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Sensitivity analysis of building energy retrofits

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Abstract

When a building is refurbished, energy conservation measures might be profitable to implement. The profitability depends, among other things, on the electricity and district-heating tariffs, the unit price for oil, etc. The cost for the retrofit is of course also important as well as the influence of the retrofit on the demand for heat in the building. By the use of a Mixed Integer Linear Programming model of a building, a number of different optimal retrofit strategies are found depending on the energy cost. The result shows that the Life-Cycle Cost for the building is subject only to small changes as long as the optimal strategies are chosen. Most important is the heating system, while building retrofits such as added insulation, are too expensive to take part in the optimal solution. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Mixed Integer Linear Programming, MILP, used for optimisation of different energy systems, is a valuable method for finding the best way to refurbish a building. During recent years, fast desktop computers have been introduced which makes it possible to optimise models with thousands of variables in just a few minutes. Commercial software for optimisation, however, use obscure input data files for the mathematical problem. These files are therefore written by use of computer programs, in our case a Windows 95 program in C. Input data can therefore be changed by the use of dialog boxes instead of recompiling the total program. All MILP models have an objective function which in our case shows the total cost and, hence, shall be minimised. The function must therefore include all the costs the proprietor pays for the building which add up to the so-called Life-Cycle Cost, LCC. There are

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also some, sometimes several hundred, constraints which ascertain that the building is provided with energy for space heating and so on. Without the constraints, the LCC would become zero and no heat could be used. This is because of the minimisation. One severe calamity with MILP programs is that they can only deal with pure linear problems. Therefore, it is not possible to multiply two variables and, thus, the problem must be divided into pieces which in turn are added to each other. The case study below will clarify the situation.

2. The model

This case deals with an existing building, which must be provided with a certain amount of heat in order to keep an indoor temperature of 21°C. The transmission factor for the building, or the sum of the multiplied U- and area values for walls, attic floor, etc., has been calculated to be 1602 W/K, while the heat lost by the use of the ventilation system is 454 W/K. In Linköping, Sweden, the average mean temperature for January, with 744 hours, is –2.9°C. The energy demand for space heating therefore becomes:

$$(1602 + 454) \times 744 \times (21 - 2.9)/1000 = 36,559\,\mathrm{kWh}.$$

In Linköping it is possible to use district heating with a price of 0.29 SEK/kWh and the cost for January therefore becomes about 10,000 SEK. (One £ equals about 10 SEK.) The building is used for several years and subsequently a present-value factor must be multiplied with the annual cost. For a 50 year period and an interest rate of 5%, this factor equals 18.26.

There are also other ways to heat the building, e.g. an oil-fired boiler with a running cost of 0.39 SEK/kWh. Now, the problem to be solved is how to use this equipment in an optimal way. Is it best to use the oil boiler, or the district-heating system or a combination of both, and what thermal sizes, P_{ob} and P_{dh}, are optimal for the equipment? In our case the first part of the objective function is

$$P_{\mathrm{oil}} \times 744 \times 0.39 \times 18.26 + P_{\mathrm{dh}} \times 744 \times 0.29 \times 18.26,$$

and an applicable constraint is

$$P_{\mathrm{ob}} \times 744 + P_{\mathrm{dh}} \times 744 \geqslant 36,559.$$

Note that the problem is linear. For this simple problem, there is no need for a computer program. When larger problems are solved, the algorithms are simpler if "$\geqslant$" or "$\leqslant$" are used instead of "$=$" in the constraints and, hence, the use of "$\geqslant$" above. The minimisation ascertains that no more energy is wasted than there is actually a need for. By adding more heating systems, all months of the year, and possibilities to reduce the need for heat for example by added extra insulation, the number of variables and constraints grow rapidly.

