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Mixed integer linear programming and building retrofits

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Abstract

When a building is subject for refurbishment it is important to add only such measures that will reduce the Life Cycle Cost (LCC), for the building. Even better is to add measures that will, not only reduce the cost, but minimise the LCC. One means for such an optimisation is to use the so called Linear Programming (LP), technique. One drawback with LP models is that they must be entirely linear and therefore two variables cannot be, for example, multiplied with each other. The costs for building retrofits are, however, not very often linear but instead 'steps' are present in their cost functions. This calamity can, at least to a part, be solved by introducing binary integers, i.e., variables that only can assume 2 values, 0 or 1. In this paper it is described how to design such a Mixed Integer Linear Programming (MILP), model of a building and how different cost elements of the climate shield influence the optimal solution.

Keywords: Building retrofits; Mixed integer liner programming (MILP); Linear programming (LP)

1. Introduction

In recent years linear programming (LP) and mixed integer linear programming (MILP) have found increased interest among researchers in applied engineering. The reason for this is partly due to the introduction of fast personal computers on everyone's desk. Problems that took hours or even days to solve can, nowadays, be solved in minutes or even seconds. It is therefore possible to design models with several thousand variables without having to wait for hours to see the optimal result. This is especially valid for MILP problems because the so-called branch and bound method must solve two LP problems for each integer that is introduced. The model is initially optimised by assuming that no variables are integers. When this is done, the problem is split into two LP problems: one problem where one of the integers is bound to a value less than or equal to zero, and another problem where the integer is set to a value greater or equal to 1. An MILP problem therefore needs substantially more time to be solved compared to an ordinary LP.

There are numerous papers about LP and MILP programming in scientific journals, see e.g., Refs. [1–5]. Papers about MILP and buildings, however, are not very common but some have been presented in recent years, see e.g., Refs. [6,7].

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2. The MILP model

All LP and MILP problems have a mathematical expression called the objective function. In our case, this function shows the total LCC for the building and the expression is, therefore, to be minimised. One way to achieve this is to set all variables to zero but if such is the case, no heating or building activity is present. A number of constraints must therefore be introduced. One constraint, for example, ascertains that enough heat is supplied to the building while others are used for finding proper thermal sizes of different heating equipment, which can be possibly installed in the building. In Ref. [7], the method is shown in detail for heating equipment and insulation measures.

The need for space heating in a building depends on the climate. It is not possible or at least very impractical to use the outdoor temperature in every moment and from this, calculate the energy cost for a long period of time. Therefore, there is need for splitting 1 year into several segments and use monthly mean temperatures as a base for the calculations. In Sweden, the electricity rates sometimes make it profitable to use heat pumps for space and hot water heating. The electricity rate is high during the winter and low at summer. The price also differs according to the time of day. Weekend rates are also low in some months. Hence, we found it practical to divide the year into 22 segments where the months November–March are split into three segments each, while each month from April to October are alloted into one segment

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each. The need for space and domestic hot water heating is presented in Fig. 1.

In January, the first segment includes 368 h where the electricity rate is high. The total amount of energy in this segment has been calculated to 16,697 kWh and the demand is 45.37 kW (see Ref. [8] for details). The need for heat must be covered by district heating, a heat pump, an oil-fired boiler or a mix of these systems. The thermal sizes of these heat sources are not known so three variables, P_{DH01}, P_{HP01}, and P_{OB01}, are introduced. The index 01 shows that the first segment is considered. The cost for district heat in Linköping, Sweden, is 0.26 SKr/kWh, the running cost for the oil-fired boiler is 0.39 SKr/kWh, while the electricity cost is 1.01 SKr/k Wh in this high-cost segment (1 US$ = 7 SKr). Each system has an efficiency which is set to 0.95 for district heating, 0.75 for the oil boiler and 3.0 for the heat pump. It is assumed that the system is used during the next 50 years and that the real discount rate is 5%, which leads to a present value factor of 18.26. The first small part of the objective function can now be elaborated:

$$[0.26\times(1/0.95)\times P_{DH01}+0.39\times(1/0.75)\times P_{OB01} +1.01\times(1/3.0)\times P_{HP01}]\times368\times18.26 \quad (1)$$

The other 21 segments are added in a similar way. The equipment must also be installed and purchased. It is assumed that the different systems costs are:

