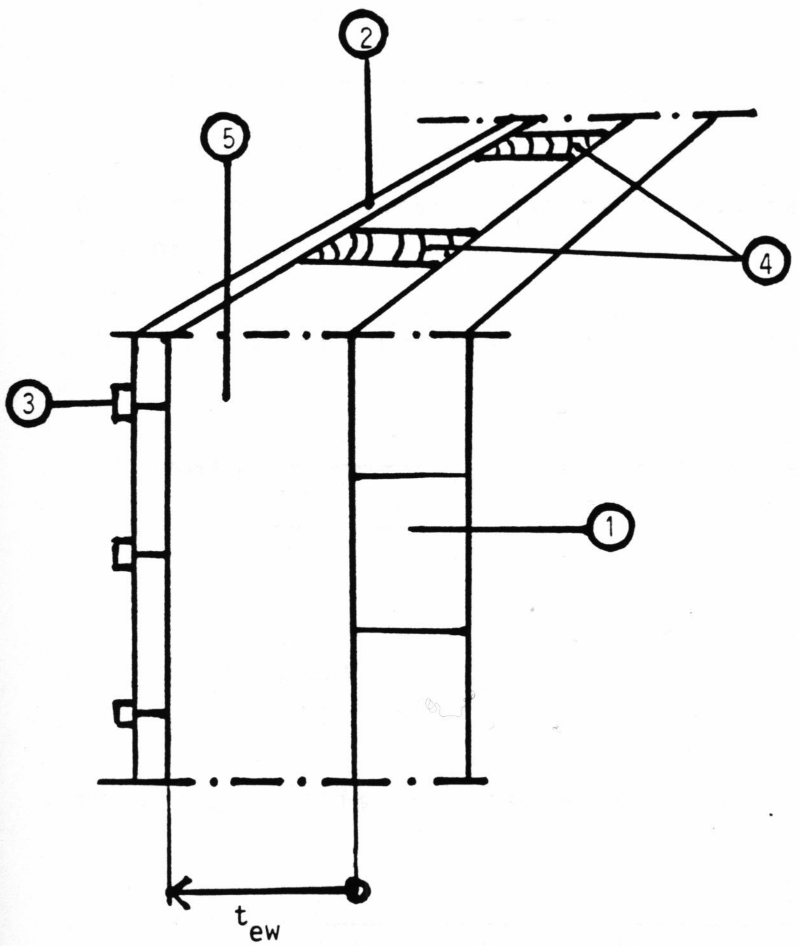
$$U_{ew,exist} = \frac{1}{\frac{0.022}{0.14} + \frac{0.075}{0.14} + 0.26} = 1.05 \quad W/m^2\,K$$

The influence of the battens has then been ignored. When putting more insulation to the external wall it is common to place this between the boarding and the timber. Figure 15 shows this.



1 = Existing timber boarding
2 = New boarding
3 = New batten
4 = New joists with $0.045 \times t_{ew}$ with 0.6 m distance
5 = New insulation e g mineral wool

Figure 15. The retrofitted external wall.

In this case the insulation laver consists of booth wood, the joists, and e g mineral wool with a big variety between their thermal properties. In (5 p 208) this is solved by calculating the "wood/mineral wool" part of the whole area.

The following calculations will explain the procedure.

The part of mineral wool in the unit area is :

$$p_I = \frac{0.6-2\times\frac{0.045}{2}}{0.6} = 0.925$$

$$p_{II} = \frac{0.6-0.6-2\times\frac{0.045}{2}}{0.6} = 0.075$$

p_{II} could, of course, also have been calculated as $p_{II} = 1 - p_I$.

The R-value for the mixed layer, ml, is:

$$R_{ml} = \frac{1}{\frac{p_I}{R_I} + \frac{p_{II}}{R_{II}}} \qquad \text{where } R_I = \frac{t_{ew}}{k_I} \quad \text{and}$$

$$R_{II} = \frac{t_{ew}}{k_{II}}$$

In our case this results in:

$$R_{ml} = \frac{1}{\frac{0.925}{\frac{t_{ew}}{0.04}} + \frac{0.075}{\frac{t_{ew}}{0.14}}} = \frac{1}{\frac{0.925\times0.04}{t_{ew}} + \frac{0.075\times0.14}{t_{ew}}} = \frac{t_{ew}}{0.0475}$$

