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Fractional Factorial Design for Energy Systems

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ABSTRACT

Nowadays, when powerful computers are on almost every one's desk, it has become more and more common to use complex energy-system models in order to predict the use of electricity and heat in buildings. At the same time, it has been harder to grasp the overall solution because of all the details implemented in such a model. A method that could help the operator to find the important parts in the model would therefore be of great interest. Traditionally this is addressed by using so-called sensitivity analyses. The most common method is then to change one input parameter a certain amount and study how much the output is influenced by this change. If the output varies significantly the parameter is supposed to be of more interest than if there is a only small change. If there is a complex model, several hundred parameters may have to be changed this way; this is very tedious. By the use of modern statistics, these calculations can be made in a more planned way and the necessary work minimized. One such method is fractional factorial design, which is used for examining a widely used Swedish energy-balance program with about 70 input data values. We have examined nine of these parameters in order to rank their importance for the output energy balance. The interactions between these nine parameters have also been studied using the same method.

INTRODUCTION

In Sweden, it is mandatory to show the authorities that the need for heat in a new building is lower or equal to that of a reference building presented in the building code. One means for showing this, is to use an energy-balance program. The program used by us, where the building is presented in the form of about 70 different values, is called ENORM,[1] which is widely used in Sweden. The program calculates the energy need and heating demand for a building on a diurnal basis, but the results are presented for 1 year. The program also includes values for the reference building and as long as the energy need and demand are lower than that for the reference building, one is allowed to use one's own building methods as far as energy conservation is concerned. Factorial design, and fractional factorial design, are statistical methods usually used for bringing down the need for experiments when you must show a scientifically significant output from, for example, a chemical process. The length of this paper does not make it possible to explain all the details and therefore the relevant in-depth knowledge must be sought elsewhere, e.g. in Ref. 2, Chapters 10–12, and Ref. 3. However, here we will show a case study where a building is analysed using the ENORM program with fractional factorial design.

CASE STUDY

Factorial design is a method for finding out what importance a production factor, or in our case a parameter for a building, has on the output result. First you choose two levels for the parameters of interest, one a low level and one a high level. These levels are shown with '–' and '+' signs respectively in a so called design matrix. The nine parameters we thought to be of major interest, before this work was utilized, are shown in Table 1. Note that the changes in U-values are considered as only one factor due to the way ENORM works. The same is valid for the air duct heat losses. (In the table, there is also a middle level, which is used later in this paper.)

TABLE 1
The Parameters Studied

Parameter	*Low level*	*Middle level*	*High level*	
U-value, attic joists plus	0·1	0·2	0·3	W/m^2 K
U-value, external wall	0·18	0·35	0·5	W/m^2 K
Indoor temperature	18·0	20·0	23·0	°C
Location (outdoor temp.)	Malmö	Jönköping	Stockholm	
Building size (area/volume)	135/324	150/360	170/408	m^2/m^3
Air renewal rate	0·5	0·5	1·0	1/h
Heat from appliances	10	12·5	15	kWh/day
Heat-recovery system	Exhaust air heat pump	Heat exchanger	Heat exchanger	
Air tightness	1	2	3	
Heat loss from air duct no. 1	0·04	0·04	0·1	W/m K
air duct no. 2	0·2	0·15	0·30	W/m K

It is not possible, in a paper of this short length, to describe all the parameters used for the building. Instead, we say only that the building is representative of modern low-energy buildings common in Sweden today. This is shown by the fact that, if all the low values above are used, the total energy needed for 1 year is about 11 000 kWh, while an average for the total building stock is about twice this value.

If ordinary factorial design was used for the nine parameters above, 2^9, i.e. 512, different runs of the energy balance program must be made (see Ref. 2, p. 306). This is a very tedious task, but, by the use of the fractional method, this number can be significantly reduced. The idea of using only a fraction of the needed 'experiments' emanates from the fact that the interaction between the variables tends to get smaller and smaller when the number of interacting variables increases. (Compare this fact with a Taylor series expansion, where terms of the third and higher order mostly are neglected.) The first thing now is to elaborate the so called design matrix, see Table 2. This table shows how the experiments, i.e. ENORM runs, are to be elaborated in order to achieve as much as possible in terms of statistical results.

