



Mixed 0–1 sequential linear programming optimization of heat distribution in a district-heating system

M. Bojic ^{a,b,*}, N. Trifunovic ^a, S.I. Gustafsson ^c

^a Masinski fakultet, Sestre Janjic 6, 34000 Kragujevac, Yugoslavia

^b Department of Building Services Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, People's Republic of China

^c IKP / Energy Systems, Institute of Technology, 581 83 Linkoping, Sweden

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Abstract

A district-heating system transports heat from the heat plant by using primary pipe network, via substation, to secondary pipe network where heat is finally distributed to buildings. When this system is designed its operational characteristics were selected to provide thermal comfort (TC) in all buildings served by this district heating system. After several years of operation, the system characteristics may change and TC in buildings deteriorates; some buildings are overheated and other buildings are underheated. The study investigates an optimum strategy to mitigate the problem caused by changes of three of system characteristics: hydraulic resistance of secondary pipe network, heat transmittance of radiators inside buildings, and heat transmittance of building envelope. A strategy of problem mitigation consists of the adjustment of hydraulic resistance of existing valves and retrofitting the local heating system with new substation heat exchanger and additional pumps. We used a steady state, bottom-up approach and mixed 0–1 sequential linear programming to find optimal mitigation strategy, i.e. optimum combination of valves' hydraulic resistances, new pumps placement and new size of substation heat exchanger. The results indicate that the calculated optimal strategy does not effectively improve TC in buildings only in cases when TC is deteriorated by higher than nominal values of heat transmittance of some building envelopes. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Heat distribution; District-heating system; Sequential linear programming

1. Introduction

As district heating and cooling systems spend great quantities of fuel at national economies level to provide heating, cooling and domestic hot water in buildings, these systems attracted great attention in the literature [1–5].

A district-heating system analyzed in this paper (Fig. 1) provides heating to three buildings. In this system, heat is generated in the heat plant, and first transported to the substation by using a primary pipe network, and then from the substation to the buildings by using a secondary pipe network. When this system was designed, its character-

istics were selected to provide thermal comfort (TC) in these buildings. Here, we studied the situation that may arise after several years from the system design when its characteristics change, causing a deterioration of TC in these buildings. Then, some buildings are overheated, while other buildings are underheated. In overheated buildings, occupants open windows to decrease the space temperature, and in underheated buildings, they turn on additional heating devices to increase space temperature. So, this situation in district-heating systems yields not only to the loss of TC, but also to higher energy consumption in heated buildings.

We investigated changes of three district system characteristics from design value: (1) the hydraulic resistance of the secondary-pipe network, (2) the heat transmittance of radiators in the heated buildings, and (3) the heat transmittance of envelope of the heated buildings. Then, to arrive at the best TC in the heated buildings, we may correct flow rates and temperatures of secondary hot water by using

* Corresponding author. Department of Building Services Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, People's Republic of China. Fax: +381-34-330-196.

E-mail address: bojic@knez.uis.kg.ac.yu (M. Bojic).

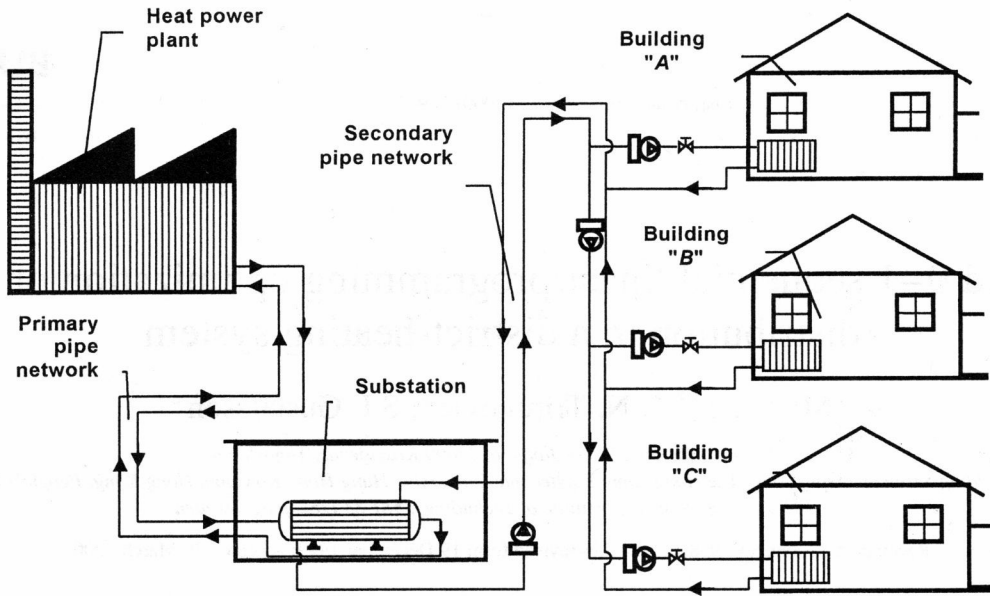


Fig. 1. Schematic of the district-heating system.

some available techniques. In previous work [6,7], we used only one technique consisting of installation of additional pumps at the proper locations of the secondary pipe network. Our research revealed that this technique only improved TC when hydraulic resistance of secondary pipe network was different from design value. To approach to TC in the building spaces when other system parameters such as the heat transmittance of radiators in heated buildings, and the heat transmittance of building envelope are

different from design values, we investigated the combination of previous techniques with two additional techniques: (1) adjustment of the hydraulic resistance of valves of the secondary pipe network, and (2) resizing of heat exchangers in substation.