In Sweden, electricity production is based on nuclear and hydro power plants with low marginal costs for an extra kWh. To some extent this is reflected in our electricity prices and therefore electrical heat-pumps are of interest for heating buildings. The electricity tariff is frequently divided into three levels. In Linköping, the high level, 0.94 SEK/kWh, applies during working days from November to March, between 06.00 and 22.00 hours. The medium price, 0.49 SEK/kWh, is used for weekends and during night time, while the low price, 0.38 SEK/kWh is valid for the other months, no matter what time of the day it is. In order to introduce the electricity tariff in the MILP program, there is therefore a need to split the winter months in to finer time segments than described above.

The model also includes a hot-water accumulator, which can be charged during medium electricity-price conditions, i.e. nights and weekends during the winter and discharged during high electricity-cost conditions. During the weekends, there is more time available for charging and subsequently these time segments must be dealt with separately. During the summer, there is no need for the accumulator because the electricity price is constant. The thermal load must therefore be divided into 22 segments, i.e. three segments during each winter month and seven segments during the summer – see Table 1. The summer segments follow the division of the climate data.

The energy demand must now be divided according to these time segments and therefore the first segment energy demand will equal

$$(1602 + 454) \times 368 \times (21 - 2.9)/1000 = 18,082\,\text{kWh}.$$

The 22 versions of this equation are used to implement the Right Hand Sides, RHSs, of constraints similar to the previous equation. The RHSs, are, however reduced by free energy from appliances, solar radiation through windows, etc. It has been assumed that this free energy emerges only during the day-time segments where these are present, see Ref. [1] for more details. The use of domestic hot water,

Table 1
Hours in each time segment used in the MILP model

Month	High price	Medium price	Low price
January	368	184	192
February	336	168	192
March	336	168	240
April	–	–	720
May	–	–	744
June	–	–	720
July	–	–	744
August	–	–	744
September	–	–	720
October	–	–	744
November	336	168	216
December	352	176	216

i.e. 3500 kWh each month, must be added here and it is assumed that this heat is used only during high electricity-cost conditions, if applicable. The resulting thermal load is shown in Fig. 1. Note that the thermal load for the summer equals the usage of hot-water energy, i.e. about 4.7 kW. The maximum thermal load for the building has been calculated to be 71.9 kW. It is then assumed that the outdoor temperature is −14°C.

In each of the 22 segments, the thermal load need must be satisfied. This could be achieved by the use of up to three different heating devices, namely an oil-fired boiler, a district-heating system and an electrical heat-pump run by the use of electricity.

The MILP program will show how many devices should be used in each time segment, and further their thermal sizes. The introduction of a heating device, however, costs money. In this study, this cost is assumed to be reflected by cost functions:

$$C_{\mathrm{ob}} = 55,000 + 60 \times P_{\mathrm{ob}} \quad \text{for the oil boiler;}$$

$$C_{\mathrm{dh}} = 40,000 + 60 \times P_{\mathrm{dh}} \quad \text{for the district} - \text{heating device,}$$

$$C_{\mathrm{hp}} = 60,000 + 5000 \times P_{\mathrm{hp}} \quad \text{for the heat pump.}$$

These costs must be added to the objective function, but they must only be part of the LCC if they are present in the optimal solution. This behaviour is fulfilled by the use of binary variables, "*A*" which can only have the value zero or unity. The objective function must therefore be appended with:

$$A_{\mathrm{ob}} \times 55,000 + 60 \times P_{\mathrm{ob}} + A_{\mathrm{dh}} \times 40,000 + 60 \times P_{\mathrm{dh}} \ldots$$

Hence, the value 55,000 only comes into operation if A_{ob} equals unity. All the integer variables for the heating devices find their values by the use of a constraint:

$$A_{\mathrm{ob}} \times M \geqslant P_{\mathrm{ob}},$$

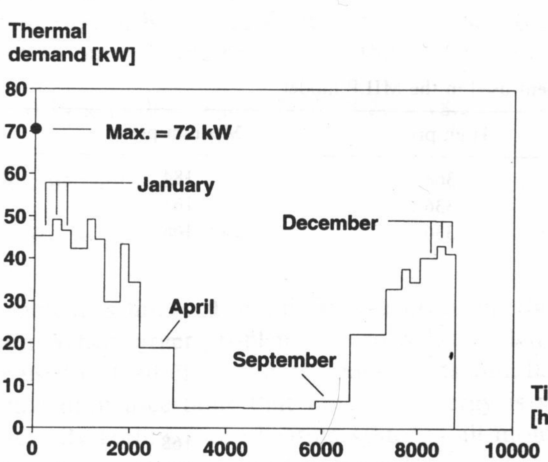

Fig. 1. Thermal demand in the studied building.

