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Optimisation and simulation of building energy systems

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Abstract

The Mixed Integer Linear Programming (MILP) technique is a useful tool for the optimisation of energy systems. However, the introduction of integers in linear models results in a severe drawback because the ranging process is no longer available. Therefore, it is not possible to study what happens to the solution if input data are changed. In this paper, we compare a MILP model of a building with a simulation model of an identical case. Both models describe a building with a number of possible retrofits. Using the MILP technique, the optimal retrofit strategy is calculated, after which certain input data are changed. The optimisation results in the lowest possible Life-Cycle Cost (LCC) of the building, and the paper describes how much the LCC will change if the property owner chooses other solutions. An increase in a particular data value may cause the LCC to increase or decrease. It may also be unchanged. Only a few data reduce the LCC when their values are increased. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Sensitivity analysis; MILP programming; Buildings; Energy systems; Life-cycle costing; Retrofitting

1. Introduction

When a building is to be retrofitted, a number of measures are possible. For example, the existing two-paned windows can be replaced with new triple-glazed types or the existing boiler can be replaced with a district heating system, if such a system is available. Several hundred combinations exist and it is not easy to choose the best strategy. The Life-Cycle Cost (LCC), which sums all costs during a certain period of time, provides a criterion for finding the best solution, i.e. when the LCC is as low as possible. The strategy is thereby optimised and no

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solution with a lower LCC will be created as long as the input data are the same. Changing these data might, however, result in a different optimal solution. It should be stressed that the result is optimised from an economic point of view. The thermal standard of the building will therefore not always change even if it is the common practice. In MILP programming, the LCC is set up in a so-called "objective function" which is to be minimised. Assume that the thermal size of the boiler is P_b kW. The cost C_b, expressed as SEK/kW, for the boiler depends on the size, and hence, the total cost will be $P_b \times C_b$ SEK. The actual size of P_b is not known, but will be a part of the result from the optimisation. A very simple solution is to choose the value zero for all the variables, such as P_b, although in this case no heating is provided in the building. A number of constraints are therefore introduced, all of which must be fulfilled in a valid solution. One such constraint can ascertain that a number of kWh are provided by using the boiler. The actual number is also the result of the optimisation. In most cases, only a few of the constraints are actually used for the specific optimal economic strategy, but it is not easy to determine them in advance. Some of the costs of building equipment are not totally linear, but instead show incremental behaviour. If a wall is retrofitted and extra insulation is to be applied, there is a "starting cost" which must be considered. Such steps are dealt with by using integers, i.e. if insulation is to be applied, a high price must be paid before the insulation retrofit actually starts. Such costs occur, for example, in the case of demolition of the existing facade. If the insulation is optimal, the integer is set to 1 and the cost is included in the optimal LCC. If the opposite is valid, the integer equals zero. LP and MILP programming from a mathematical viewpoint is dealt with in [1] or [2], and is therefore not covered here in greater detail.

2. Optimisation

The cost of heating a building varies according to the climate during the year. If electricity is used, which is common in countries, such as Sweden and Norway, the cost might also depend on the time of day. High price periods apply on winter working days, while a lower price is charged in summer weekends. District heating tariffs may also be divided into such time-of-use tariffs. Since the Linear Programming (LP) method is used, the model must be linear and it is not possible to multiply two variables. The energy need has to be split up into a number of segments and the energy in that segment multiplied by the applicable cost. The energy cost is incurred every year, and thus, a present-value factor must be introduced. For a 50-year project life and an interest rate of 5%, this factor will be 18.26. Our objective function, therefore, includes the following expression, where P_{1hdh}, P_{1hhp} and P_{1hob} indicate the thermal need in kW for a district heating system, a heat pump and an oil-boiler in the January high cost segment.

$$\left(P_{1hdh} \times 0.26/0.95 + P_{1hhp} \times 0.94/3.0 + P_{1hob} \times 0.39/0.7\right) \times 368 \times 18.26$$