We used a steady-state, bottom-up approach to design an energy module network corresponding to the district-heating system, and then derive a set of equations that describes the behavior of the system [8]. Furthermore, we

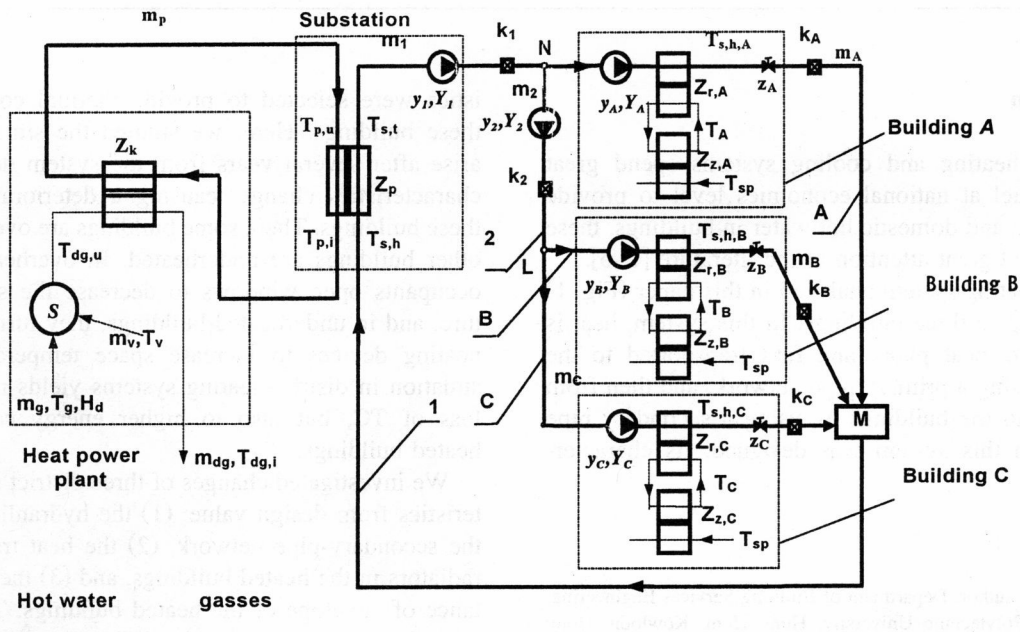


Fig. 2. Network of energy-object modules.

developed a computer program that uses the mixed 0–1 linear programming method and solved the optimization problem [9–12]. The similar methodology has been successfully applied to the optimization of other kinds of energy systems [13–21].

2. Mathematical model

2.1. Description of the system

The district-heating system considered in this paper is shown in Fig. 1 and its energy-objects are presented in Fig. 2. In this system heat is transferred from the heat plant to building by using two pipe networks: primary and secondary. In the primary pipe network, heat goes from the heat plant to the substation by using the hot water. In the substation heat exchanger, this water is cooled from the temperature $T_{p,u}$ to the temperature $T_{p,i}$, and water of in the secondary pipe network is heated from $T_{s,h}$ to $T_{s,t}$. Water of the secondary pipe network is cooled in radiators of the three buildings to the temperatures $T_{s,h,A}$, $T_{s,h,B}$ and $T_{s,h,C}$. The three water flows mix and return to the substation heat exchanger. In all three branches of the secondary pipe network, there are valves to enlarge the hydraulic resistance of the branches.

2.2. Equations

For this district-heating system, we designed its energy-object network and established hydraulic equations, energy equations, inequalities and objective function.

For this energy-module network, six hydraulic equations are written as:

$$y_1 Y_1 - k_1 m_1^2 + y_A Y_A - (k_A + z_A) m_A^2 = 0, \quad (1)$$

$$y_1 Y_1 - k_1 m_1^2 + y_2 Y_2 - k_2 m_2^2 + y_B Y_B - (k_B + z_B) m_B^2 = 0, \quad (2)$$

$$y_1 Y_1 - k_1 m_1^2 + y_2 Y_2 - k_2 m_2^2 + y_C Y_C - (k_C + z_C) m_C^2 = 0, \quad (3)$$

$$m_1 = m_2 + m_A, \quad (4)$$

$$m_2 = m_B + m_C, \quad (5)$$

$$Y_i = A_i + B_i m_i + C_i m_i^2, \quad i = (1, 2, A, B, C). \quad (6)$$

Eq. (1) presents the first law of thermodynamics for flow in branches 1 and A, Eq. (2) presents the first law of thermodynamics for flow in branches 1, 2 and B, Eq. (3) presents the first law of thermodynamics for flow in branches 1, 2 and C, Eq. (4) presents the mass balance for node N, Eq. (5) presents the mass balance in node L, and Eq. (6) presents the performance equations of pumps 1, 2, A, B and C.