$40{,}000+60\times P_{DH}$	For district heating
$55{,}000+60\times P_{OB}$	For the oil boiler
$60{,}000+5000\times P_{HP}$	For the heat pump

The costs, however, must also be calculated as present values. The practice life for the district heating system is assumed to be 25 years while the oil boiler and the heat pump is thought to be 15 years. Further, assuming a total project life of 50 years and a real discount rate of 5%, the present value for the district heating system will become:

$$(40{,}000+60\times P_{DH})\times(1+(1+0.05)^{25}) =51{,}812+77.72\times P_{DH} \quad (2)$$

Note that P_{DH}, etc., are now presented without indices and therefore shows the maximum thermal size of the equipment. So, the model must include expressions for finding the maximum need for, e.g., district heat in all the 22 segments. This is implemented through the use of 22 constraints for each heating device, and one is shown here:

$$(1/0.95)\times P_{DH01}-P_{DH}\leq0.0 \quad (3)$$

As can be seen above, the cost for the district heating equipment starts with a step, i.e., 40,000 SKr. This cost must be present in the objective function but only if the district heating is optimal to use. This behaviour is achieved by implementing a binary variable A_1, which only can assume the values of 0 or 1, and by introducing one more constraint:

$$A_1\times M-P_{DH}\geq0.0 \quad (4)$$

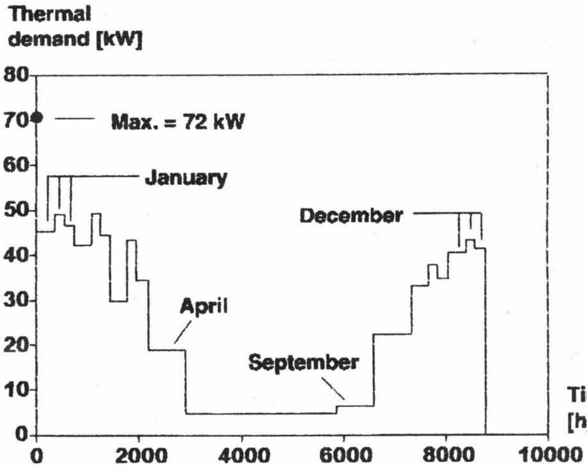

Fig. 1. Thermal demand for the studied building, [8].

M is set to a value higher than P_{DH} might ever take, e.g. 200. Note that the maximum demand is about 72 kW in Fig. 1. If P_{DH} has a value greater than zero, A_1 must be equal to 1. A_1 must also be present in the objective function and then multiplied by the cost 51,812 SKr. Because of the minimisation, A_1 becomes 0 if $P_{DH}=0$ as well. This part of the objective function therefore becomes:

$$A_1\times51{,}812+77.72\times P_{DH}+A_2\times56{,}260+61.33 \times P_{OB}+A_3\times105{,}933+8827\times P_{HP} \quad (5)$$

A sufficient amount of heat must be supplied to the building. In the first time segment, this amount has been calculated to 16,697 kWh. By using 22 constraints, of which the first is:

$$(P_{DH01}+P_{OB01}+P_{HP01})\times368\geq16{,}697 \quad (6)$$

this is achieved.

It is also possible to affect the energy need in the building by applying extra insulation and better windows. The method used for extra insulation is partly presented in Ref. [9] and is therefore only described briefly here. The new U-value for an extra insulated wall can be calculated as:

$$U_{NEW}=k_{NEW}\times U_{EXI}/(k_{NEW}+U_{EXI}\times t) \quad (7)$$

where k_{NEW} is the conductivity for the new insulation in W/m² °C, U_{EXI} is the existing U-value in W/m °C, and t, the thickness of the added insulation in metres. Unfortunately, Eq. (7) is not linear and thus, the so-called stepwise linearisation must be used. A number of binary integers (11 variables were used in our case) must be introduced. The first integer, IS_0, is applied for 0.02 m of extra insulation, the second one for 0.04 m and so on. Only one of the integers can be 1 while the others must be 0. If all integers are 0, it is not optimal to add insulation at all, see constraint (8).

$$IS_0+IS_1+IS_2+IS_3+\cdots+IS_{10}\leq1 \quad (8)$$

After this, the integers IS are coupled to the cost for extra insulation and are added to the objective function. By Eq. (7), they are also coupled to the decrease of energy demand in the building and are added to expression (5). Insulation is useless outside of the heating season. An energy balance for the building shows that four segments, (viz. 11–14) only need energy for domestic hot water heating. The IS variables are, therefore, not present for those segments.