The district-heating price is 0.26 SEK/kWh and the efficiency is 0.95. The other values refer to the heat pump and the oil-boiler. Only one time segment is shown here and it has a length of 368 h. Note that we do not know the actual values of P_{1hdh} etc., which instead are set by the optimisation. The value of the objective function will be zero if $P_{1\mathrm{hdh}}$, $P_{1\mathrm{hhp}}$ and $P_{1\mathrm{hob}}$ all equal to zero. This will, of course, be unacceptable and a constraint is therefore introduced:

$$(P_{1\mathrm{hdh}} + P_{1\mathrm{hhp}} + P_{1\mathrm{hob}}) \times 368 \geq 17 \times 10^3 \text{ kWh}$$

This constraint ascertains that at least 17,000 kWh is delivered from the boiler, or boilers if that is cheaper. The needed amount of heat has been calculated from values showing the climate in January and *U*-values, and areas for the external walls, floor, windows etc. for the existing building. The right-hand side of the constraint will also be augmented by an expression which shows how much the thermal need is decreased for different amounts of extra insulation.

The model contains 22 time segments. The reason for this is firstly that monthly mean values are used for the climate. Twelve segments are used for this purpose. Secondly, according to the electricity tariff, the five winter months have a high price and a low price segment. In order to properly represent the use of a hot water thermal storage system, weekends must be treated separately and therefore each winter month, from November to March, is divided into three segments. The model is shown in detail in [3] and [4], and in its present state, it contains 183 variables and 152 constraints. Of the variables, 75 are binary integers. These variables can only assume the values 0 or 1, see below. The actual optimisation process is dealt with by using special software e.g. the ZOOM, LAMPS or CPLEX programs. For pedagogic reasons, we start with a case where no building retrofits are performed on the climatic shield or ventilation system. In order to achieve this, we have set high values for the costs of such measures. The optimisation result can be studied in Fig. 1, which shows that the oil-fired boiler should be used only in two segments, i.e. January and February nights between Monday and Friday.

The oil-boiler is also used for covering thermal peaks up to 72 kW. The heat pump should operate throughout the year. This is partly a consequence of the Swedish electricity tariffs of 1.01, 0.56 and 0.45 SEK/kWh for winter working days, winter nights and summer, respectively. The price of oil in this study is set to 0.39 SEK/kWh (1 USD equals about 8.3 SEK). The

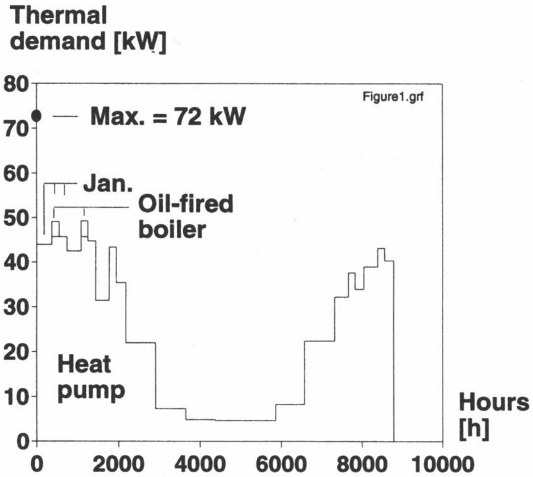

Fig. 1. Thermal demand in the studied building and optimal use of the oil-boiler and heat pump.