Also, 12 energy-balance equations are written:

$$c_{p,v} m_v T_v + c_{p,g} m_g T_g + m_g H_a = c_{p,dg} m_{dg} T_{dg,u}, \quad (7)$$

$$c_{p,dg} m_{dg} (T_{dg,u} - T_{dg,i}) = c_w m_p (T_{p,u} - T_{p,i}), \quad (8)$$

$$c_w m_p (T_{p,u} - T_{p,i}) = 0.5 Z_k (T_{dg,u} + T_{dg,i} - T_{p,u} - T_{p,i}), \quad (9)$$

$$c_w m_p (T_{p,u} - T_{p,i}) = c_w m_i (T_{s,t} - T_{s,h}), \quad (10)$$

$$c_w m_p (T_{p,u} - T_{p,i}) = 0.5 Z_p (T_{p,u} + T_{p,i} - T_{s,t} - T_{s,h}), \quad (11)$$

$$c_w m_A (T_{s,t} - T_{s,h,A}) = 0.5 Z_{r,A} (T_{s,t} + T_{s,h,A} - 2T_A), \quad (12)$$

$$0.5 Z_{r,A} (T_{s,t} + T_{s,h,A} - 2T_A) = Z_{z,A} (T_A - T_{sp}), \quad (13)$$

$$c_w m_B (T_{s,t} - T_{s,h,B}) = 0.5 Z_{r,B} (T_{s,t} + T_{s,h,B} - 2T_B), \quad (14)$$

$$0.5 Z_{r,B} (T_{s,t} + T_{s,h,B} - 2T_B) = Z_{z,B} (T_B - T_{sp}), \quad (15)$$

$$c_w m_C (T_{s,t} - T_{s,h,C}) = 0.5 Z_{r,C} (T_{s,t} + T_{s,h,C} - 2T_C), \quad (16)$$

$$0.5 Z_{r,C} (T_{s,t} + T_{s,h,C} - 2T_C) = Z_{z,C} (T_C - T_{sp}), \quad (17)$$

$$m_1 T_{s,h} = m_A T_{s,h,A} + m_B T_{s,h,B} + m_C T_{s,h,C}. \quad (18)$$

Eq. (7) presents combustion energy balance in the heat plant, Eqs. (8) and (9) present the heat transfer in the heat plant from boiler hot gases to the hot water of primary pipe network, Eqs. (10) and (11) present the heat transfer in the substation from the hot water of the primary pipe network to the water of the secondary pipe network, Eq. (12) presents transfer of heat at radiators from the water of the secondary pipe network to the space of building A, Eq. (13) presents transfer of heat through the envelope of building A from the space of building A to the outside space, Eq. (14) presents transfer of heat at radiators from the water of the secondary pipe network to the space of building B, Eq. (15) presents transfer of heat through the envelope of building B from the space of building B to the outside space, Eq. (16) presents transfer of heat at radiators from the water of the secondary pipe network to the space of building C, Eq. (17) presents transfer of heat through the envelope of building C from the space of building C to the outside space, Eq. (18) presents energy balance in node M. Moreover, three temperature equations are written as:

$$T_A + \delta T_{neg,A} - \delta T_{pos,A} = T_{TC}, \quad (19)$$

$$T_B + \delta T_{neg,B} - \delta T_{pos,B} = T_{TC}, \quad (20)$$

$$T_C + \delta T_{neg,C} - \delta T_{pos,C} = T_{TC}. \quad (21)$$

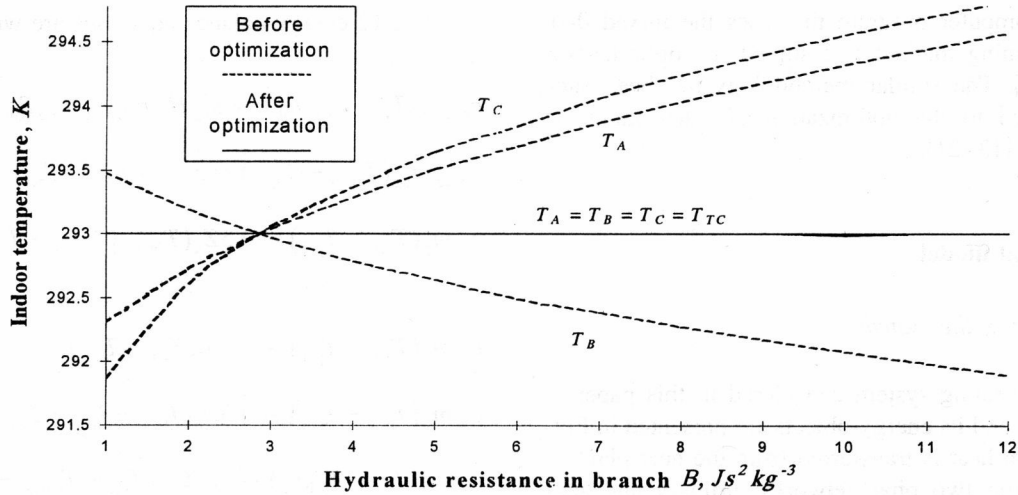


Fig. 3. Indoor temperatures of buildings A, B and C vs. hydraulic resistance in branch B.

