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Production Control of Cross Cut Operations at Wood Manufacturing Industries

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Abstract

Cross cut operations in the wood industry is an important industrial process. The cutting is based on the contents and location of different defects on the wood surface. This information is then used by an optimization module that cuts each individual board in an optimal fashion given specific product values. However, most production processes have a fixed demand of each product and this aspect is not considered in the cross cut optimization. We describe and investigate some control strategies that guide the process so that the total number of boards used can be kept as low as possible while simultaneously maintaining a smooth production. These strategies continuously change the relative product values used in the optimization module. We perform numerical analysis on a number of different scenarios and these results show that the proposed control strategies substantially can reduce the number of boards used.

Preface

The study described in this paper is a part of a larger project called *Optimization of wood manufacturing industries* involving the County of Kalmar, Linköping University and some wood manufacturing industries in Kalmar county. The overall aim of the project is to develop production techniques and strategies for wood manufacturing industries in order to increase their competiveness. We have chosen an overall approach which is based on mathematical modelling of the real applications and then use optimization techniques to come up with efficient strategies. Close cooperation with companies are an important part of the project. Companies involved in the study are members of a consortium called *Consortium Wood Industry Kalmar* but this membership has not been mandatory to participate in the project. Two factories are studied here, *Bringholz furniture* in Ruda and *Mörlunda chair and furniture* in Mörlunda both sited about 300 km south of Stockholm. The project is financed by the County of Kalmar and the European Community, through *Task 5b*. We acknowledge the valuable cooperation with Barbro Molinder at the administration of the County of Kalmar. The work in this particular paper is about production control strategies which can be used in any wood manufacturing industry where there is an unknown quality of the raw material used to manufacture particular products.

1 Cross cut operations

Traditionally, cross cutting of boards in the secondary wood industry has been a manual task requiring experienced labour. The operator tries to cut out an optimal mix of fixed-length products from boards containing defects such as knots, splits, resin pockets etc. The operator is thus manually performing all the three tasks, inspection, optimization and cutting. Nowadays, most cutting is done using optimizing saws where only the inspection task is performed manually. The operator uses a special fluorescent crayon and partitions the board into sections which are given different quality labels based on the contents of different defects. The markings are read by a special camera and an optimal saw pattern is calculated based on a given cutting bill, which is a list of products to cut. This list includes a description of the products, for example, dimensions, demand and quality restrictions. An important aspect is also the associated value for each product. The overall objective of the process is to cut a specified number of each product in a cutting bill with as least number of boards as possible. An important observation is that this objective is not equivalent to the problem where each board is to be used in an optimal fashion given product values. This distinctive difference makes the overall process into a much more complex problem than it seems in the first place.

There are many industrial processes where the production is controlled by setting product prices or values. This is typically the case when raw material is of unknown quality and when there is a specific demand of products. These products, in turn, have specific quality requirements. Products in this application are then used in further manufacturing processes into e.g. chair arms, chair legs etc. The overall aim is to use the raw material in an optimal way. A problem that arise is the fact that routines are used to optimize individual boards. These routines are unbeatable when it comes to optimize single boards. However, this is only a local view of the problem and the global view when a certain demand profile is given can not be considered.

A simple example is as follows. Any optimization routine requires product values in order to optimize and we can suppose that these are known in advance. Now, suppose the optimal use of boards give a proportion 60% product A and 40% product B. However, the demand may be given such that the proportions should be 20% and 80%, respectively. To control the previous example we need instruct the optimization routine that product B is relatively more important than A as compared to the initial values. This weighting of products can be viewed as an internal weighting to compensate for the demand profile. An optimal weighting is the one which gives a production distribution that correspond to the actual demand with the use of as few boards as possible. The problem we focus upon is how to find this weighting.

Various cross cutting applications for individual boards has attracted a lot of attention in the literature. Examples of this is Brunner *et al.* [3], Klinkhachorn *et al.* [10] and Carnieri *et al.* [5]. Carnieri *et al.* [6] adopted an approach to the problem together with heuristic procedures for the rip-first and crosscut first cutting strategies. Most of these deal only with the optimal use of clear areas in the presence of defects which must be removed. Carnieri *et al.* [5] consider an application with one defect. Defects are handled in a more general way by Sarker [15] who discusses value as a function of defect contents and Sweeney and Haessler [16] who introduce multiple quality grades. Although none of them addresses cutting of wood directly, the problem discussed by Sweeney and Haessler is actually identical to the old crayon marking crosscut problem. In Rönnqvist [12] a general mathematical model for the cross cutting with different quality zones are presented. This model was later used in Åstrand and Rönnqvist [1] and Rönnqvist anÅstrand [13] where approaches for integrating the defect analysis and model formulation are presented.

