem must be linear. Only expressions of the type

$$C_1 \cdot A + C_2 \cdot B + C_3 \cdot D = C_4$$

where $C_1 \ldots$ are different constants and A, B, D are different variables that can be dealt with. Note that the equality sign could be an inequality ($\leq$ or $\geq$) instead. On the other hand, several thousand equations could be included in the problem and some of the variables may only take the values zero or unity. This means that non-linear problems can often be transformed to linear ones and solved by use of the traditional Simplex and Branch-and-Bound methods.[3] In linear programs, there is an objective function that is to be minimized or maximized. In the case of energy-system optimization, this function often shows the total LCC of the system and the problem will be to find the minimum LCC. The thermal and electrical loads will be part of the objective function because the total cost is a result of, among other things, using different fuels in the boilers. One detailed example of how to design the objective function, and the constraints, in a mixed-integer program can be found in Ref. 4.

THE DISTRICT-HEATING LOAD

In order to visualize the situation, we will use an example from Kalmar in the south of Sweden. The district-heating load has been monitored once per hour for almost a full year, from 1400 h on 19 March 1990, to 0700 h on 1 March 1991. Figure 1 shows a duration graph, the values of the load have been plotted in falling magnitude. The maximum load in the district heating net is 77·2 MW but this load only occurred for a short period during one hour. The rest of the peak is very steep and has likewise a very short duration (see Fig. 1). The amount of energy that must be produced at above a 60 MW

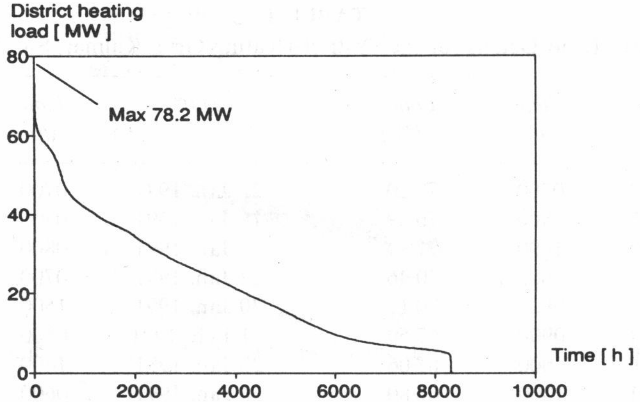

Fig. 1. Duration graph of district heating load in Kalmar, monitored values.

rating is minute, and therefore could not influence the cost very much. One important fact, however, is that the utility must be able to produce 78·2 MW or there will be a shortage of district heat. This means that it is essential to install boilers, etc., that are able to produce the total demand and while there might be a substantial cost for this equipment, it must be included in the objective function.

Normally it is estimated that the peak load and its adjacent values emerge in the same time interval, e.g. between 0600 and 2200 h in December. A closer look at the peak is presented in Table 1. The highest value emerged for 30 October 1990 at 0700 h and the next highest for 31 January 1991 at 0800 h. Unfortunately, the first value seems to be in error. The two adjacent values in the original data file are much lower, so the peak load must probably be set to 76·89 instead of 78·20 MW. Fortunately, the error for this hour does not influence the model load in a significant way.

It is important to note that the peak contains values from the first two months January and February, except for the first one which probably is an error. The second-highest value in Table 1 occurred at 0800 h on 31 January, while the third value emerged almost one month later.

A closer look at the data set for the district-heating load reveals that not all hours during the full year were included in the file. Of the 8760 hours that should be present, only 8312 are rewarded. Most of the hours lacking (439) occur between 0100 h on 1 March and 1400 h on 19 March, but still about ten hours are missing. It is important that the model should be consistent and subsequently contain 8760 hours. In order to achieve this, about nine days before 1 March and nine days after 19 March are copied and added instead of the missing interval. The peak load, as found in Table 1 above, will not be affected by these added hours, and thus those values could be used to examine how they will fit into the two types of models frequently used.