In Eq. (19), $\delta T_{pos,A}$ presents an increase of space temperature T_A of building A above T_{TC} and $\delta T_{neg,A}$ presents a decrease of T_A beneath T_{TC} . If $\delta T_{pos,A} > 0$ then $\delta T_{neg,A} = 0$ and if $\delta T_{pos,A} < 0$, then $\delta T_{neg,A} = 0$. In Eq. (20), $\delta T_{pos,B}$ presents an increase of space temperature T_B of building B above T_{TC} and $\delta T_{neg,B}$ presents a decrease of T_B beneath T_{TC} . If $\delta T_{pos,B} > 0$ then $\delta T_{neg,B} = 0$ and if $\delta T_{pos,B} < 0$, then $\delta T_{neg,B} = 0$. In Eq. (21), $\delta T_{pos,C}$ presents an increase of space temperature T_C of building C above T_{TC} and $\delta T_{neg,C}$ presents a decrease of T_C beneath T_{TC} . If $\delta T_{pos,C} > 0$ then $\delta T_{neg,C} = 0$ and if $\delta T_{pos,C} < 0$, then $\delta T_{neg,C} = 0$.

2.3. Inequalities

For efficient use of mixed 0–1 sequential programming, it is necessary to include:

$$Z_p - \Delta Z_p \leq Z_p \leq Z_p + \Delta Z_p, \quad (22)$$

$$z_i - \Delta z_i \leq z_i \leq z_i + \Delta z_i, \quad (i = A, B, C), \quad (23)$$

$$18 K \leq T_{s,t} - T_{s,h} \leq 22 K. \quad (24)$$

The inequality (22) restrains the substation heat exchanger to its minimum and maximum size, and inequality

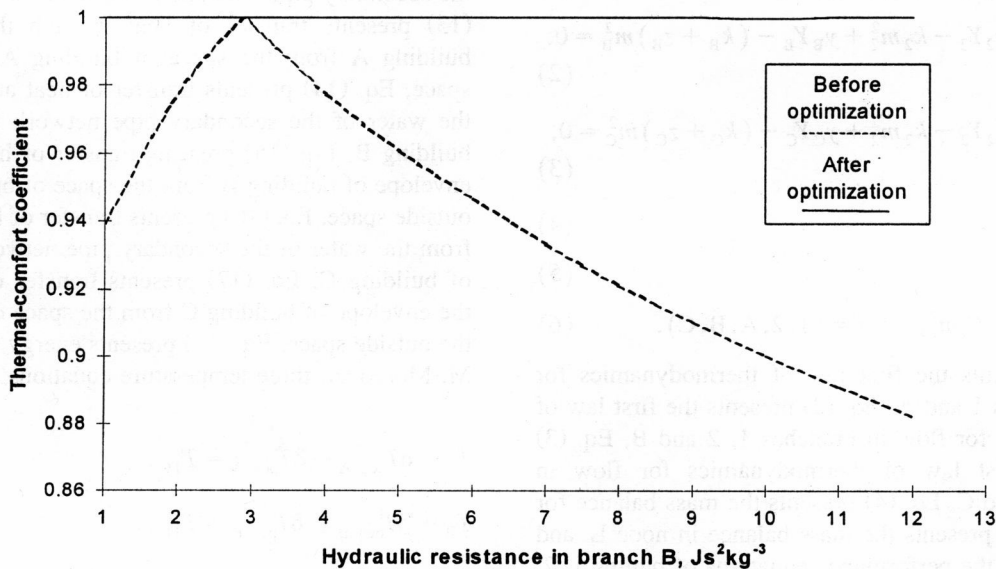


Fig. 4. TC coefficient vs. hydraulic resistance in branch B.

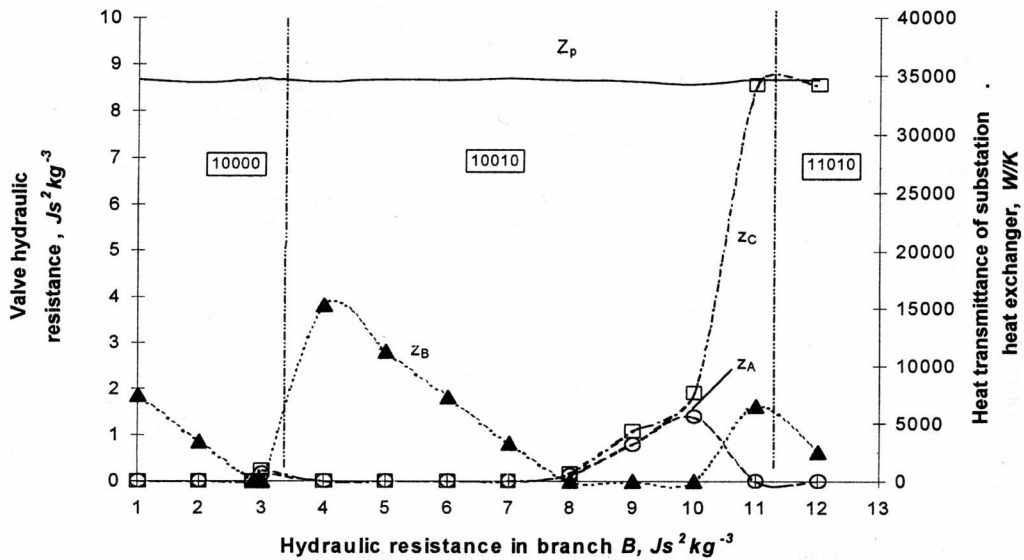


Fig. 5. Control diagram as a function of hydraulic resistance in branch B.