The top left mark in the matrix shows us the level of variable number 1, e.g. the U-value for the attic joists. Here, this is a '–' sign and subsequently the U-value must equal 0·1 W/m^2 K in the first energy-balance

TABLE 2
Design Matrix for a 2^{9-5} Fractional Factorial Design

Run number	*Set-up of levels*														
	1	*2*	*3*	*4*	*1·2*	*1·3*	*1·4*	*2·3*	*2·4*	*3·4*	*5*	*6*	*7*	*8*	*9*
1	–	–	–	–	+	+	+	+	+	+	–	–	–	–	+
2	+	–	–	–	–	–	–	+	+	+	+	+	+	–	–
3	–	+	–	–	–	+	+	–	–	+	+	+	–	+	–
4	+	+	–	–	+	–	–	–	–	+	–	–	+	+	+
5	–	–	+	–	+	–	+	–	+	–	+	–	+	+	–
6	+	–	+	–	–	+	–	–	+	–	–	+	–	+	+
7	–	+	+	–	–	–	+	+	–	–	–	+	+	–	+
8	+	+	+	–	+	+	–	+	–	–	+	–	–	–	–
9	–	–	–	+	+	+	–	+	–	–	–	+	+	+	–
10	+	–	–	+	–	–	+	+	–	–	+	–	–	+	+
11	–	+	–	+	–	+	–	–	+	–	+	–	+	–	+
12	+	+	–	+	+	–	+	–	+	–	–	+	–	–	–
13	–	–	+	+	+	–	–	–	–	+	+	+	–	–	+
14	+	–	+	+	–	+	+	–	–	+	–	–	+	–	–
15	–	+	+	+	–	–	–	+	+	+	–	–	–	+	–
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

calculation, see Table 1. The second mark is also a '–' sign and this will likewise lead to a low level for parameter number 2, i.e. the *U*-value for the external wall, and so on up to column number 4. If we were going to elaborate an ordinary factorial design for four variables, column number 5 would depict the combination of levels for the numbers 1 and 2 columns, i.e. multiply the two signs which will result in a '+' sign. This row is later used for finding out if there is a combination effect between the numbers 1 and 2 parameters. Still assuming we are only calculating for four variables, the column that follows the one marked with 3·4, would show the combination of the levels for parameters 1, 2 and 3. Multiplying these levels results in a '–' sign. However, these combinations of three or more levels are neglected in the fractional version of the method. Instead we insert the parameter number 5, i.e. the air renewal rate in that position. The important thing is that we must still use the low level here because of the calculated '–' sign. The method will thus result in a '+' sign for the ninth parameter because this column would have been the result of multiplying the four first levels, which all have '–' signs. Our first experiment must therefore be elaborated by using ENORM with low levels for all parameters, except for the last one, i.e. the rate of heat loss from the ventilation ducts. In order to deal with all the combinations for four parameters, we need 2^4 experiments, i.e. 16 different runs. We have also neglected the combinations of more than

TABLE 3
The Calculated Need for Purchased Electricity in the Building, Setting the Nine Parameters According to the Design Matrix in Table 2

Run number	*Results (kWh/year)*
1	11 127
2	22 323
3	19 246
4	24 343
5	22 819
6	24 172
7	20 703
8	29 175
9	18 560
10	25 856
11	21 641
12	19 416
13	25 970
14	27 363
15	19 616
16	38 935

two parameters and thus five different possibilities are withdrawn. The procedure is therefore called a 2^{9-5} fractional factorial design because we have nine parameters with two levels, while five possibilities are neglected (see Ref. 2 p. 378 and the following, for the details).

In Table 3, the need for energy is shown for all the 16 different ENORM experiments. In experiment number 1, all the parameters but one were at their low levels so resulting in a need for 11 127 kWh for 1 year. In experiment number 2 resulting in 22 323 kWh, parameters number 1, 5, 6 and 7 were high, while the others were low according to Table 2.

Now the so-called main and interaction effects are to be calculated. This is fulfilled by using both Tables 2 and 3. According to Ref. 2, p. 309, these effects are the same as the differences between the mean average for the values in Table 3 as long as the signs in Table 2 are taken into proper account. The first value in Table 3 is 11 127. For parameter number 1, in Table 2, this value should be considered as negative because there is a '–' sign in the top left position. For the same parameter, the next negative values are found in row numbers 3, 5, 7, etc., in Table 2. Thus we add all the 'positive' values, calculate the average, add all the 'negative' values calculate their average and then subtract these values. The procedure is shown in detail in the following expression:

$$(22\,323 + 24\,343 + 24\,172 + 29\,175 + 25\,856 + 19\,416 + 27\,363 + 38\,935)\,/\,8 - (11\,127 + 19\,256 + 22\,819 + 20\,703 + 18\,560 + 21\,641 + 25\,970 + 19\,616)\,/\,8 = 51\,901$$