The same procedure is valid for windows but here, the different window constructions are coupled to a set of binary integers. The model must deal with insulation measures for the attic, the floor and the walls, which can be insulated both on the outside and on the inside of the house. Three types of window retrofits are dealt with: triple-glazed windows, windows with low emissivity coatings and gas filled windows. Four different orientations (north, east, south and west) are also present. So, there are about 75 binary integers present in the model.

One very important factor to deal with is the current state of existing windows or the facade of the building. If the windows for example, are affected by rot they must be changed immediately to new ones while the remaining life of existing windows is set to null. If this is not the case, they have a salvage value which must be considered. This is dealt with by the use of a so-called unavoidable, or inevitable retrofit cost. In our case study, 27 windows are oriented to the east. Each window has an area of 2.8 m^2 and the cost for a new window of the same type in current costs is assumed to be 1100 SKr/m^2. If no measures are made for thermal reasons, 83,160 SKr must be invested in order to change the poor existing windows to new ones of the same type. Assume that new windows last 30 years before they have to be changed again. A present value calculation for 50 years and a 5% discount rate shows:

$$83{,}160+83{,}160\times(1+0.05)^{-30}-(83{,}160/3)\times(1+0.05)^{-50}$$
$$=99{,}984 \text{ SKr}$$

If original windows are in perfect condition, no investment will have to be made for 30 years and the expression would have looked like:

$$83{,}160\times(1+0.05)^{-30}-(83{,}160/3)\times(1+0.05)^{-50}$$
$$=16{,}824 \text{ SKr}$$

If triple-glazed windows (cost: 1300 SKr/m^2) are installed at year 0, the present value becomes 118,162 SKr. In the first case with poor windows, the better thermal behaviour must save 18,178 SKr before triple-glazed windows are profitable while they have to save 101,338 SKr if the original windows are in perfect shape. The same procedure must be considered for all the building measures in the model. Table 1 shows the inevitable costs for the building if no thermal improvement is made and for cases where better windows and added insulation are applied.

To the unavoidable cost above, the actual cost for the retrofit must be added which, in turn, depends on what solution is optimal. If none of the 'thermal' retrofits are optimal, 407,632 SKr must be added to the objective function. If triple-glazed, east-oriented windowst are optimal, then the unavoidable cost is 307,648 SKr while new windows cost 118,162 SKr or a total cost of 425,810 SKr. This sum must, therefore, be compared with the unavoidable cost when no retrofits at all are present and the difference, i.e., 18,178 SKr, which is coupled to a binary variable and added to the objective function. The model must therefore include a new set of constraints where the first sets the unavoidable cost if no retrofits are optimal:

Table 1
Unavoidable or inevitable retrofit costs (SKr) for the building

Measure	Cost (SKr)
No retrofit	407,632
Attic floor insulation	407,632
Floor insulation	407,632
External wall insulation, outside	222,832
External wall insulation, inside	376,832
Windows	
North	407,632
East	307,648
South	407,638
West	315,584

$$\mathrm{IS}_0+\mathrm{IS}_1+\cdots+\mathrm{IS}_{10}+F_{00}+F_{02}+\cdots+F_{23}+\mathrm{NOR}\geq 1 \quad (9)$$

The binary integers F_{00} to F_{23} shows that window retrofits are optimal if the values equal 1. If all the IS and F integers equal 0, NOR (for no retrofit) must be 1 and this binary integer is then coupled with the cost 407,632 SKr above and inserted in the objective function. The second constraint adds the same value if one of the retrofits are chosen:

$$\mathrm{IS}_0+\mathrm{IS}_1+\cdots+\mathrm{IS}_{10}+F_{00}+F_{01}+\cdots+F_{23}+M\times R-\geq M+1 \quad (10)$$

M is set here to a value higher than the possible sum of all retrofit integers, in our case 200. If one or more retrofits are optimal, R (for retrofit) must be 1 and the R variable is coupled to the unavoidable cost and inserted in the objective function. This awkward way is needed because it is not possible to add just a value to the objective function. The unavoidable cost is needed here in order to achieve the accurate LCC but it does not affect the optimal solution. The next four constraints are needed because it should not be possible to add both triple-glazed windows and windows with better thermal performance in the same orientation at the same time. One constraint is:

$$F_{00}+F_{10}+F_{20}\leq 1 \quad (11)$$

The first figure in the index 00 shows that it is triple-glazed windows while the second figure shows the orientation where 0 means north, 1 means east and so on.