where M is a value larger than P_{ob} might ever assume, say 200 kW. See Ref. [2], p. 179, for more details about this so-called fixed charge problem. If the oil boiler is used in any segment, P_{ob} must have a value larger than zero and therefore 22 constraints of the following type must be added for each type of heating system:

$$P_{1ob}/0.75 - P_{ob} \leqslant 0,$$

where 0.75 is the efficiency of the oil boiler system. The subscript 1 shows that this variable applies only for time-segment number 1. The model has been described in more detail in Ref. [1].

3. Optimisation

The model in its present state has 107 constraints and 98 unique variables and therefore a computerised method is needed for the optimisation. Here, the ZOOM

Table 2
Thermal energy need and cost in each time segment

Month	Segment number	Hours (h)	Oil boiler			Heat pump			Total cost (SEK)
			Power (kW)	Energy (kWh)	Energy cost (SEK)	Power (kW)	Energy (kWh)	Energy cost (SEK)	
January	0	368	–	–	–	45.37	16,696	5231	5231
	1	184	2.51	462	240	46.63	8578	1401	1641
	2	192	–	–	–	46.63	8953	1462	1462
February	3	336	–	–	–	42.28	14,206	4451	4451
	4	168	2.72	457	238	46.63	7834	1280	1518
	5	192	–	–	–	44.64	8751	1400	1400
March	6	336	–	–	–	29.78	10,006	3135	3135
	7	168	–	–	–	43.38	7288	1190	1190
	8	240	–	–	–	34.32	8237	1345	1345
April	9	720	–	–	–	18.86	13,579	1720	1720
May	10	744	–	–	–	4.70	3497	443	443
June	11	720	–	–	–	4.86	3499	443	443
July	12	744	–	–	–	4.70	3497	443	443
August	13	744	–	–	–	4.70	3497	443	443
September	14	720	–	–	–	6.46	4651	589	589
October	15	744	–	–	–	22.16	16,487	2088	2088
November	16	336	–	–	–	32.98	11,081	3472	3472
	17	168	–	–	–	37.62	6320	1032	1032
	18	216	–	–	–	34.53	7458	1218	1218
December	19	352	–	–	–	40.26	14172	4440	4440
	20	176	–	–	–	43.17	7598	1241	1241
	21	216	–	–	–	41.23	8906	1455	1455
Total	–	–	–	919	478	–	194,791	39,922	40,400

program Ref. [3], has been used: it solves the problem in just a few seconds on an ordinary PC. The solution is presented in Table 2.

From Table 2, it is obvious that the oil-fired boiler is used only for very short periods of time, see segments 1 and 4. However, it satisfies part of the thermal peak-demand of the building, i.e. 71.9 kW. This can be found by studying the variables P_{ob} and P_{hp} in the output of the program, which are 33.8 and 15.53, respectively. P_{hp} is the electric power and in our case the value must be multiplied by the Coefficient Of Performance, COP, which was set to 3.0. In the same way, P_{ob} must be multiplied by the oil boiler is efficiency of 0.75. The thermal powers of the two systems will then add up to 71.9 kW. District heating is not a part of the optimal solution.

In this case study, it is assumed that the oil boiler already exists in the building and that it has a remaining life of 10 years. The heat pump, however, must be installed at the beginning of the calculation period. The new life of the oil boiler and the heat pump is assumed to be 15 years. Using the interest rate and project life makes it possible to calculate the present value for the devices. The remaining contribution to the LCC is the subscription fee from the electricity tariff, which is 1100 SEK each year. The total LCC is shown in Table 3, which differs only by 100 SEK from the value calculated by ZOOM.