Coefficient Of Performance (COP) for the heat pump is set to 3.0 which might be applicable for a device coupled to pipes buried in the ground. The efficiency of the oil-boiler is 0.75. The running cost of the heat pump is, therefore, always lower than for the oil-fired boiler. The reason for using the boiler is that large heat pumps are very expensive compared to oil-boilers. In this study, the costs have been set to $55{,}000 + 60 \times P_{\mathrm{ob}}$ SEK for the boiler and $60{,}000 + 5000 \times P_{\mathrm{hp}}$ SEK for the heat pump. These costs must, of course, be calculated as present values before they are inserted in the objective function, see [4] for details. A binary variable is now multiplied with the value 55,000 and another one is multiplied with 60,000. If P_{hp} is larger than zero, the present value of $60{,}000 + 5000 \times P_{\mathrm{hp}}$ SEK should be present in the objective function. The binary variable is therefore set to the value one. If the heat pump is not present in the optimal solution, the variable is set to zero. P_{hp} is now zero and no cost will be present for the heat pump in the objective function. In the studied case, the thermal size of the boiler i.e. the demand for heating the building, is 35.03 kW, while the heat pump is optimised to 15.23 kW. If the COP for the heat pump and the efficiency for the oil-fired boiler are considered, the total thermal capacity installed in the building is 71.96 kW, which is the actual need, see Fig. 1. We have used two decimal figures here just to show that the values add up. The total LCC is calculated at 1.566 MSEK.

3. Simulation

Having come so far we have a solution which minimises the LCC. What happens if the property owner chooses for instance a heat pump of a different size? For the sake of consistency, the boiler should be changed accordingly. Fig. 2 shows the resulting LCC for different electrical sizes of the heat pump. If the property owner chooses not to use a heat pump at all, the LCC will be 2.39 MSEK. The incremental cost of the heat pump will thus not be present in the LCC. If only a very small pump is used, for example, 1 kW as shown in Fig. 2, the incremental cost is present, but at the same time, the energy cost is reduced and the

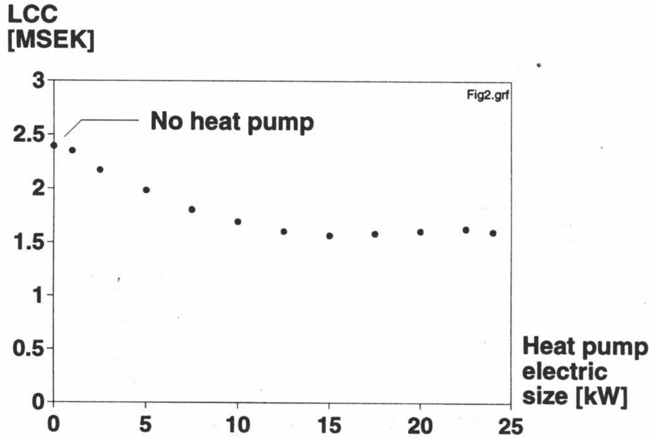

Fig. 2. LCC in MSEK of the studied building for different electrical sizes of a heat pump.

LCC becomes 2.35 MSEK. However, the slope of the curve clearly changes at that point. If a theoretical heat pump with a size of 0.001 kW is chosen, the LCC will be 2.50 MSEK. This point is, however, not present in Fig. 2. The same effect can be seen at the other end of the graph. When the size of the heat pump exceeds 23.99 kW, there is no need for an oil-fired boiler and hence, the slope changes again. This results in a LCC of 1.60 MSEK. The optimal LCC is, however, about 0.04 MSEK lower. From Figs. 1 and 2, it is obvious that the size of the heat pump is not very interesting, when the LCC is considered, on the right-hand side of the optimum point, as long as its size, together with the size of the oil-fired boiler, is sufficient for meeting the thermal peak in the building.

If a too small heat pump is chosen, the result will be unsatisfactory because the slope of the curve is much steeper on the left-hand side of the optimum. It is assumed that proper sizes of heat pumps are available which might not reflect the reality.