In our case, the demand charge in the electricity tariff depends on the fuse that must be used, see Table 2.

The model must therefore include expressions that calculates the current and which set proper values in the objective

Table 2
Fuse tariff for Linköping Sweden

Fuse size (A)	16	20	25	35	50	63	80	100
Annual cost (SKr)	1025	1165	1375	1863	2713	3475	4525	6338

function. In this case study, electricity can only be used for running two heat pumps. The first one, P_{HP}, is used as a normal heating device taking the heat from the ground water while the other one, P_{EA}, takes it from the exhaust air. (It is not plausible that both heat pumps are optimal but we do not know in advance which solution is preferrable.) The efficiency for the first one is set to 3.0 while the second one is set to 2.0. Hence, the following constraints are used:

$$(1/3.0)\times P_{HP01}-P_{HP}\geq 0$$

Compare with expression no. (2) (12)

$$(1/2.0)P_{EA01}-P_{EA}\geq 0 \quad (13)$$

$$P_{HP}+P_{EA}-P_{EL}\leq 0 \quad (14)$$

$$-(1000/380\times 3^{0.5})\times P_{EL}+\mathrm{CU}=0 \quad (15)$$

Eq. (15) is only used for calculating the current when we know the demand for electricity and the voltage, which is 380 V. If the current is lower than 35 A, but higher than 25 A, an annual cost of 1863 SKr must be present in the objective function (see Table 2). This is achieved by using 8 new binary integers, E, and 8 integers Y. In each set, only one of the integers can be 1. Eight new constraints must be present in the model and the first one is presented below:

$$CU-16\times E_0+M1\times Y_0\leq M1 \quad (16)$$

$M1$ is a large value (in our case 10,000). If Y_0 is set equal to 1, and CU is lower than 16 (which is the first fuse size in Table 2), E_0 finds the value 1. E_0 is then coupled to the annual cost 1025 SKr, which is present in the objective function as a present value. If CU is larger than 16, the E_0 must be set to 0.

The model now includes 150 constraints and 182 variables where 74 are binary integers.

3. Optimisation

The model above is implemented in a Windows 95 program and written in classic C. The program writes the mathematical problem to a so-called MPS-file, which is an often used standard. Several optimisation codes can read such files, e.g., CPLEX or LAMPS, but we have used the ZOOM program [10] because we have some experience in that product. By using ZOOM, it is possible to find the optimal way to heat the building. First an oil-fired boiler with thermal size 21.3 kW, must be combined with a 37.8-kW heat pump, which add up to 59.1 kW. Two building retrofits were also optimal viz., low emissivity triple-glazed windows and weather-stripping. The first measure decreases the demand from 71.96 to 61.80 kW while the second lowers the demand to 59.1 kW. The heating equipment is therefore sufficient in thermal size. In Table 3, the energy need for the optimal solution is shown in detail.

Table 3
Optimal demand (kW) energy need (kW h) and costs (SKr) for energy use in the studied building

Segment no.	No. of h	Oil-boiler			Heat Pump			Total costs
		Demand	Energy	Cost	Demand	Energy	Cost	
1	368	–	–	–	36.62	13,476	4536	4536
2	184	2.51	462	240	37.88	6970	1308	1548
3	192	–	–	–	37.88	7272	1365	1365
4	336	–	–	–	33.49	11,253	3788	3788
5	168	2.68	450	234	37.88	6364	1194	1428
6	192	–	–	–	35.85	6883	1292	1292
7	336	–	–	–	22.06	7412	2495	2495
8	168	–	–	–	35.66	5990	1124	1124
9	240	–	–	–	26.59	6382	1198	1198
10	720	–	–	–	13.11	9439	1428	1428
11	744	–	–	–	4.70	3496	529	529
12	720	–	–	–	4.86	3499	529	529
13	744	–	–	–	4.70	3496	529	529
14	744	–	–	–	4.70	3499	529	529
15	720	–	–	–	3.23	2326	352	352
16	744	–	–	–	17.07	12700	1922	1922
17	336	–	–	–	26.28	8830	2973	2973
18	168	–	–	–	30.93	5196	975	975
19	216	–	–	–	27.83	6011	1128	1128
20	352	–	–	–	32.58	11,468	3861	3861
21	176	–	–	–	35.49	6246	1172	1172
22	216	–	–	–	33.55	7247	1359	1359
Sum	8784	–	912	474	–	155,455	35,586	36,060