4. Sensitivity analysis

In the base case above, the oil price was 0.39 SEK/kWh. In order to study the retrofit strategy and the resulting LCC, eight new optimisations have been elaborated – see Table 4 – where different oil prices have been considered.

When the oil price is low, so is the total LCC – see Table 4 for an oil price of 0.1 SEK/kWh. The optimal solution was to use solely the oil-fired boiler. No investment in a heat pump was profitable. When the oil price increased by 0.1 SEK/kWh, this was no longer true. A heat pump with a thermal power of 35 kW must be chosen for optimal conditions to prevail. The oil boiler's thermal size was at the same time reduced by 10 kW, but the heat produced in the oil boiler decreased to less than half its original value. The increase in LCC is also substantial. If the oil price is once again raised by 0.1 SEK/kWh, to 0.3 SEK/kWh, the LCC increases but far from the amount found earlier. The heat pump should be increased in thermal size and now the heat demand from the oil boiler has almost vanished. Further increments in the

Table 3
Present-value cost elements for the studied building, base case

Energy cost	40,400 SEK each year	737,704 SEK
Electricity subscription fee	1100 SEK each year	20,086 SEK
Heat pump cost	137,650 SEK each installation year	243,028 SEK
Oil boiler cost	57,030 SEK each installation year	58,295 SEK
Total LCC		1,059,113 SEK

Table 4
Minimised LCC in MSEK and optimal strategy for a varying oil price

Oil Price (SEK/kWh)	LCC (kSEK)	Thermal size oil boiler (kW)	Heat from the oil boiler (MWh)	Thermal size heat pump (kW)	Heat from the heat pump (MWh)
0.1	538.1	71.94	195.7	–	–
0.2	989.9	60.48	80.6	34.47	115.0
0.3	1056.4	35.51	1.6	45.30	194.1
0.4	1059.2	33.84	0.9	46.59	194.8
0.5	1060.5	30.49	0.3	49.08	195.4
0.6	1060.6	30.49	0.3	49.08	195.4
0.7	1060.7	30.49	0.3	49.08	195.4
0.8	1060.8	30.49	0.3	49.08	195.4

oil prices are of almost no significance for the proprietor, because the LCC is from now almost constant. So is the optimal solution. Even if no oil at all was to be used in the building, this small reduction could not pay for yet another heating system. No building retrofit measures were optimal.

Suppose the oil boiler was totally worn out. This will increase the LCC for the installation of a new oil boiler and the total LCC has been calculated by ZOOM to be 1069 kSEK. Now the optimal way to heat the building is to use the heat pump alone, i.e. no oil boiler should be used. The same LCC and the same strategy is optimal if the existing oil boiler has five years left of its life. This is so because the oil boiler is still a part of the optimal solution. If 10 years apply, the LCC decreases to 1059 kSEK and using both the oil boiler and the heat pump is optimal – see Tables 2 and 3. An even longer existing life, now 15 years, will decrease the LCC to 1.044 kSEK but the optimal strategy is identical to that of an existing life of 10 years. For a change in the remaining life of the existing heating system, the strategy therefore changed but the LCC is fairly constant for the different values. The LCC only changed by 2.3% for the examined interval.

If the oil boiler-efficiency is higher, the LCC must be lower when the oil boiler is part of the optimal solution. Subsequently, for a low efficiency, the LCC must increase for those cases as long as the boiler is present and be constant if only the heat pump is optimal. In Table 5, the optimal solutions are shown for varying efficiencies from 0.5 to 1.0 of the oil boiler.

Also here the total thermal power coming out of the heating systems adds up to 71.9 kW. The LCC varies very little for substantial changes in the oil boiler's efficiency, as long as the optimal conditions prevail. When efficiencies of 0.5 or 0.6 are assumed, only one of the time segments showed oil heating as the preferred choice. When an efficiency of 1.0 is used, the four first segments should at least be part satisfied by the oil-fired boiler.