TABLE 1
Peak Load Details for the District-Heating Grid, Kalmar, Sweden

Date	*Time (h)*	*Load (MW)*	*Date*	*Time (h)*	*Load (MW)*
30 Oct. 1990	0700	78·20	22 Jan. 1991	0700	66·54
31 Jan. 1991	0800	76·89	18 Jan. 1991	0700	66·18
28 Feb. 1991	1600	71·84	22 Jan. 1991	0800	66·00
18 Jan. 1991	0800	70·46	27 Jan. 1991	0700	65·42
26 Jan. 1991	0800	70·17	30 Jan. 1991	1500	65·35
1 Feb. 1991	0900	67·59	1 Feb. 1991	0800	65·25
29 Jan. 1991	1000	67·06	30 Jan. 1991	1400	65·08
28 Feb. 1991	1000	66·80	24 Jan. 1991	0600	64·42
13 Feb. 1991	1600	66·77	18 Feb. 1991	0700	64·30
28 Feb. 1991	0800	66·76	21 Jan. 1991	0900	64·27

In the first type of model, it is assumed that the climate is of major importance for the district-heating load. One example of such a model, used for a study of Malmö, Sweden, is shown in Fig. 2.[5] The district-heating load has, in that case, been calculated assuming that the load originated from a gigantic building. The transmission factor was calculated to be 14·39 MW K^{-1}, the thermal losses from ventilating the 'building' were assumed to be 5·07 MW K^{-1}, while the heating of domestic hot-water was assumed to require 350 GWh for a full year. Using climatic data for Malmö, it was possible to calculate the load found in Fig. 2.

One very big advantage of the above procedure is that if the 'building' is to be affected by retrofit measures, it is possible to calculate exactly how much the load would be decreased by such a measure. If Table 1 and Fig. 2 are compared, it is obvious that the finer details of a real load will not emerge in Fig. 2. The load for one hour in December might be much higher than one in

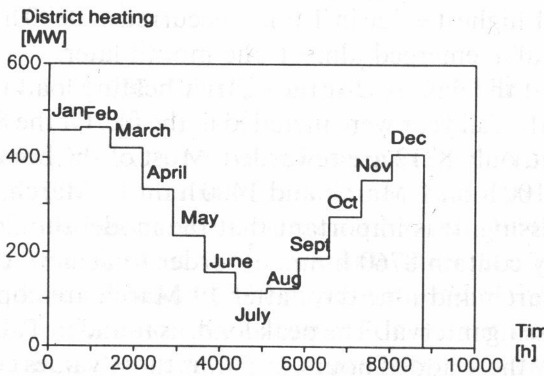

Fig. 2. District heating load for 12 months, Malmö, Sweden.[5]

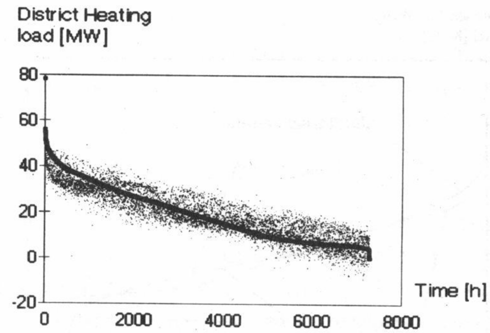

Fig. 3. Monitored and calculated values for the district-heating load in Kalmar.

February even if the model says that this is not the case. On the other hand, the real load for one year will probably not be representative for all future years that are under consideration. This applies for the model in Fig. 2, where meteorological mean values for the outside temperatures were used. In Ref. 6, the real district and electricity demands of Kalmar have been studied and some statistical investigations have been elaborated. The result shows that there is a fairly good correlation between the outdoor temperature and the district-heating load. The correlation coefficient was 0·895 for the total data set and the correlation was not improved if only certain selected values were used instead of the total set. The 'best-fit equation' for the district-heating demand (in MW) was calculated to:

$$\mathrm{DH}_{\mathrm{load}} = 5{\cdot}6 + 1{\cdot}525t \tag{1}$$

where t equals the indoor temperature (20°C) minus the outdoor temperature. In Fig. 3 the values calculated from eqn (1) are shown as dots while the monitored values emerge as a line. If Figs 1 and 3 are compared, it is apparent that some values are missing; only about 7500 h are present in Fig. 3, whereas about 8200 h are dealt with in Fig. 1. This is because the monitored temperatures included some 700 values which could not be used due to registration errors. It must also be noted that the monitored values have been sorted in descending order, so that each monitored value corresponds to the calculated value for the same specific hour. The calculated values will therefore not be sorted in descending order, but instead emerge as scattered dots. In Fig. 4 these calculated values are also sorted, but two points vertically aligned on the separate curves will now not correspond to the same hour of the year. However as Fig. 4 shows, the correlation between the calculated values and the monitored values has been improved substantially by this procedure.

The calculated values underestimate the demand for very cold days but will yield higher values during part of the summer. For very warm days, i.e.

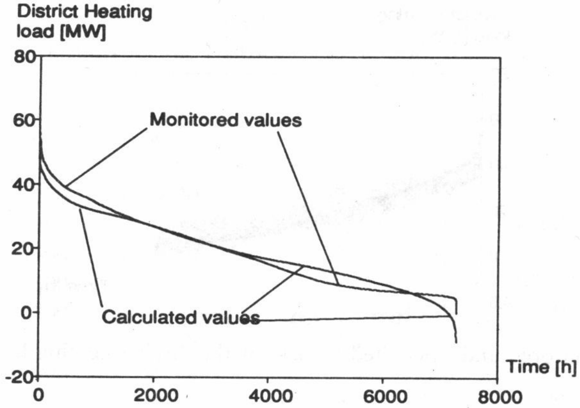

Fig. 4. Monitored and calculated values for the district-heating load in Kalmar.

when there is no real climate load at all, the demand appears to be negative, which of course is not logical. These values could be excluded when the final model is designed. The total amounts of energy under the two curves are almost exactly the same and thus it seems that the method shown will result in an acceptable model for predicting the district-heating demand.

THE ELECTRICITY LOAD

The electricity demand does not show the same high correlation with the climate as does district heating. The correlation coefficient has been calculated to be 0·326 for the total data set.[6] If only the minimum demand each night was used, it reduced the influence of industrial and other activities during daytime, and this increased the correlation coefficient to 0·748.

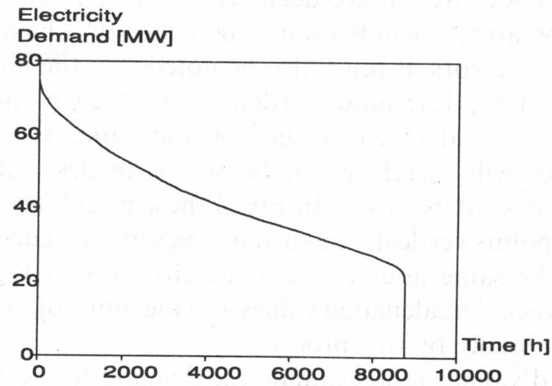

Fig. 5. The electricity demand in Kalmar, Sweden, during 1990.

Exclusion of the summer months, however, did not improve this value. In Fig. 5 the electricity demand is shown for the total data set in the form of a duration graph. The peak load is about 75 MW while the lowest load is about 22 MW. The climatic load was calculated[6] as:

$$E_{\text{load}} = 20{\cdot}58 + 0{\cdot}809t \qquad (2)$$

where t equals the indoor temperature (20°C) minus the outdoor temperature. If eqn (2) is used for calculating the electric space-heating load, the result would be similar to that shown in Fig. 3. The problem will then be the same, i.e. should the monitored values be sorted according to their strict order of magnitude or is it better to sort them into separate time segments, no matter what their level?