4. Comparing simulation and optimisation

One retrofit which is almost always profitable is weatherstripping. Hitherto, this type of retrofit has been prohibited by the very high cost, which we set to 25,000 SEK, for sealing each window or door. A more acceptable value would be 250 SEK. In Sweden, it is necessary, according to the building code, to ascertain a ventilation flow of 0.5 renewals/h. In the existing building, we have assumed that a value of 0.6 is present and, hence, there is a gap which can be utilized. Weatherstripping, with this lower cost, is included in the optimal solution and the LCC becomes 1.558 MSEK, i.e. slightly lower than before. At the same time, the new optimal sizes of the heat pump and oil-boiler become 14.63 and 33.90 kW, respectively. In Fig. 3, the change in LCC is shown for different values of the sealing cost. The squares show the LCC when weatherstripping is implemented, whatever the cost, while the dots show the result of optimisation. For low sealing costs, the two LCCs are identical, but when the weatherstripping cost exceeds 307 SEK per sealed item, the retrofit is excluded from the optimal solution. The

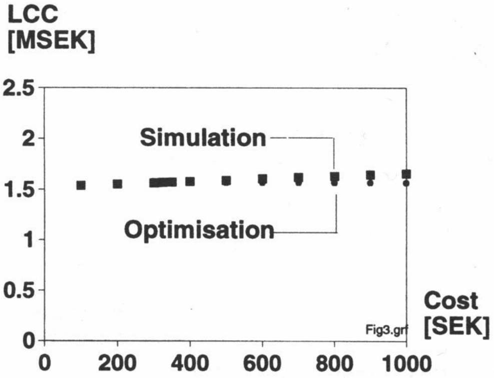

Fig. 3. LCC values for different costs of weatherstripping.

dots in Fig. 3 are subsequently located on a horizontal straight line, while the squares are located on a straight, but ascending, line.

The same behaviour can be seen for fenestration retrofits. Until now, they have also been excluded from the optimal solution because of the set high retrofit costs. Now a change from ordinary double-paned windows, with a cost of 1100 SEK/m^2, to triple-glazed ones is assumed to cost 1300 SEK/m^2.

If windows with a Low Emissivity (LE), coating are chosen, the cost increases to 1500 SEK/m^2. The *U*-values for the three types are set to 3.0, 1.5 and 1.2 W/°C m^2, respectively. The optimisation results in a strategy where LE windows are optimal. The sizes of the heat pump and oil-fired boiler are 12.32 and 29.60 kW, respectively, and the optimal LCC is reduced to 1.467 MSEK. If the cost of changing to LE windows is reduced, the LCC must also decrease because it is always optimal to install them. If, however, the cost is increased see Fig. 4, the next best solution will be optimal, i.e. to install triple-glazed windows without a LE coating.

In Fig. 4, the LE windows are abandoned at a cost of 1600 SEK/m^2 and the LCC subsequently shows a horizontal line. Instead, triple glazed windows without LE become optimal. At the same time, a slightly larger oil-fired boiler and heat pump become optimal. If the cost of the now optimal windows is increased, two possibilities exist. LE windows may once again become optimal or double-paned windows should be used. The overall behaviour is, however, the same for weatherstripping and fenestration.

For our original data set, extra insulation on the attic floor was not optimal. The cost was assumed to be $(260 + 530 \times t)$ SEK/m^2, where t is the extra thickness in metres. The first value shows an increment in the cost, which does not affect the actual amount of extra insulation, but instead may indicate whether this amount is optimal or not, see Fig. 5.

When the incremental cost is lower than about 250 SEK/m^2, it is optimal to apply an extra 0.16 m of mineral wool on the attic floor. At the same time, the optimal size of the boiler will decrease to 28.96 kW and the heat pump size to 10.53 kW.

If the other part of the insulation cost varies, there is a change in behaviour. As the cost decreases, more and more insulation should be added, which affects the total thermal need in the building. At a cost of 150 SEK/m for each square metre, 0.34 m extra insulation should be

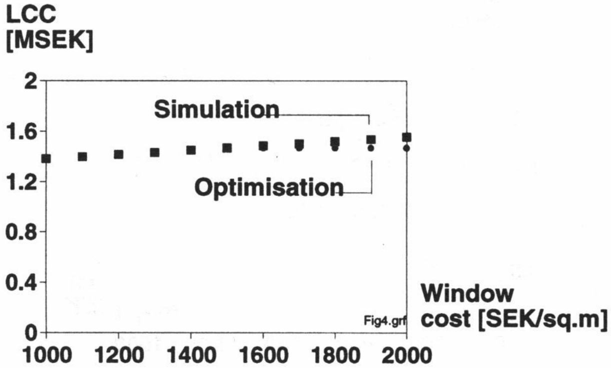

Fig. 4. LCC values for a varying LE-window retrofit cost.