In our model, the oil boiler cost is assumed to be reflected by two values C_1 and C_2, where C_1 was set to 55,000 SEK, and C_2 to 60 SEK/kW in the base case. The

Table 5
LCC and optimal strategy for varying efficiency of the oil boiler

Efficiency (1)	LCC (kSEK)	Thermal size	
		Oil boiler (kW)	Heat pump (kW)
0.5	1062	45.74	49.08
0.6	1061	38.11	49.08
0.7	1060	36.25	46.56
0.8	1058	31.72	46.56
0.9	1057	29.59	45.33
1.0	1055	27.36	44.58

installation cost C_1 of the boiler is important, but not until the cost is so high that the optimal strategy totally changes, or in other words when the oil boiler falls out of the optimal strategy. This is examined in Table 6.

For low C_1 costs, the LCC varies slightly while the optimal strategies remain constant. This is so because the C_1 cost does not influence the sizes of the oil boiler or the heat pump. This constant only raises the LCC. When the cost exceeds over a certain value, the cost is so high that the oil boiler is abandoned in the optimal solution and after this the LCC will not change at all.

This is also valid for the C_2 constant as can be found in Table 7.

The result was however, not expected. Changing the C_2-value should influence both the oil boiler and the heat pump thermal sizes. This behaviour might be explained by the fact that the optimisation only can result in equipment which can vary only in discrete steps. The model does not show a continuous function. In order to examine this, also the C_2 constant for the heat pump has been examined – see Table 8.

From Table 8, it is obvious that changes in the C_2 constant influence the sizes of the heating equipment. The changes are, however, small even for significant changes of the constant. A further increase of C_2, to 12,000 SEK/kW, results in a new optimal strategy, i.e. the district-heating system comes into operation. When this happens, district heating is the only heating system that should be used, but it should be combined with adding an extra 0.06 m mineral wool to the attic-floor insulation.

Table 6
LCC and optimal strategy for varying costs C_1 for the oil boiler

Oil boiler cost C_1 (SEK)	LCC (kSEK)	Thermal size	
		Oil boiler (kW)	Heat pump (kW)
40,000	1044	33.83	46.56
50,000	1054	33.83	46.56
60,000	1064	33.83	46.56
70,000	1069	–	71.94
80,000	1069	–	71.94

Table 7
LCC and optimal strategy for varying costs C_2 for the oil boiler

Oil boiler cost C_2 (SEK/kW)	LCC (kSEK)	Thermal size	
		Oil boiler (kW)	Heat pump (kW)
100	1060	33.83	46.56
200	1064	33.83	46.56
300	1067	33.83	46.56
400	1069	–	71.94
500	1069	–	71.94

Table 8
LCC and optimal strategy for varying costs C_2 for the heat pump

Heat pump cost C_2 (SEK/kW)	LCC (kSEK)	Thermal size	
		Oil boiler (kW)	Heat pump (kW)
6000	1086	33.83	46.56
7000	1113	35.51	45.33
8000	1140	35.51	45.33
9000	1166	36.49	44.58
10,000	1190	33.92	40.41

For the base case the district-heating energy price is as low as 0.26 SEK/kWh. It may seem peculiar that this system is not optimal from the start. There are, however, more elements in the tariff that influence the cost for district heating, e.g. an annual fee of 4000 SEK and a subscription fee of 260 SEK/kW. If the annual fee is reduced to 1 SEK, the optimal solution includes both the district-heating system and the heat pump. District heating is then used for covering the peak. A reduction of the subscription fee only, to 1 SEK/kW, will however not change the optimal solution. The running cost for district heating is therefore very important for the optimal strategy, but the resulting LCC is fairly constant due to the change in heating systems. This can be found in Table 9.

The influence of the unit electricity price can be shown by changing the COP of the heat pump. In the original case, this value is set to 3.0 which might be valid for a ground-water coupled device. In Table 10, the COP has been varied from 2.0 to 3.0.