THE COST OF PRODUCTION OR PURCHASE OF ENERGY

The utility can produce electricity and heat in its own facilities or it can buy electricity from the national grid. The tariff that is used is split up into time segments. During working days, the high-price hours range from 0600 to 2200, while a cheaper price is used during rest of the day and on Saturdays and Sundays. The price will often differ depending on the month of the year. During summer, the unit price is lower than it is during winter. The models are used for minimization of the life-cycle cost and therefore it is important to use a strong connection between the actual use of energy and the applicable price in the tariff. In Table 2, the monitored demand has been split up according to the month and time of day. In January, for example, there are 368 high-price and 376 low-price hours, The use of electricity during high-price conditions are 22·1 GWh: this implies an average demand of 60 MW. For the low-price segment, the corresponding values are 17·8 GWh and 47·3 MW. The district-heating use is 16·5 GWh (average demand 44·9 MW) for the high-cost segment and 14·9 GWh (average demand 39·5 MW) during the low-cost segment.

The electricity demand in Table 2 must now be arranged in descending order, so that it can be compared with the situation found in Fig. 5. This has been done in Fig. 6.

Unfortunately, it has not been possible to show both the curve in Fig. 5 and the graph in Fig. 6 in the same diagram. The calculation process used for producing Fig. 6 ascertains that the same amount of electric energy is shown in both figures. It is also obvious that Fig. 6 does not show the peak load which is apparent in Fig. 5. Because of the very small area, i.e. energy, in the peak, the difference in electricity cost will be very little between the two graphs shown. There is, however, a possibility that the size of the equipment

TABLE 2
Electricity and District-Heating Demands During 1990 in Kalmar, Sweden

Month	*No. of hours*		*Electricity*				*District heating*			
	High	*Low*	*High*		*Low*		*High*		*Low*	
			Energy[a]	*Load*[b]	*Energy*[a]	*Load*[b]	*Energy*[a]	*Load*[b]	*Energy*[a]	*Load*[b]
Jan.	368	376	22·1	60·0	17·8	47·3	16·5	44·9	14·9	39·5
Feb.	320	352	18·1	56·7	14·0	39·7	17·0	53·2	16·3	46·4
Mar.	352	392	18·8	53·4	14·2	36·2	12·1	34·3	11·4	29·1
Apr.	336	384	16·1	47·9	12·6	32·8	7·9	23·5	8·1	21·2
May	368	376	15·8	43·0	10·6	28·1	5·0	13·5	5·0	13·3
Jun.	336	384	14·3	42·6	10·5	27·4	2·7	8·0	2·9	7·6
Jul.	352	392	13·8	39·2	10·9	27·7	2·7	7·6	2·6	6·7
Aug.	368	376	16·7	45·5	11·1	29·6	2·5	6·9	2·4	6·5
Sep.	320	400	16·4	51·2	13·5	33·6	5·4	17·0	5·4	13·6
Oct.	368	376	20·6	55·9	14·4	38·4	9·2	25·0	7·8	20·7
Nov.	352	368	21·4	60·8	15·6	42·0	12·3	35·0	10·4	28·3
Dec.	336	408	20·0	59·5	18·3	45·0	13·1	39·0	13·4	32·8
Total			214·1		163·5		104·4		106·4	

[a] In GWh.
[b] In MW.

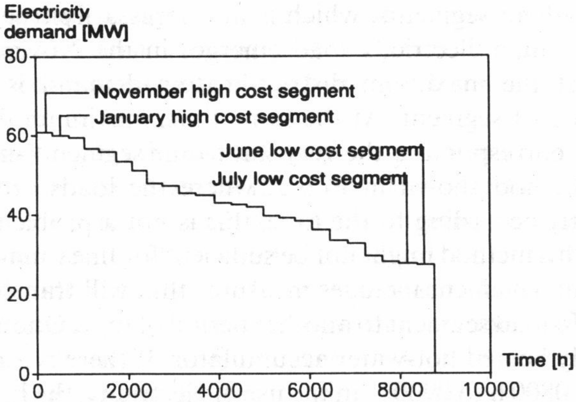

Fig. 6. Duration graph of the 24-segment electricity demand, Kalmar, Sweden.

needed will be misjudged if only Fig. 6 is used. In Ref. 4, this has been dealt with by the use of a 25th segment showing the maximum demand for the months November–March, which are the months of concern due to the demand part of the tariff.