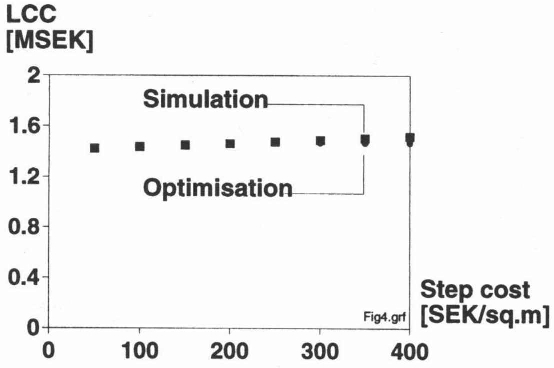

Fig. 5. LCC for varying incremental costs of extra insulation.

added, while 0.18 m is optimal at a cost of 450 SEK/m^2 m. When the cost exceeds 500, insulation is excluded from the optimal solution, see Fig. 6. To the left of the value 500, the dots have a small slope, while the LCC is constant to the right of that point. For the values 150 and 500, the LCCs are calculated at 1.446 and 1.467 MSEK, respectively. The optimal heat pump size is constant, while the size of the boiler varies between 27.84 and 28.75 kW. As long as the property owner acts in an optimal way, the LCC can be held almost constant regardless of the cost of the extra insulation. Note that only optimal LCCs are present in Fig. 6. If 0.34 m of extra insulation is applied and its cost is at the highest level in Fig. 6, the LCC becomes 1.497 MSEK.

The same behaviour can also be found if varying energy prices are considered. In the case above, an electrical heat pump was found to be optimal. Fig. 7 shows how the LCC varies for different electricity prices.

The electricity price differs according to the time of day. On working days during the winter, the price is 1.01 SEK/kWh, but in Fig. 7 it varies from 0.5 to 1.5 SEK/kWh. For the lowest price, only triple-glazed windows are optimal. For 0.6, weatherstripping is added and for 0.8,

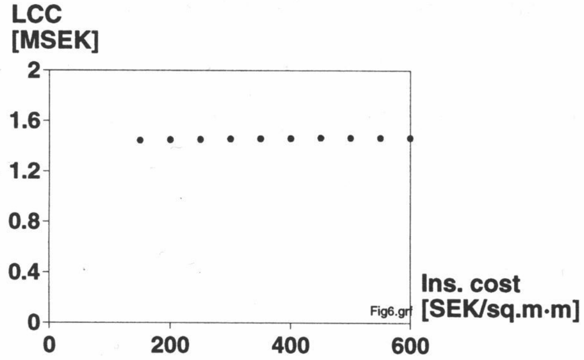

Fig. 6. LCC versus cost for extra insulation.

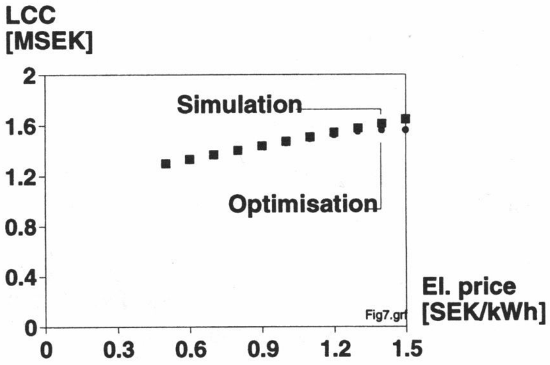

Fig. 7. LCC versus electricity price on winter working days.