Table 9
LCC and optimal strategy for a varying energy cost for district heating

District heating price (SEK/kWh)	LCC (kSEK)	Thermal size		
		Oil boiler (kW)	Heat pump (kW)	District heating (kW)
0.19	0.986	–	–	75.74
0.20	1.024	–	–	75.74
0.21	1.059	33.83	46.56	–
0.22	1.059	33.83	46.56	–

Table 10
LCC and the optimal strategy for a varying COP for the heat pump

COP (1)	LCC (kSEK)	Thermal size			Insulation (m)
		Oil boiler (kW)	Heat pump (kW)	District heating (kW)	
2.0	1227	–	–	69.31	0.06
2.2	1227	–	–	69.31	0.06
2.4	1227	–	–	69.31	0.06
2.6	1189	32.81	41.23	–	0.06
2.8	1121	33.80	46.59	–	–
3.0	1059	33.83	46.56	–	–

When the COP is low, district heating is used. Further, an extra 0.06 m insulation should be added to the attic floor. This strategy will not change until the COP becomes 2.6 when the district heating is abandoned and the oil boiler and heat pump are used instead. Still, extra insulation is optimal. When the COP is 2.8, this further insulation is not necessary and the optimal solution is to use only the "dual fuel" system.

The insulation measures in the model use three constants for showing the actual building cost. The first cost D_1 is applied for showing the cost for scaffolding etc. or measures not directly coupled with the extra insulation. D_2 shows the cost which is introduced when the first few centimetres of insulation are applied while the third constant D_3 is a cost coupled to the thickness of insulation. D_1 and D_2 are expressed in SEK/m^2 while D_3 shows the cost in SEK/m^2 m. The total cost for an insulation measure is therefore

$$\text{Insulation cost} = [D_1 + D_2 + D_3 t]A,$$

where "t" equals the thickness of the new insulation and "A" the area of the building asset. In this case study, it has been assumed that D_1 has a value of zero because an attic-floor insulation is considered. D_2 has an assumed value of 260 SEK/m^2 while D_3 is set to 530 SEK/m^2 m. As mentioned above, the constants not multiplied by a variable only affect the level of the cost and not the actual thickness of optimal insulation. However, they might have an importance for deciding if insulation should be applied or not. In Table 11, the optimal solutions are shown for various levels of the D_3 cost. When the insulation cost is low, about 0.12 m of new insulation should be added to the attic floor. Naturally, less insulation should be applied if the cost increases. It is important to note, however, that for a certain level of the cost, the measure is abandoned from the optimal solution. Adding e.g. 0.05 m of insulation is therefore not profitable and the best solution is to leave the building shield as it is. If the insulation cost is increased above this level, the LCC will not change at all. It shall also be noted here that the MILP model deals with the insulation in discrete steps of 0.02 m. It is therefore not possible to achieve a solution with for

Table 11
LCC and optimal strategy for a varying building cost for extra insulation

D_3 (SEK/m^2 m)	LCC (kSEK)	Thermal size			Insulation (m)
		Oil boiler (kW)	Heat pump (kW)	District heating (kW)	
50	1035	30.86	41.79	–	0.12
100	1041	30.94	41.91	–	0.10
150	1046	31.06	42.11	–	0.08
200	1050	31.06	42.11	–	0.08
250	1054	31.24	42.42	–	0.06
300	1057	31.24	42.42	–	0.06
350	1059	33.83	46.56	–	–

example a layer of 0.11 m. See Ref. [4] for details on how insulation measures are dealt with in MILP programs.

5. Conclusions

MILP models are a suitable way for optimising building retrofits. The sensitivity analyses show that a change of input data not always affect the resulting LCC in a way that might be expected. If, for example, the oil price is increased this will be important only inside a certain interval. When this is not valid, the LCC is constant because the oil boiler is abandoned by the optimisation. Even if the LCC is increased, this is many times less than expected because different optimal solutions come to the rescue. The technique always finds the cheapest way to heat the building, many times in ways which are very hard to find by way of experience or traditional calculations.

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