THE LOAD-MANAGEMENT PROBLEM

Load-management equipment in residential buildings and industry is nowadays available in order to cut peak loads. Possible examples are addition of hot-water accumulators in buildings or disconnection of kilns for short periods of time in industry. In Ref. 7, this is dealt with by use of maximum-load days or by lumping of days. It should then be possible to ascertain that the amount in the cut-off peak for the high cost segment will later emerge in the low cost or at least later in the high-cost segment. Energy is not saved but the peak is reduced and the same amount of energy is used but when the risk for a peak is low or does not exist at all. Figure 5 shows that some 200 hours have higher demands than the peak of approximately 60 MV that is found in Fig. 6. It will not be possible in practical use to add all these hours to a model as separate segments and therefore they are lumped in smaller segments as shown in Ref. 7. The method used for producing Fig. 6 is not sufficient for this fine tuning of the model and thus it would be important to split the left-hand side of the load into smaller segments. Likewise, it is important to ascertain that the electricity and the district-heating loads coincide with one another. The selected maximum electricity-load segments must therefore correspond to the very same time segments for the district-heating load. In Refs 7 and 8, this is not fulfilled, instead, the maximum electricity-load segments are *assumed* to correspond to maximum district-heating load segments, which is not necessarily true—see Table 2, where the maximum electricity load emerges in the November high-cost segment, whilst the maximum district-heating demand is found in the February high-cost segment. At the same time, maximum district-heating segments must correspond to the very same time segments in the electricity load. In the method shown in Fig. 2, where the loads are split up into segments strictly according to the time, this is not a problem; however, as shown above, this method might not be sufficient for fine tuning of the model. Electric load management includes measures that will transfer some of the energy in a peak-load segment to another period of time. One example of this is an electrically heated hot-water accumulator. If there is a risk for a peak hour, say from 0800 h to 0900 h, in the use of electricity, the heater should be disconnected for that particular period of time. The energy that was going to be used must now instead be transferred by the model to the hour between 0900 h to 1000 h. The possibility of producing district heat, using CHP, is at the same time reduced during the first hour and increased in the second hour.

Everyone understands that the process for evaluating all the possibilities is a major task and we do not think that it will be possible to design a model that is able to achieve an accurate answer for all the different kinds of load-management measures. One example can be shown from the data in Table 1. If we could have used a load-management system to decrease the load by, say, 5 MW at 0800 h on the 31 January 1991, this would have been a fine measure. (The second value is used here because the first one was probably wrong.) These five megawatts could be transferred to any one of the hours coming later that day, because the same data do not reappear in the table. However, the transfer of 5 MW from 0800 h on 28 February to 1000 h on the same day would not be a good idea because then a new peak might result.

CONCLUSIONS

Neither of the two common methods for modelling electricity and district-heating loads will be sufficient for proper use in linear programming methods. The first kind of model splits the loads due to the different tariff elements. In Sweden, this means that the loads are split into high- and low-cost segments for every month during one year: this will lead to 24 time segments. There is also a single segment showing the maximum demand for electricity due to the cost for this demand. The first method, however, will calculate the average in all segments and thus the finer details in the load variation will be obliterated. One major advantage with the method is that the model is consistent, i.e. each time segment will correspond exactly for the two loads. Load-management equipment could not be modelled properly as long as part of the energy in the high-cost segment is not transferred completely to the low-cost segment, and vice versa. It will therefore not be possible to model a transfer of energy from one hour to a subsequent one without special measures.

The other model splits the loads in a more versatile manner, this model, however, it not consistent because it has not been ascertained that the time segments for the electricity and the district-heating loads are identical. Instead, it is assumed that the district heating and the electricity loads have their maxima simultaneously. We have shown here that this is not always true. In these types of models it is not possible to add load-management measures because the model does not keep track of subsequent hours in the correct order. The model of the loads will, however, show a better correlation with the real ones. It seems that the first method is the preferable one, because of the consistency in time segments, but more time elements should be added to model the peak in a more accurate manner.

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