The new U-value for the extra insulated wool will be:

$$U_{ew} = \frac{1}{\dfrac{1}{U_{exist}} + \dfrac{t_{ew}}{0.0475}} = \frac{U_{ew,exist} x 0.0475}{0.0475 + U_{exist} x t_{ew}} =$$

Inserting $U_{ew,exist}$ = 1.05 W/m^2 ^oC will give :

$$U_{ew} = \frac{0.0499}{0.0475+1.05xt_{ew}} \qquad (F13)$$

In (34 p 95) a slightly different method is presented, which in our case, however, gives almost the same result.

$$U_{ml} = \sum \frac{U_n x A_n}{A_n} = \frac{\dfrac{0.14}{t_{ew}} x0.045 + \dfrac{0.04}{t_{ew}} x(0.6-0.045)}{0.6}$$

$$= \frac{0.0063+0.022}{0.6xt} = \frac{0.0472}{t}$$

4.1.2.2 The external wall. Retrofit cost

In 31 Figure 7:43 we can find the cost for retrofitting this kind of wall. The procedure is the same as for the attic floor.

Table III Retrofit cost for the external wall

	Material	Time (h)	Wages	Sum
A. Scaffold	15.20	0.25	14.50	29.70
B. 0.022 m existing boarding. (Demolished)	-	0.15	8.70	8.70
C. New boarding	47.20	0.98	56.84	104.40
D. Mineral wool thickness (t_{ew})	$230 x t_{ew}$	0.12	6.96	$6.96+230 x t_{ew}$
E. New joists	$260 x t_{ew}$	0.34	19.72	$19.72 x 260 x t_{ew}$

I Sum A + B + C	62.40	1.38	80.04	142.44
II Sum D + E	$490 x t_{ew}$	0.45	26.68	$26.68+490 x t_{ew}$

Indirect costs 181 % part I (wages)	146.32
Indirect costs 181 % part II (wages)	48.29
Sum part I	288.76
Sum part II	$74.97+490 x t_{ew}$
Taxes part I 12.87 %	37.16
Taxes part II 12.87 %	$9.64+63.06 x t_{ew}$
Total sum part I	325.99 (SEK/m^2)
Total sum part II	$84.61+553 x t_{ew}$(SEK/m^2)

From Table III it appears that each time the outer part of the wall has to be substituted, it will cost about 326 SEK/m^2. This cost is called the inevitable renovation cost, because it appears even if no energy retrofits are made to the building. If the wall shall be extra insulated it will cost approximately (85 + 553 x t_{ew}) (SEK/m^2) more, where t_{ew} is the thickness of the extra insulation added to the wall.

The cost for retrofitting the wall is thus A_{ew}(326 + 85 + + 553 x t_{ew}) or with the area of the external wall included:

$$228\ 200 + 59\ 500 + 387\ 100 \times t_{ew} \quad \text{SEK}$$

As mentioned before the remaining life for the facade of the wall was 10 years and the new facade has an assumed life-cycle of 30 years. This means that after 10 and 40 years about 228 000 SEK has to be invested, whether no extra insulation is added to the wall or not. If the wall shall be insulated a new facade is built and the next time it has to be replaced occurs after 30 years. It is obvious that this has to be considered when calculating the LCC for the wall. Figure 16 shows the procedure.