For the overall production control of cross cut operations there is essentially nothing reported. In an automated system using optimization there is a need to heuristically and explicit adjust objective coefficients in order to meet product demands. Figure 1 illustrate the overall control process. It is obvious that a poor internal weighting may lead to poor results in an automatic process where there is none or little manual interaction. If internal prices is not dynamically adjusted the optimization process will create an off-balance production in relation to what is wanted. A strategy that is based on the global process will lead to a better raw material useage. Does it exist an optimal price setting policy? That is a hard question and depends on the actual application. If it is possible to obtain information about the raw material and the underlying processes it is possible to customize control strategies.

Existing strategies, if any exist, can be very crude. One is simply to set a fixed product value which is kept and then continuously remove each product from the cutting list once its demand is satisfied. Cutting solutions towards the end will obviously give rise to a lot of wastage as the number of possible combinations will drop drastically with the decreasing number of products. Generally cost coefficients would be initialized using different criteria based on experience. In other all are set to the same initial value or there may be a scaling depending on e.g. the length of the products.

In this paper we concentrate on the standard problem with crayon marking mentioned earlier. The quality restrictions are simple; each product must be cut from a defect free

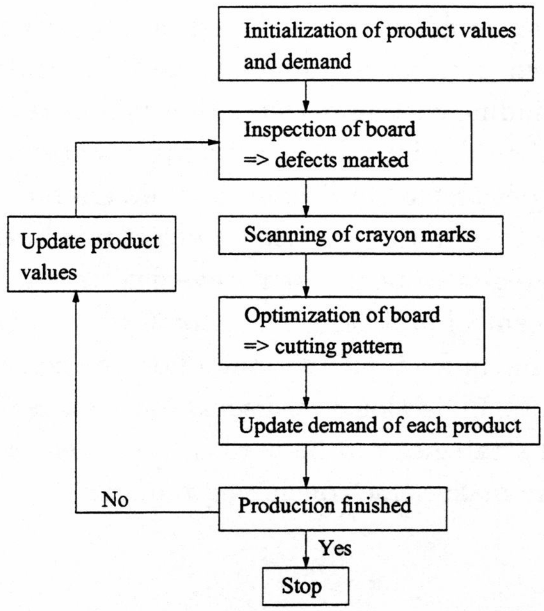

Figure 1: Overall control process.

part of the board. In Figure 2 an illustration of the main steps of this cross cutting process is given. Wooden boards are transported on a conveyer belt from storage piles to the operators who identifies and mark all defects. Thereafter, this information together with the cutting bill is used to define and solve an optimization problem which gives the cutting pattern. The solution is then used to generate sawing instructions for the saw. Once the board is cut, it is sorted into the correct product pile.

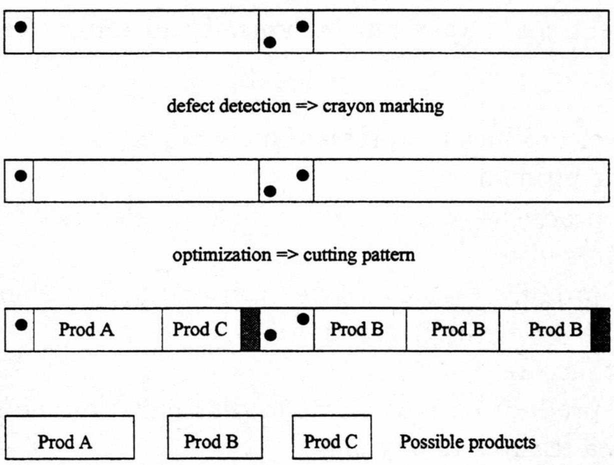

Figure 2: Illustration of the main steps of the cross cutting process.