The other effects are shown in Table 4. The problem is now to find out which of the effects are important. When dealing with ordinary experiments, this may be found by comparing the effects to the one found for a normal distribution, i.e. a totally random result. If the same method is used here, we must examine if some of the effects in Table 4 are clearly

TABLE 4
Effects for the Fractional Factorial Design

Factor	*Effect*	*Factor*	*Effect*
1	51 901	9	−11 167
2	14 885	10	6 581
3	46 241	11	40 665
4	23 449	12	7 385
5	9 425	13	22 109
6	9 173	14	15 829
7	−335	15	14 229
8	1 325		

outside this random behaviour. The mean average for the values in Table 4 equals 23 204, while the standard deviation is 5794.

Assuming that the values that are outside four standard deviations from the average are of interest, we can identify the factors 1, 3, 7 and 9, which are the *U*-values for the external wall and attic joists, the outdoor temperature, interaction between the *U*-values and the size of the building, and interaction between the indoor temperature and the size of the house, see Tables 1 and 2.

It is somewhat strange that the indoor temperatures (factor 2) do not influence the result more than is found in Table 4, even if there is a strong interaction between the indoor temperature and the building size—see factor 9 in Table 4. Sometimes, effects from those parameters, which are included as 'extras', i.e. number 5–9 (see Table 2) are overwhelmed by the others. In order to solve this, we have elaborated an ordinary factorial design with only those parameters found important above, i.e. the *U*-values, the indoor temperature, the outdoor temperature and the size of the building—see Ref. 3 for a detailed factorial design dealing with an energy model. This time we have only four parameters to examine and subsequently it is possible to use the same design matrix as shown in Table 1. We only need to change the head line figures: 5 will now become 1·2·3, 6 will become 1·2·4, 7 will become 1·3·4, 8 will be 2·3·4, while the fifteenth column will represent the interaction between all the four

TABLE 5

Results from 16 ENORM Runs for Different Levels of Two Envelopes and Two Temperature Parameters and their Main and Interaction Effects

Run number	*Result (kWh/year)*	*Factor*	*Effect*
1	16 695	1	43 382
2	20 438	2	54 780
3	21 455	3	14 996
4	27 546	4	19 980
5	17 998	5	10 348
6	21 858	6	598
7	23 476	7	3 476
8	29 735	8	3 188
9	18 216	9	4 334
10	22 607	10	1 100
11	23 792	11	100
12	30 959	12	854
13	19 733	13	28
14	24 256	14	214
15	26 135	15	−2
16	33 483		

factors, i.e. 1·2·3·4. All but these four parameters are set to the middle levels. In Table 5, the resulting energy need and the calculated main and interaction effects are presented from the sixteen new runs of ENORM. The average of these effects equals 12 267, while the standard deviation is 17 256. If the same criterion as before, i.e. four standard deviations, is used to depict the factors of interest, none stands out. The same is valid for three intervals, while the adoption of two standard deviations selects factor number 2 and almost number 1, i.e. indoor temperature and *U*-values. From Table 5, it is also obvious that interactions between the factors are not of high importance, the values dwell within plus/minus one standard deviation.

From these two factorial designs it is obvious that the *U*-value for the building envelope and the temperature difference between the in- and out-door temperatures are most important for the energy balance of a building. Both the fractional and the ordinary factorial design show this. In the fractional design, it was possible to include five extra parameters which, however, were not investigated to the same extent because of the tedious calculation effort needed. Further, the fractional factorial design revealed that the ventilation-air renewal rate probably was important, but this effect could be the result of interactions with other parameters.

CONCLUSIONS

The paper shows that it is possible to use statistical methods, such as factorial design, in order to reveal the relative importances of different input data in computer-simulation models. By the use of factorial design, the so-called main and interaction effects can be calculated: these are measures of their individual and combined influences on the output from a computer program. However, by using a lot of input data in the models, even factorial design becomes a very tedious process. Fortunately, this drawback may, at least to some extent, be overcome by the use of the fractional method, where the interactions between three or more levels are neglected. By using such a method for an energy-balance program for buildings, we were able to show that the *U*-value for the building envelope had the highest importance followed by the difference between the in- and out-door temperatures. There is also an indication that the ventilation-air renewal rate was of major importance for the resulting energy need of the building. For the rest of the parameters studied, for example the type of heat-recovery unit, the analysis showed that they were of minor importance, or the analysis was inconclusive.

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