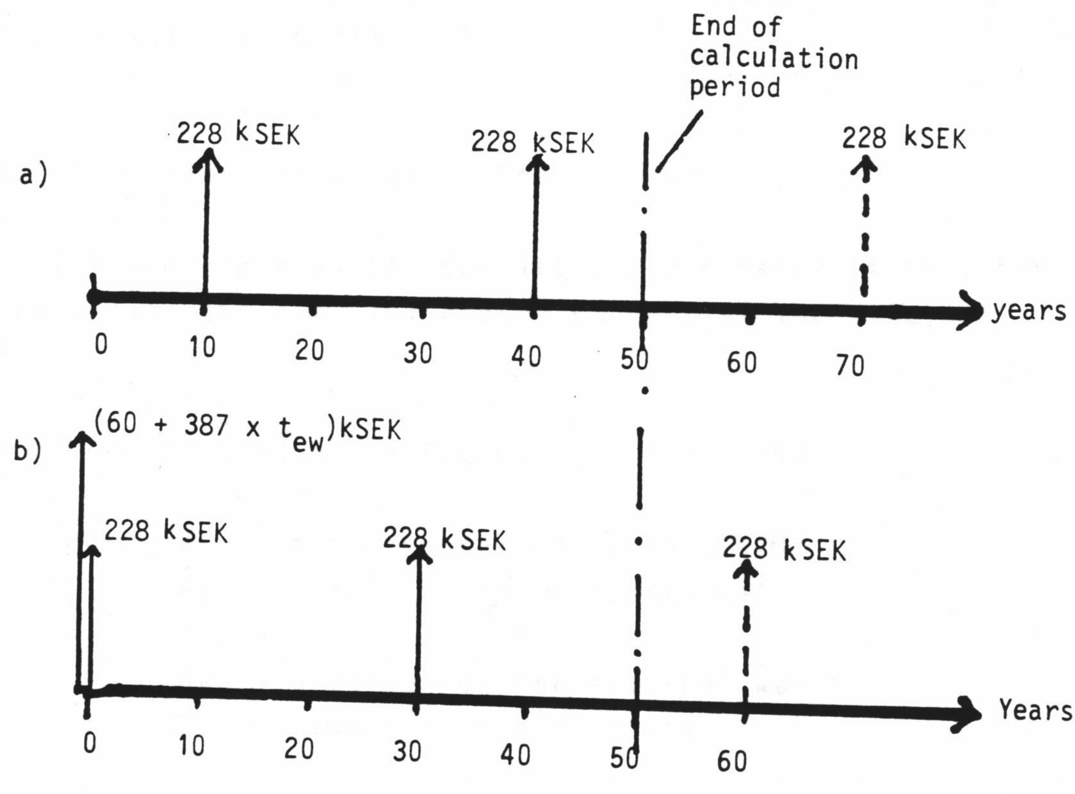


Figure 16. Recurring costs without a) and b) with external wall insulation retrofit.

Calculating the present value for the first case gives with the use of formula (F2).

$$228\ 200(1 + 0.05)^{-10} + 228\ 200(1 + 0.05)^{-40} - 2/3(228\ 200(1 + 0.05)^{-50}) = 140\ 095 + 32\ 414 - 13\ 266 = 159\ 243 \text{ SEK} \quad (F14)$$

The second case results in:

$$RC_{ew} = 228\ 200 + 59\ 500 + 387\ 100 \times t_{ew} + 228\ 200(1 + 0.05)^{-30} - 1/3 \times 228\ 200(1 + 0.05)^{-50} = 287\ 700 + 52\ 800 - 6\ 663 + 387\ 100 \times t_{ew} = 333\ 867 + 387\ 100 \times t_{ew} \quad (F15)$$

The fact that the renovation of the facade now comes earlier than necessary makes the retrofit 333 867 - 59 500 - - 159 243 = 115 124 SEK more expensive. This extra cost must result in a reduced energy cost of that size, if the energy retrofit shall be profitable.

4.1.2.3 The external wall. Energy cost

The same procedure as for the attic floor makes it possible to calculate the cost for energy concerning the external wall.

Annual energy cost = $e \times A_{ew} \times U_{ew} \times D \times 3\ 600$

where e = the energy price in SEK/MJ
A_{ew} = area of the external wall

U_{ew} = U-value for the external wall
D = number of degree hours

With the figures from our numerical example this results in

$$\text{the annual energy cost} = \frac{0.30}{3.6} \times 700 \times \frac{0.0499}{0.0475+1.05 \times t_{ew}} \times 105\ 241 \times 10^{-3} = \frac{1\ 102.8}{0.0475+1.05 \times t_{ew}}$$

Multiplying this with the adequate present value factor 18.26 gives us the life-cycle energy cost:

$$EC_{ew} = \frac{20\ 137.5}{0.0475+1.05 \times t_{ew}} \qquad (F16)$$

4.1.2.4 The external wall, LCC, optimization

As shown above the external wall has a LCC either if any energy retrofits or not are done to the wall. This LCC is (F14) and (F16).

$$\text{Existing } LCC_{ew} = 159\ 243 + \frac{20\ 137.5}{0.0475+1.05 \times 0} = 583\ 190 \text{ SEK}$$

Using the same technique as for the attic floor gives LCC with extra insulation to:

$$LCC_{ew} = 333\ 867 + 387\ 100 \times t_{ew} + \frac{20\ 137.5}{0.0475+1.05 \times t_{ew}}$$

With (F12) the miminum can be calculated as:

$$t_{ew} = -\frac{0.0475}{1.05} + \sqrt{\frac{20\ 137.5}{387\ 100 \times 1.05}} = -\ 0.0452 + 0.223 = 0.18 \text{ m}$$

This makes the minimum LCC_{ew} to 489 708 SEK.