(23) restricts hydraulic-resistance coefficient of valves A, B and C, and inequality (23) restricts difference of heating temperature of water of the secondary pipe network in the substation.

2.4. Objective function

Our objective is to minimize the departure of space temperatures from TC temperatures T_{TC} in different buildings.

$$F_{min} = \delta T_{neg,A} + \delta T_{neg,B} + \delta T_{neg,C} + \delta T_{pos,A} + \delta T_{pos,B} + \delta T_{pos,C} \quad (25)$$

In Appendix A, nominal values for the system parameters are given. For these values, optimal TC is achieved i.e. $F_{min} = 0$.

3. Results

In this paper, we considered three cases: change of hydraulic resistance of pipe network, change of heat transmittance of radiators inside buildings and change of heat transmittance of building envelope.

For each case considered in this paper, we present three figures. The first figure shows the indoor temperatures of buildings A, B and C before and after optimization of heat distribution. The second figure shows the TC coefficient

$$TCT = 1 - \frac{|T_A - T_{TC}| + |T_B - T_{TC}| + |T_C - T_{TC}|}{T_{TC} - T_{sp}} \quad (26)$$

before and after optimization of heat distribution. For nominal system parameters $T_A = T_B = T_C = T_{TC}$, we have

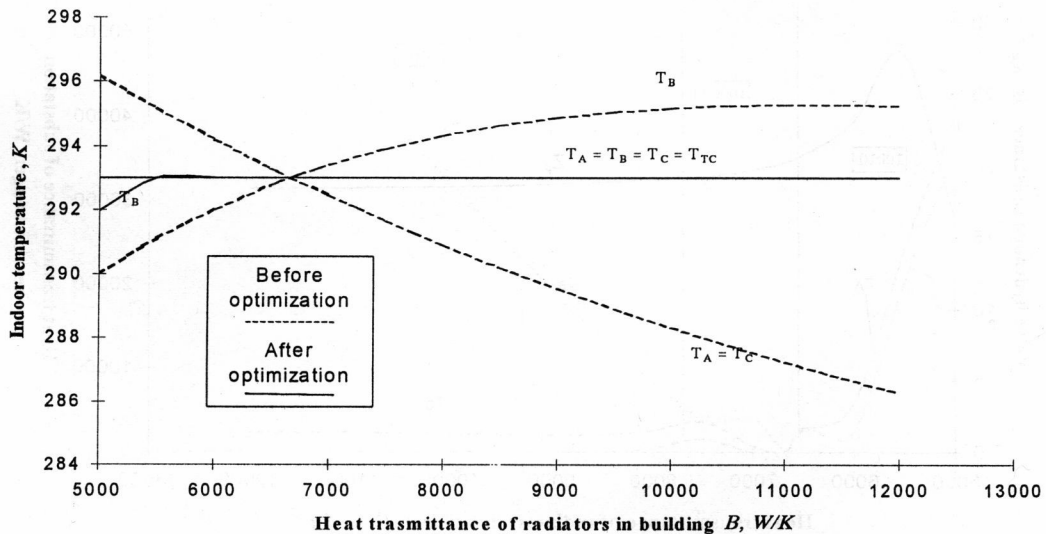


Fig. 6. Indoor temperatures of buildings A, B, and C vs. heat transmittance of B-building radiators.

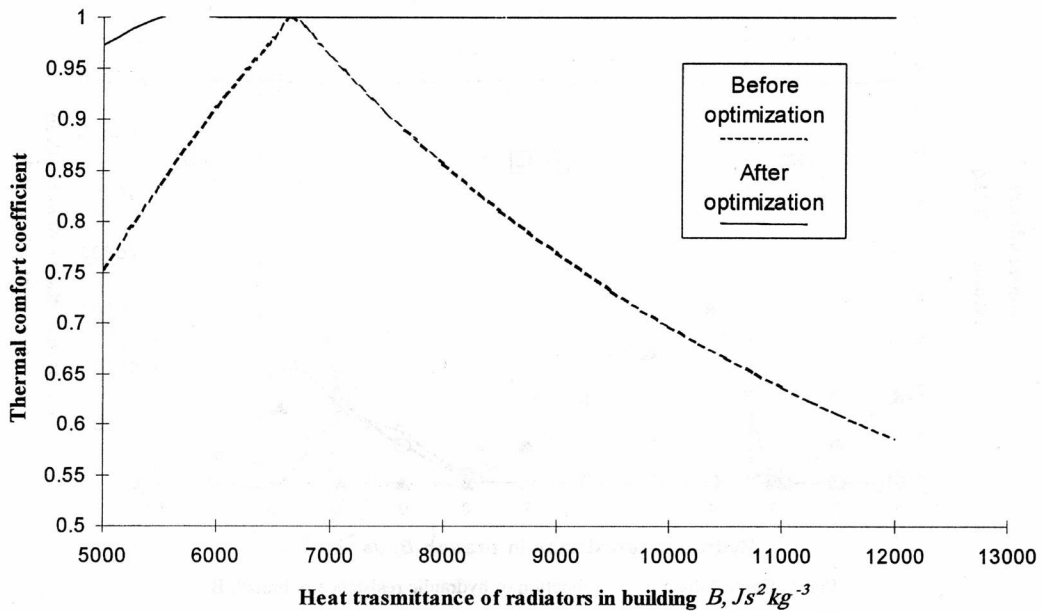


Fig. 7. TC coefficient vs. heat transmittance of B-building radiators.