LE-windows are used. When the price reaches 1.2 SEK/kWh, 0.18 m extra insulation on the attic floor should be added. The heat pump is abandoned for winter working days at a price of 1.40 SEK/kWh and district heating is used instead. At the same time, extra insulation is excluded from the optimal solution. After this, the LCC is constant.

Because of the dramatic change in optimal strategy when the electricity price is varied, three cases will be studied in more detail. For the situation where the electricity price is 1.10 SEK/kWh, the LCC includes the items in Table 1. We have shown the costs, as they are calculated from the optimisation and simulation programs, in close detail because it is then possible to analyse the results.

An existing building must be refurbished time to time. The windows must be replaced when they become dilapidated. This must be done even if the new windows have the same thermal performance as the old ones. All such costs, calculated as present values, are gathered in the unavoidable cost in Table 1. The largest cost item in the table is, however, the energy cost. When the cost of electricity is increased, another optimal solution comes into play as shown in Table 2.

If Tables 1 and 2 are compared, it is obvious that the energy cost in the latter case is lower. The same applies to the heat pump, the oil-fired boiler and the demand fee for electricity.

Table 1
LCC details when the high cost segment for electricity is 1.10 SEK/kWh (all costs in SEK)

Unavoidable retrofit cost	407,633
Triple-glazed windows with LE coating	69,830
Weatherstripping	33,099
Energy cost	693,144
Heat pump cost (12.32 kW)	214,692
Oil-boiler cost (29.59 kW)	58,035
Insulation cost	–
Demand fee for electricity	21,268
LCC	1,497,701

Table 2
LCC details when the high cost segment for electricity is 1.20 SEK/kWh (all costs in SEK)

Unavoidable retrofit cost	407,633
Triple-glazed windows with LE coating	69,830
Weatherstripping	33,099
Energy cost	648,488
Heat pump cost (10.53 kW)	198,889
Oil-boiler cost (28.75 kW)	57,984
Insulation cost (0.18 m)	97,024
Demand fee for electricity	18,713
LCC	1,531,661

Instead, a large sum is spent on insulation of the attic floor, where 0.18 m of extra mineral wool must be added.

If the electricity price is increased to 1.40 SEK/kWh, the strategy in Table 3 becomes optimal.

The heat pump is no longer used throughout the year because of the high electricity cost on winter working days. Instead, district heating is used for those segments because of a lower energy cost than for oil, see Table 4. The optimal thermal size of the district heating system was found to be 35.29 kW in the January high cost segment. This segment includes 368 h, and hence, 12,986 kWh are used. The cost will be 3554 SEK.

The annual energy cost, 33,793 SEK, must now be multiplied by the present value factor, 18.26, in order to obtain the cost for 50 years, which is found in Table 3. If Tables 1–3 are compared, the energy cost is at its lowest value in Table 3. However, high subscription fees must be paid to the district heating utility, and therefore, the lower running cost must balance these fees if the district heating system is to be included in the optimal solution. When only a small amount of oil is needed (see Fig. 1), this is not the case and the boiler is a better choice than the district heating system.

When extra insulation or better windows are added to the climate shield, the peak load of the building will decrease because of lower heat transmission (cf. e.g. Tables 1 and 2). The

Table 3
LCC details when the high cost segment for electricity is 1.40 SEK/kWh (all costs in SEK)

Unavoidable retrofit cost	407,633
Triple-glazed windows with LE coating	69,830
Weatherstripping	33,099
Energy cost	617,060
Heat pump cost (10.53 kW)	198,889
District heating equipment (37.16 kW)	54,700
District heating subscription costs	136,183
Insulation cost	–
Demand fee for electricity	18,713
Salvage value of discarded boiler	38,909
LCC in SEK	1,575,016

influence of the so-called thermal lag effects will also, at least theoretically, further improve the situation.