We see that the extra insulated wall has a lower LCC than the existent one, so also in this case it is profitable to insulate.

4.1.2.5 The combination of the attic floor and the external wall. Optimization

To find the LCC for both the attic floor and the external wall I only have to sum the expressions (F11) and (F16). This results in

$$LCC_{ew,af} = 125\ 000 + 300\ 000\ t_{af} + \frac{18\ 438.5}{0.04+0.8 \times t_{af}} +$$

$$+ 333\ 867 + 387\ 100 \times t_{ew} + \frac{20\ 137.5}{0.0475+1.05 \times t_{ew}} \qquad (F17)$$

This expression can be formulated like:

$$f(t_{af},\ t_{ew}) = A + B \times t_{af} + \frac{C}{D+E \times t_{af}} + F \times t_{ew} +$$

$$+ \frac{G}{H+I \times t_{ew}} \qquad (F18)$$

Now the minimum can be found by derivating $f(t_{af},\ t_{ew})$ first with the emphasis on t_{af} and after that with the emphasis on t_{ew}. This is easy done here because the t_{ew} is a constant, when the emphasis is put on t_{af} and vice versa.

The expression (18) thus has its minimum when:

$$t_{af} = -\frac{D}{E} \begin{matrix}+\\(-)\end{matrix} \sqrt{\frac{C}{B \times E}}$$

$$t_{ew} = -\frac{H}{I} \begin{matrix}+\\(-)\end{matrix} \sqrt{\frac{G}{F \times I}}$$

Unfortunately, we have the constraint that the LCC for the insulated attic floor and the insulated external wall shall not be higher than for the existent building parts. Therefore, it is not quite adequate to sum the constants into A in (F18). The new optimization problem therefore can be written:

$$\text{minimize: } Y_1\left(125\ 000 + 300\ 000 \times t_{af} + \frac{18\ 438.5}{0.04+0.8 \times t_{af}}\right) +$$

$$+ Y_2 \times 460\ 962 + Y_3\left(333\ 867 + 387\ 100 \times t_{ew} + \right.$$

$$\left. + \frac{20\ 137.5}{0.0475+1.05t_{ew}}\right) + Y_4 \times 583\ 190$$

subject to: t_{af}, $t_{ew} > 0$

Y_1, Y_2, Y_3, Y_4 = 1 or 0 integers

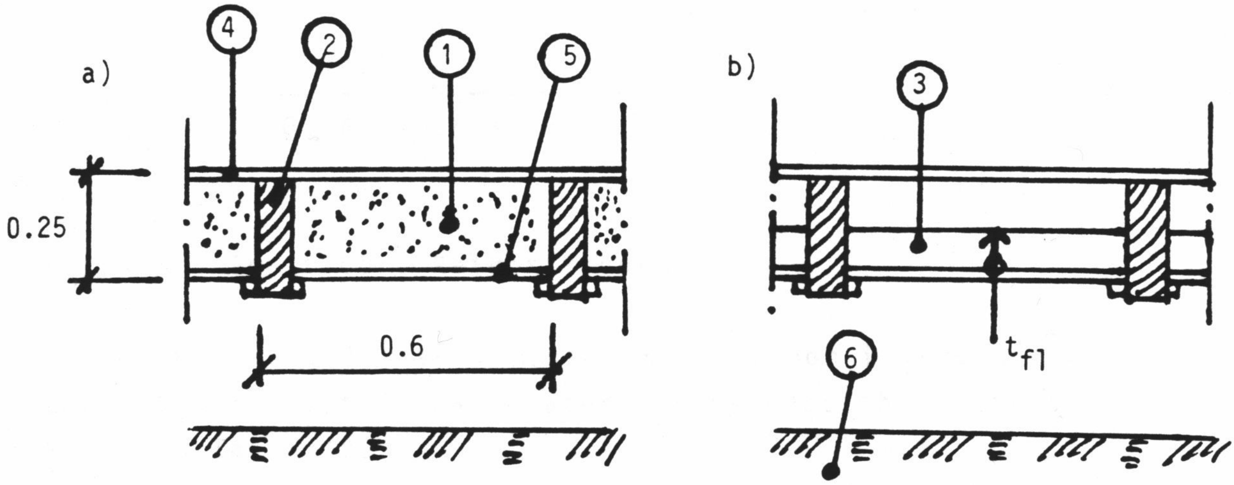
$Y_1 + Y_2 = 1$
$Y_3 + Y_4 = 1$

If $Y_2 = 1$ then $t_{af} = 0$
If $Y_4 = 1$ then $t_{ew} = 0$ (F19)