optimal TC in all buildings, and $TCT = 1$. If the indoor temperature of any of the buildings differs from T_{TC} , TC is not optimal, and $TCT < 1$. The third figure shows values of nine system parameters when TC in buildings is optimized: the hydraulic resistances of three valves, the heat transmittance of its substation heat exchanger, and presence of pumps in five locations of its secondary pipe network. The presence of a pump is denoted with 1, whereas the non-presence of a pump is denoted with 0. In this figure, the presence of the pumps in five locations is shown by using a five-digit binary number. The first digit presents pump in branch 1, the second pump in branch 2,

the third pump in branch A, the fourth pump in branch B and the fifth pump in branch C.

Figs. 3–5 relate the case when the hydraulic resistance of branch B k_B changes. Figs. 6–8 describe the case when the heat transmittance $Z_{r,B}$ of the radiators in the building B changes. Figs. 9–11 describe the case when the heat transmittance $Z_{Z,B}$ of the building B envelope changes.

3.1. Figs. 3–5

The hydraulic resistance of branch B less than nominal $k_B = 2.87 J s^2 kg^{-3}$ yields greater hot-water flow rate in

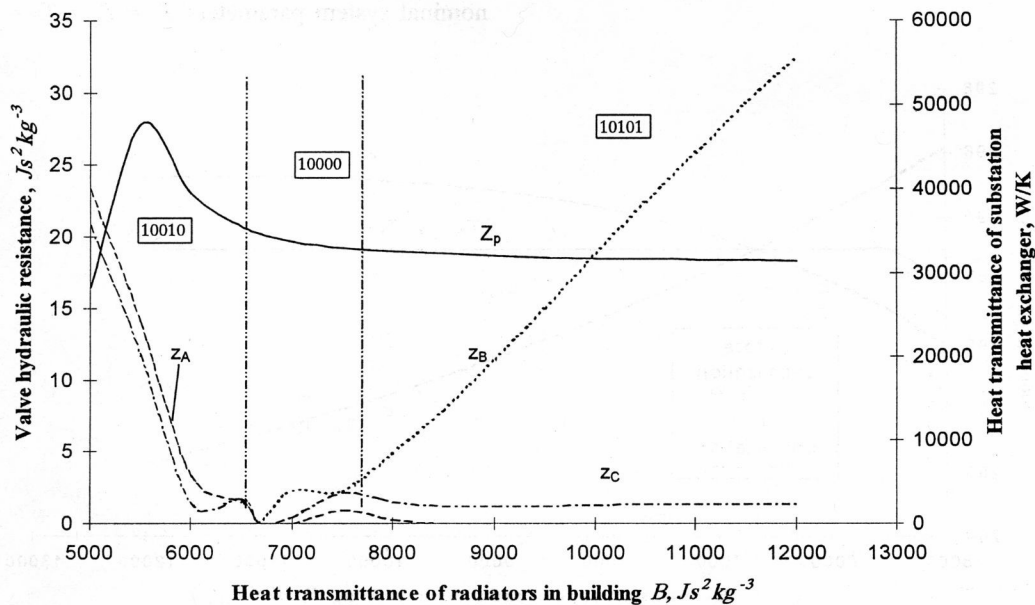


Fig. 8. Control diagram as a function of heat transmittance of B-building radiators.