From Table 3, the average energy cost for each kWh can be calculated, i.e., 0.23 SKr. This value is due to the heat pump. This is also the reason for why there are only two building retrofits being present in the optimal solution. It shall be noted here that time segment number 15 only shows 3.23 kW where instead it should have been 4.86 kW. The higher value must be present in order to provide the building with domestic hot water. The reason for the wrong value is due to the energy balance for the existing building where no retrofits were implemented. The balance shows that the segments 11–14 are only used for hot water heating while segment 15 must use heat also for space heating. The binary variables coupled to window retrofits and weather-stripping for segments 11–14 are therefore not present in the in the MILP model. Segment 15, however, has such integers and therefore the model saves heat due to the retrofits even if the value falls lower than 3500 k Wh/month. The heating season will shrink if more retrofits are optimal but the phenomenon is present only in one time segment here. The total energy cost in Table 3, which occur every year, must now be calculated as a present value. The cost is therefore multiplied with 18.26, see expression (1). The heating equipment cost can be calculated by use of expression (5). It was optimal to choose triple-glazed windows with low emissivity coating. The cost for such windows is assumed to be 1500 SKr/m^2 while weather-stripping has a cost of 14,000 SKr and a life span of 10 years. In Table 4, all these costs are presented as present values and the sum represents the total LCC.

It was not found profitable to add extra insulation to the climate shield. Experience from the OPERA-model shows, however, that at least extra attic floor insulation (many times) is a profitable retrofit, [11]. When MILP is used, it is not possible to use the so-called ranging method, i.e., to determine in which interval a variable becomes optimal. Therefore, it is necessary to change a variable in the input data and optimise the problem once again. One suitable parameter to change is present in the cost function for insulation. The cost for all insulation measures is presented in the following form:

$$C_{ins}=C_1+C_2+C_3\times t \quad (17)$$

where C_1 shows the unavoidable cost in SKr/m^2, C_2 the 'step' cost for the insulation in SKr/m^2, C_3 the cost in SKr/(m$^2\times$m) and t the added amount of new insulation in metres. If the cost C_2 is changed it will only affect the possibility for insulation to be optimal, not the amount of insulation that should be added which is the case if C_3 is changed. In the original case, C_2 was set to 260 SKr/m^2 and this is now changed to 200. The optimisation now shows that 0.14-m attic floor insulation must be added. Because of this, a slightly smaller heat pump should be used and, further, the total LCC is reduced to 1.454 MSKr. (The shift between extra insulation or not seems to emerge for a C_2 cost of about 240 SKr/m^2).

As shown above, it was found that triple-glazed windows were optimal. This is so because the original double-glazed windows were worn out and their remaining life was set to 0 years. Suppose they have 20 years left before they must be changed. A new optimisation shows that window retrofits will no longer be optimal, but instead 0.16 m of extra insulation should be added to the attic floor. Some extra optimisations show that window retrofits fall out from the solution if the original ones have approximately 15 years left of their remaining life.

Table 4
Present value costs and LCC (SKr) for the studied building

Oil-boiler (28.4 kW)	58,001
Heat pump (12.6 kW)	217,153
Fuse tariff (20 A)	21,273
Energy	658,456
Windows, east	136,341
Windows, west	125,521
Weather-stripping	33,099
Unavoidable retrofit cost	215,600
Life-cycle cost	1,465,444

4. Conclusions

It is shown that a building can be described mathematically in the form of a Mixed Integer Linear Program (MILP) model. The integers are very important because 'steps' in the cost functions can be dealt with. Small changes in these steps might result in different optimal solutions. Fortunately, the optimisation results in solutions that differs very little from each other in terms of the minimised Life-Cycle Cost (LCC). Small errors in input data therefore do not necessarily lead to hazardous solutions as long as the proprietor acts in an optimal way. However, if combinations of measures that do not fit together are chosen, the result is likely to be an expensive experience.

Acknowledgements

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