As shown above the problem is solved when $t_{aw} = 0.23$, $t_{ew} = 0.18$, $Y_1 = 1$, $Y_2 = 0$, $Y_3 = 1$, $Y_4 = 0$ and then the objective function has the value 766 022, which equals the LCC for the attic floor and the external wall together.

4.1.3.1 The floor. Thermal performance

The existent U-value for the floor is 0.14/0.25 = = 0.56 $W/m^2\ {}^oC$. The construction is depicted in Figure 17.



1 = saw dust
2 = joist
3 = mineral wool
4 = wooden floor
5 = boarding
6 = earth

Figure 17. Existing[a)] and retrofitted floor[b)].

Because of the foundation columns the temperature will be slightly higher in the space between the earth and the floor and subsequently it is not quite adequate to use the outside temperature as a parameter to predict the heat transferred through the floor. In this thesis it is beyond the scope to examine these type of boundary conditions and in spite of the facts above I will use the same calculation procedure as for the external wall. In (5 p 203) there has been a modification of the allowable U-value for this kind of spaces, but only if the insulation is better than the code in some other places in the building.

Thermally, the floor thus is considered exactly as the external wall and because of this it will be proper to use the same calculation procedure viz:

$$U_{fl} = \frac{U_{fl,exist} \times 0.0475}{0.0475 + U_{fl,exist} \times t_{fl}} = \frac{0.56 \times 0.0475}{0.0475 + 0.56 \times t_{fl}} =$$

$$= \frac{0.0266}{0.0475 + 0.56 \times t_{fl}} \quad \text{(F20)}$$

4.1.3.2 The floor. Retrofit cost

In (31 Figure 9:22) an almost similar floor is treated and the retrofit cost is calculated. In my case the cost is shown in Table IV.

Table IV Retrofit cost for the floor

	Material	Time	Wages	Sum
A. 0.022 m existing flooring boards. Demolished		0.25	14.50	14.50
B. 0.200 m existing sawdust. Demolished	-	0.12	6.96	6.96
C. Blind floor demolished	-	0.05	2.90	2.90
D. Rubbing and lacquering of the flooring[1]	60.0	-	-	60.0
E. 0.022 New flooring	62.70	0.36	20.88	83.58
F. Mineral wool	$220 \times t_{fl}$	0.16	9.28	$9.28 + 220 \times t_{fl}$
G. Wind protection	3.85	0.04	2.32	6.16
H. 0.013+0.013 boarding	26.70	0.25	14.50	41.20
I. 0.025+0.025 blind flooring batten	5.20	0.17	9.85	15.05

I Sum except for F			71.92	230.37
II Sum only F			9.28	$9.28 + 2.20 \times t_{fl}$

Indirect costs 181 % part I (wages)	130.17
Indirect costs 181 % part II (wages)	16.79
13 % on part D	7.80
Sum part I	368.34
Sum part II	$26.07+220 \times t_{f1}$
Taxes part I 12.87 %	47.40
Taxes part II 12.87 %	$3.35+28.31 \times t_{f1}$
Total sum part I	415.74
Total sum part II	$29.42+248 \times t_{f1}$

Footnote 1. Made by conctractors

The cost for the maintenance of the floor is A + D + E. The cost for wages is 35.38 SEK/m^2 and for material 122.70. Together with indirect costs and taxes the cost will become 250.70 SEK/m^2.

Our mathematical expression for the floor retrofit thus is $(250 + 195 + 250\ t_{f1})$ SEK/m^2.