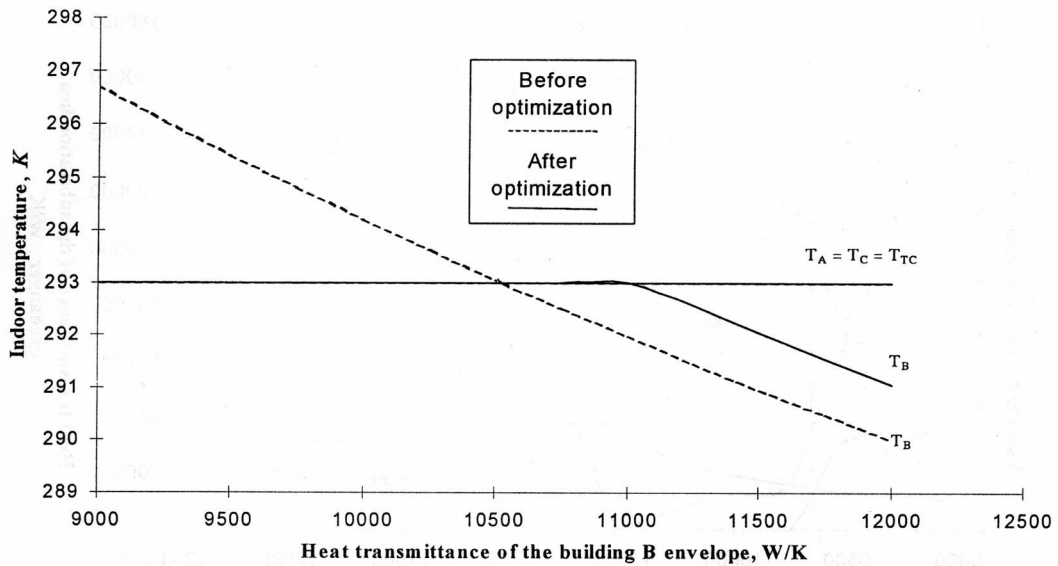


Fig. 9. Indoor temperatures of buildings A, B, and C distribution vs. heat transmittance of B-building envelope.

branch B than nominal, and higher indoor temperature of building B than T_{TC} . Here, the optimal value of TC may be reached by adjusting the valves without resizing the heat exchanger or placing additional pumps.

For k_B higher than nominal, placing an additional pump in branch B is suggested. This pump should compensate for high k_B of the flow of hot water in this branch. If k_B exceeds 11, also placing of pump in branch 2 is suggested.

3.2. Figs. 6–8

If the heat transmittance is less than nominal $Z_{r,B} = 6667$ W/K, insufficient heat is transferred from radiators to the air in the building B. Then, the indoor temperature of building B is not optimal, and buildings A and C are

overheated. Placing an additional pump in branch B, adjusting valves and retrofitting the heat exchanger in the substations yield the optimal TC.

For $Z_{r,B}$ near nominal, additional pumps are not needed. Optimal TC is reached by adjusting valves.

When $Z_{r,B}$ is higher than nominal, building B is overheated, while buildings A and C are underheated. To achieve optimal TC, additional pumps in branches A and C should be placed.

3.3. Figs. 9–11

For the $Z_{Z,B}$ less than nominal $Z_{Z,B} = 10526$ W/K, the heat loss of building B through its envelope is smaller,

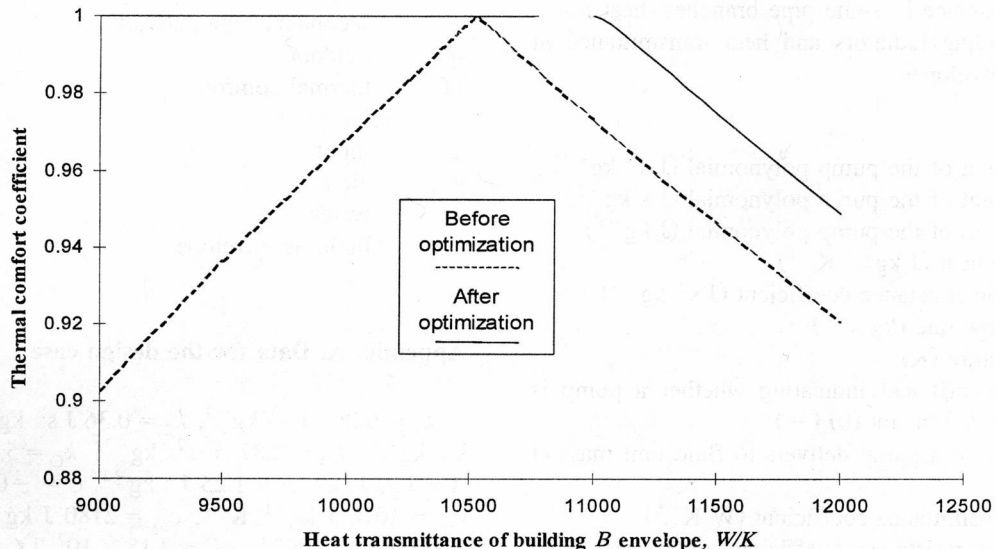


Fig. 10. TC coefficient vs. heat transmittance of B-building envelope.