If the building is assumed to react as a "lumped-heat-capacity" system (see [5], p. 142), the temperature, T, could be calculated as:

$$(T - T_{\infty})/(T_0 - T_{\infty}) = \mathrm{e}^{-B}$$

where $B = [hA/\rho cV]\tau$, T_{∞} is the surrounding temperature, T_0 is the initial temperature, h is the heat transfer coefficient, A the area of the lump, ρ the density, c the heat capacity and V the volume. For a building, hA could be changed to $UA + V_1 c_a$, where UA is the sum of all U-values multiplied by the applicable areas A, V_1 is the ventilation mass flow and c_a is the heat capacity for air. ρcV will correspond to the mass of the building multiplied with the building heat capacity. When better windows, extra insulation and a lower ventilation flow are the result of refurbishment, B will become lower and subsequently the temperature rate, and hence, T will increase. If the outdoor temperature falls, it would therefore take some time before the space heating demand increases and a so-called thermal lag is present. This lag is, however, difficult to observe in real-world buildings or it is so short that it has small implications on the

Table 4
Heat capacity in kW, usage in kWh and cost in SEK of district heating and the heat pump system

Month	Hours	District heating			Heat pump			Total cost
		Capacity	Usage	Cost	Capacity	Usage	Cost	
January	368	35.29	12,986	3554	–	–	–	3554
	184	8.76	1612	441	31.63	5819	1086	1527
	192	5.36	1029	282	31.63	6073	1134	1416
February	336	33.67	11,316	3097	–	–	–	3097
	168	8.93	1500	410	31.63	5314	992	1402
	192	4.34	833	228	31.63	6073	1134	1362
March	336	23.68	7956	2178	–	–	–	2178
	168	4.03	677	185	31.63	5314	992	1177
	240	–	–	–	27.67	6640	1240	1240
April	720	–	–	–	16.24	11,662	1754	1754
May	744	–	–	–	4.70	3496	525	525
June	720	–	–	–	4.86	3499	525	525
July	744	–	–	–	4.70	3496	525	525
August	744	–	–	–	4.70	3496	525	525
September	720	–	–	–	5.11	3679	551	551
October	744	–	–	–	17.35	12,908	1936	1936
November	336	30.92	10,389	2843	–	–	–	2843
	168	–	–	–	30.92	5194	969	969
	216	–	–	–	27.33	5903	1102	1102
December	352	31.35	11,035	3020	–	–	–	3020
	176	3.86	679	186	31.63	5567	1039	1225
	216	1.1	237	65	31.63	6832	1275	1340
Total	8784	–	60,249	16,489	–	100,965	17,304	33,793

LCC. See, for example, [6] where an office building, specially designed to utilise this thermal lag, has been studied. In the Swedish building code, however, it is possible to take some advantage of such "heavy" buildings because the design outdoor temperature can be set to a higher value. This will decrease the boiler cost, but as found in e.g. Table 1, this will not influence the LCC very much.

5. Conclusions

By using a mixed integer linear programming model, we have optimised the renovation strategy for an existing building. The existing oil-fired boiler should be combined with a heat pump run on electricity. In addition, weatherstripping and low emissivity triple-glazed windows were included in the optimal solution. The boiler was almost entirely used for covering the thermal peak during cold winter days. The heat pump should be used throughout the year. If the costs of the climate shield retrofits are increased, the life-cycle cost of the building increases to a certain level where the retrofit is excluded from the optimal solution. After this, the life-cycle cost becomes constant.

The use of incremental cost functions for the retrofit measures significantly changes the optimal solution. An extra amount of insulation on the attic floor, for example, will only be profitable if this amount exceeds a certain level. The same behaviour is shown for the heating system. If the utility uses a time-of-use rate for electricity, higher prices in one time segment support insulation of the attic floor, but when further increases are made the oil-fired boiler is excluded from the system and district heating is used instead. The heat pump is abandoned for the high price segments and insulation is no longer profitable.

By using a simulation program, we have also examined what happens to the life-cycle cost if the property owner chooses other than optimal solutions. For moderate discrepancies, the difference between these costs is small, but if, for example, the wrong heating system is used, significant divergences may occur.

Acknowledgements

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