The outline of this paper is as follows. In Section 2 we describe the underlying process including the optimization problem that we want to control. In Section 3 we discuss a production planning problem that is based on the assumption that all boards are defect free. It is then possible to formulate an optimization problem that finds an optimal strategy in finding cutting patterns that minimizes the total number of boards used. In this section we also introduce an example that will be used as an illustration of the various strategies tested. In Section 4 we describe a number of different control strategies. These are then initially tested on defect free boards. In Section 5 we apply the same control strategies on boards that have defects which is the realistic case and a considerably harder control problem. In Section 6 we investigate some basic properties of the control problem. In Section 7 we study how the control strategies behave under various input scenarios. In Section 8 we discuss some issues raised throughout the paper and how the control strategies can be used in other related and similar applications. In the last Section we make some concluding remarks.

2 Optimization problem to control

To develop control strategies for the overall planning problem it is important to be familiar with the optimization problem. In this section we formulate this optimization problem that can be used in general cross cut operations. This section is not a prerequisite for the remaining part of the paper but it gives a background to the optimization problem and its solution methods.

To formulate the mathematical model we discretize the board into n elements. The size of each element could vary but is typically 10 mm. We introduce the following notation.

l_i = length of product i expressed in elements.

c_i = value for product i.

$$x_{ij} = \begin{cases} 1 & \text{if product } i \text{ starts in discretization element } j \\ 0 & \text{otherwise} \end{cases}$$

$$a_{ijk} = \begin{cases} 1 & \text{if product } i \text{ which starts in discretization element } j \\ & \text{also covers discretization } k \\ 0 & \text{otherwise} \end{cases}$$

$$b_{ij} = \begin{cases} 1 & \text{if product } i \text{ which starts in discretization element } j \\ & \text{is a feasible allocation.} \\ 0 & \text{otherwise} \end{cases}$$

The coefficients a_{ijk} are easily determined by

$$a_{ijk} = \begin{cases} 1 & \text{if } j \leq k \leq j + l_i, \; i = 1, \ldots, m, \; j = 1, \ldots, n, \; k = 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

To find values for the coefficients b_{ij} we need to check against the defects in each board. The model can now be stated as

$$[\mathbf{P}] \qquad \max \sum_{i=1}^{m} \sum_{j=1}^{n} c_i b_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \sum_{k=1}^{p} a_{ijk} b_{ij} x_{ij} \leq 1 \qquad j = 1, \ldots, n$$

$$x_{ij} \in \{0, 1\} \quad i = 1, \ldots, m \quad j = 1, \ldots, n$$

The n constraints ensure that any position (element) is covered by at most one product. The b_{ij} coefficients are determined by the quality restrictions and therefore we need to check each variable against all its restrictions. The problem is a so called set-packing problem which is a well-known integer programming problem. Each product makes a contribution to the constraint set in the form of a column. The coefficients in the column is either 0 or 1. Furthermore, all 1's are adjacent. These two properties makes it possible to reformulate problem [**P**] as a longest path problem in an acyclic network. The arcs in the network (after the filtering process) represent feasible products and the nodes represent potential cut positions. This final problem can efficiently be solved using a Dynamic Programming (DP) procedure.

The method to solve the model is illustrated in Figure 3. In the upper figure we have two products with lengths three and five (elements) and values 3 and 6 respectively. We start by assuming that all allocations are feasible. In the middle part we have tested all quality restrictions and the pruned graph include all feasible allocations. This network is then used to solve the longest path problem where the optimal path is given in the lower network. The corresponding physical cutting pattern is given at the bottom of the figure.

The cross cutting problem can be solved by classical DP which essentially decomposes a large problem into a series of more tractable smaller problems. General references on DP are e.g. Dreyfus and Law [8] and Martello and Toth [11]. In DP nomenclature the problem has the following characteristics: The problem is divided into stages, represented by the discretization elements at which a decision is required. At every stage, the model can be in a number of states. States provide information needed to make an optimal decision at every stage. The decision made at every stage describes how the state at the current stage is transformed into the state of the next stage. Most

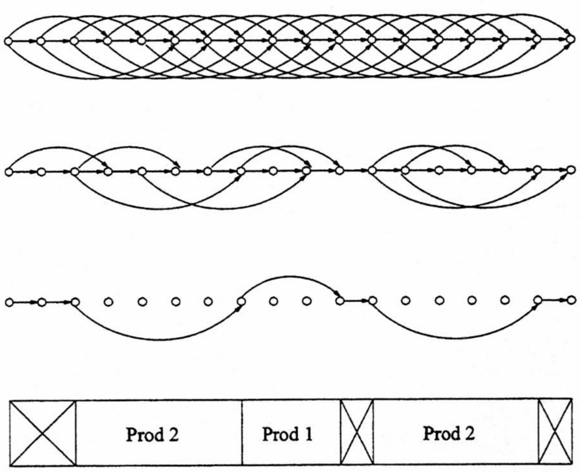

Figure 3: An illustration of the solution approach.