For the floor in our case, where the floor area is 1 000 m^2:

$$250\ 000 + 195\ 000 + 250\ 000 \times t_{f1}$$

The remaining life for the existing floor is 20 years and thus the present value for the retrofit will be, without insulation and assuming, that the new floor will last for 35 years:

$$250\ 000 \times 1.05^{-20} - 5/35 \times 250\ 000 \times 1.05^{-50} = 94\ 222 -$$

$$- 3\ 114 = 91108 \text{ SEK} \qquad \text{(F21)}$$

For the case of an energy retrofit at the base year:

$$RC_{f1} = 445\ 000 + 250\ 000 \times t_{f1} + 250\ 000 \times 1.05^{-35} -$$
$$- 20/35 \times 250\ 000 \times 1.05^{-50} = 477\ 865 + 250\ 000 \times t_{f1} \qquad \text{(F22)}$$

4.1.3.3 The floor, energy cost, LCC and optimization

In (F20) I have shown the new U-value for the floor. The energy cost will therefore become (see 4.1.2.3):

$$18.26 \times 0.30 \times 1\ 000 \times \frac{0.0266}{0.0475+0.56 \times t_{fl}} \times 105\ 241 \times 10^{-3}$$

$$= \frac{15\ 335}{0.0475+0.56 \times t_{fl}} \qquad \text{(F23)}$$

With $t_{fl} = 0$ the cost for energy during the life-cycle will be 322 842 SEK.

Together with the inevitable retrofit cost (F21) we get 322 842 + 91 108 = 413 950 SEK, which is the LCC without any extra insulation to the floor.

With the extra insulation we have to find the minimum of (F22) + (F23). This can be calculated with:

$$t_{fl} = -\frac{0.0475}{0.56} \underset{(-)}{+} \sqrt{\frac{15\ 335}{0.56 \times 250\ 000}} = -\ 0.0848\ +$$

$$+\ 0.331 = 0.25$$

$$LCC_{fl} = 477\ 865 + 250\ 000 \times 0.25 + \frac{15\ 335}{0.0475+0.56 \times 0.25} =$$

$$= 681\ 007 \text{ SEK}$$

In this case it obvious that it is more expensive to insulate the floor than leaving it as it is, the LCC for insulating is almost 270 000 SEK higher than for the other alternative.

It has to be noted that if the extra insulated floor had been the most profitable, 0.25 m insulation should be put into the floor. A look at the Figure 17 shows that this is not possible without making the existent joists higher. This procedure makes the mathematical expression (F22) invalid, because this had not been estimated in the calculations in Table IV. However, it is possible to solve this problem by making the expression nonlinear as depicted in Figure 18.

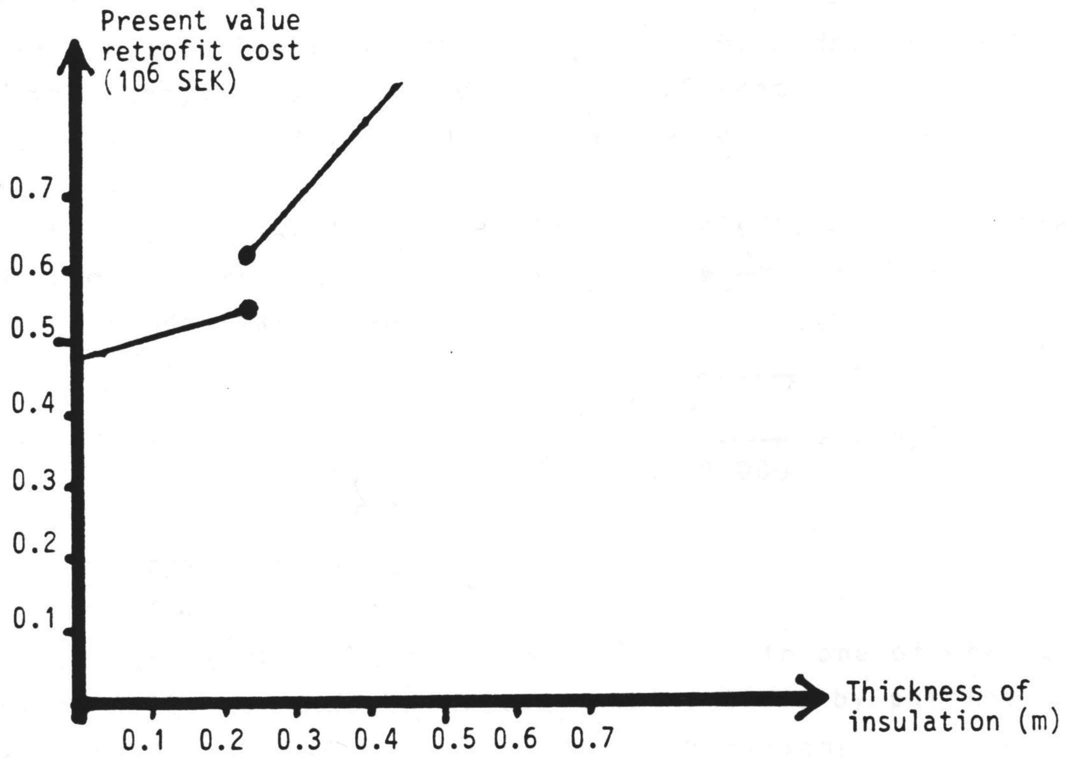


Figure 18. The present value of the retrofit for the floor.