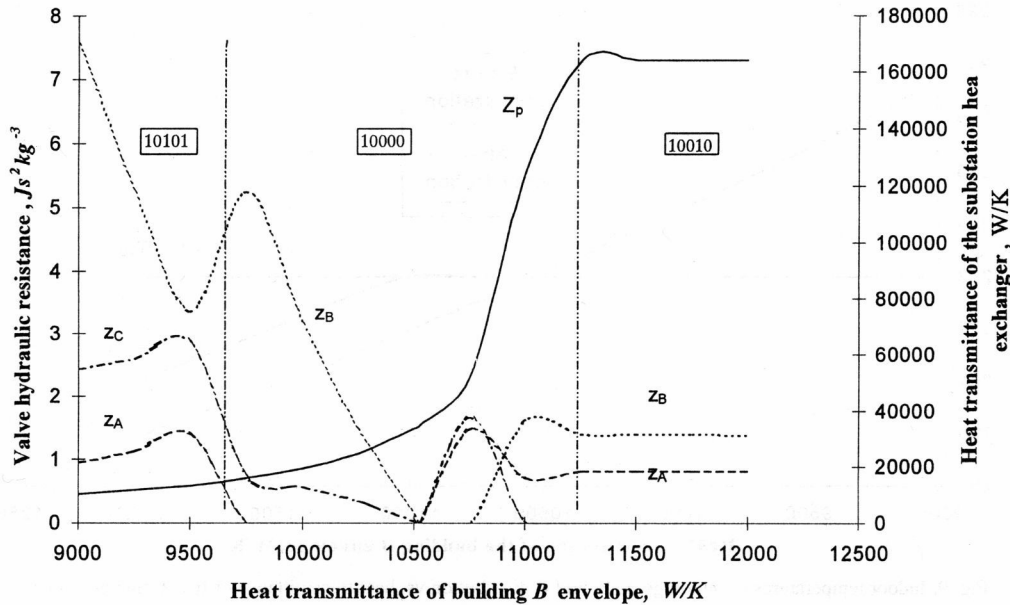


Fig. 11. Control diagram as a function of heat transmittance of B-building envelope.

leading to overheating of the building. The TC can be efficiently restored by placing pumps in branches A and C, reducing the heat transmittance of the heat exchanger in the substation, and adjusting the valves in all branches (Fig. 11).

For the high values of $Z_{Z,B}$, the indoor temperature of building B is smaller than T_{TC} . This indoor temperature of building B can be increased by placing a pump in branch B, retrofitting the substation heat exchanger and adjusting the valves.

4. Conclusions

Results show that significant improvement of TC can be obtained when the TC deterioration is caused by variations of hydraulic resistance in some pipe branches, heat transmittance of building radiators and heat transmittance of some building envelopes.

Nomenclature

<i>A</i>	coefficient of the pump polynomial ($J s^2 kg^{-3}$)
<i>B</i>	coefficient of the pump polynomial ($J s kg^{-2}$)
<i>C</i>	coefficient of the pump polynomial ($J kg^{-1}$)
<i>c</i>	specific heat ($J kg^{-1} K^{-1}$)
<i>k</i>	hydraulic-resistance coefficient ($J s^2 kg^{-3}$)
<i>m</i>	mass-flow rate ($kg s^{-1}$)
<i>T</i>	temperature (K)
<i>y</i>	binary coefficient indicating whether a pump is installed (1) or not (0) (–)
<i>Y</i>	energy that a pump delivers to fluid unit mass ($J kg^{-1}$)
<i>Z</i>	heat-transmittance coefficient ($W K^{-1}$)
<i>z</i>	hydraulic-resistance coefficient of valves ($J s^2 kg^{-3}$)

∂T the difference between indoor-air temperature and T_{TC}

Indices

A	building A
B	building B
C	building C
dg	fume gas
g	fuel
h	cooled
i	output
k	boiler
neg	negative
p	primary pipe network
pos	positive
r	radiators
s	secondary pipe network
sp	outdoor
TC	thermal comfort
t	hot
u	input
v	air
w	water
z	building envelope

Appendix A. Data for the design case

$k_1 = 0.064 J s^2 kg^{-3}$, $k_2 = 0.36 J s^2 kg^{-3}$, $k_A = 5.18 J s^2 kg^{-3}$, $k_B = 2.87 J s^2 kg^{-3}$, $k_C = 5.10 J s^2 kg^{-3}$, $A = 110 J/kg$, $B = 1.25 J s kg^{-2}$, $C = -0.164 J s^2 kg^{-3}$, $c_{pv} = 1010 J kg^{-1} K^{-1}$, $c_{pg} = 2180 J kg^{-1} K^{-1}$, $c_{pdg} = 1200 J kg^{-1} K^{-1}$, $H_d = 3.18 \times 10^7 J/kg$, $m_v = 0.855 kg/s$, $m_g = 0.045 kg/s$, $m_{dg} = 0.9 kg/s$, $T_v = T_g = 283 K$,

$T_{sp} = 255 \text{ K}$, $T_{p,u} = 383 \text{ K}$, $T_{p,i} = 363 \text{ K}$, $T_{s,h} = 343 \text{ K}$,
 $T_{s,t} = 363 \text{ K}$, $T_{s,h,A} = 343 \text{ K}$, $T_{s,h,B} = 343 \text{ K}$, $T_{s,h,C} = 343 \text{ K}$,
 $m_p = 4,17 \text{ kg/s}$, $c_w = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$, $Z_k = 1484 \text{ W/K}$,
 $Z_{r,A} = 5833 \text{ W/K}$, $Z_{r,B} = 6667 \text{ W/K}$, $Z_{r,C} = 5000 \text{ W/K}$,
 $Z_{z,A} = 9211 \text{ W/K}$, $Z_{z,B} = 10526 \text{ W/K}$, $Z_{z,C} = 7895 \text{ W/K}$,
 $y_1 = 1$, $y_2 = 0$, $y_A = 0$, $y_B = 0$, $y_C = 0$, $Z_p = 34500 \text{ W/K}$,
 $z_A = 0 \text{ J s}^2 \text{ kg}^{-3}$, $z_B = 0 \text{ J s}^2 \text{ kg}^{-3}$, $z_C = 0 \text{ J s}^2 \text{ kg}^{-3}$.

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