After the point, where the entire space in the existent floor has been filled with mineral wool, the joists have to be made higher. This can be made either by reducing the heigth of the room or the space between the floor and the earth. Of course, this will imply a discontinuous step in the retrofit cost function. The necessary insulation procedure is a little similar to that of insulating an external wall (see 4.1.2.1) and I, therefore, will assume that the retrofit cost function is:

$$477\ 865 + 250\ 000\ t_{fl} \quad \text{when} =< t_{fl} < 0.23 \text{ m}$$

$$620\ 000 + 663\ 000\ t_{fl} \quad \text{when } t_{fl} > 0.23 \text{ m} \qquad \text{(F24)}$$

(The latest expression can be calculated as:

$$477\ 865 + 250\ 000 \times 0.23 + 85\ 000 + 553\ 000 \times t_{f1})$$

Unfortunately, this means that it is no longer possible to use the derivative technique to find the minimum, the function is discontinuous. In (7 p 25 - 62) so called "direct search methods" are described. In these methods, the values of the function is examined and by different procedures, an approximation of the minimum is calculated.

In this case, however, it is also possible to calculate the minimum for the steeper cost function (when $t > 0.23$ m) using the derivative technique.

This makes $t_{f1} = -\frac{0.0475}{0.56} \underset{(-)}{+} \sqrt{\frac{15\ 335}{0.56 \times 553\ 000}} = -0.0848 +$

$+ 0.2225 = 0.13$ m

It is obvious that the minimum is located in one of the lag points 0.23 - or 0.23 +. We can examine these by putting $t_{f1} = 0.23$ m in the two different LCC functions.

In the first case the minimum will be:

$$LCC_1 = 477\ 865 + 250\ 000 \times 0.23 + \frac{15\ 335}{0.0475+0.56\times0.23} =$$

$= 622\ 347$ SEK

and in the other:

$$LCC_2 = 620\ 000 + 553\ 000 \times 0.23 + \frac{15\ 335}{0.0475+0.56\times0.23} =$$

$= 834\ 172$ SEK

It is obvious that LCC_1 is lowest, but as mentioned above the optimal solution for the floor was to leave it as it is.

4.1.3.4 The combination of the attic floor, external walls and the floor

Exactlv as before it is now possible to add the expressions for the floor to the earlier model (F19).

Thus:

$$\text{minimize: } Y_1(125\ 000 + 300\ 000 \times t_{af} + \frac{18\ 438.5}{0.04+0.8 \times t_{af}}) +$$

$$+ Y_2 \times 460\ 962 + Y_3(333\ 857 + 387\ 100 \times t_{ew} +$$

$$+ \frac{20\ 137.5}{0.0475+1.05 \times t_{ew}}) + Y_4 \times 583\ 190 + Y_5\ (X_1(477\ 865 +$$

$$+ 250\ 000 \times t_{fl}) + X_2(620\ 000 + 553\ 000 \times t_{fl}) +$$

$$+ \frac{15\ 335}{0.0475+0.56 \times t_{fl}}) + Y_6 \times 413\ 950$$

Subject to: $t_{af}, t_{ew}, t_{fl} > 0$

$Y_{1-6} = 1$ or 0 integers

$Y_1 + Y_2 = 1$, $Y_3 + Y_4 = 1$, $Y_5 + Y_6 = 1$

$X_1, X_2 = 1$ or 0 integers

$X_1 + X_2 = 1$

If $Y_2 = 1$ then $t_{af} = 0$

$Y_4 = 1$ then $t_{ew} = 0$

$Y_6 = 1$ then $t_{fl} = 0$ (F25)

As is shown above solving this problem makes $Y_5 = 0$, $Y_6 = 1$, $X_1 = 1$ or 0 (does not matter), $t_{fl} = 0$. The value of the object function will be 760 022 + 413 950 = 1 179 972. The rest of the solution is mentioned under (F19).