importantly, the decision made at the current state is independent of previously reached states or previously chosen decisions. This is called the *Principle of Optimality*. In this application it is a one-to-one correspondence between stages and states. Finally, there will be a recursion which relates the reward gained from the current stage to that of previous stages. At each stage the decisions made to reach each state is recorded. A link or arc from one stage to another correspond to a product, and the value assigned to that link correspond to the product value. The recursive relationship using a forward DP formulation of the problem can be written as

$$f(j) = \max_{i \in I_j} \{c_{ij} + f(j - l_i)\} .$$

Here, $f(j)$ is the optimal policy function value at stage j (or element j) and $I_j \subseteq \{0, 1, \ldots, m\}$ is an index set defined by which cut-options that are feasible at stage j. Note that, in order to enable gaps between consecutive products, an artificial product with length $l_0 = 1$ and value $c_0 = 0$ is introduced. We also need to ensure that $1 \leq j - l_i \leq n$. We let $y(j)$ record the optimal decision how each stage were reached. Each $y(j)$ is therefore simply a back pointer to where the cut-option started. The initial condition for the problem is $f(0) = 0$. The optimal objective function is found at stage n, i.e. $f(n)$. Once $f(n)$ is found, the optimal solution is identified by backtracking using the back pointers y.

3 Production planning

The aim of the production control is besides using as few boards possible also to achieve a production that is *smooth*. By this we mean a production where each product is produced in about the same pace and that the demand is reached in about the same time. In Figure 4 we illustrate production of two products, A (dashed line) and B (solid line). One production profile (left) illustrates a production (left) which we regard as smooth whereas the oher (right) gives a off-balanced production i.e. non-smooth. An important property of a smooth production is that it will use less boards. The main reason is that it is possible to find efficient cutting patterns throughout the production with many products available. In a non-smooth production there will be less products available allowing less cutting patterns.

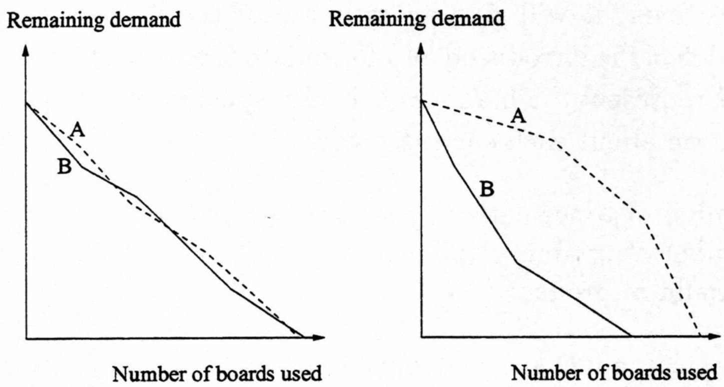

Figure 4: Illustration of a smooth (left) production of two products and a non-smooth production (right).

3.1 Case study

In our study of the control processes we will use one standard example throughout the paper. The cutting bill has six products, denoted A to F. Information regarding product length and demand is given in Table 1. Each product requires a defect free area corresponding to its length.

3.2 Mathematical models

In some cases when the quality is known in advance, for example, with boards of the same lengths and defect free it is possible to formulate a mathematical model that

Product	Length (mm)	Demand
A	2500	1000
B	2100	1000
C	1800	1000
D	1500	1000
E	1400	1000
F	900	1000

Table 1: Cutting bill with information about length and demand for the six products in the case study.

providse an optimal solution i.e. a solution which requires a minimum number of boards. We start to describe this problem as it gives some insight into the general problem. Moreover, it will give an estimate of the minimum number of boards required when defects is introduced. To formulate the mathematical model we introduce variables that represent the number of times a specific cutting pattern is used. We also need information about each cutting pattern.

$$\begin{aligned} y_j &= \text{Number of times pattern } j \text{ is used.} \\ a_{ij} &= \text{Number of product } i \text{ in pattern } j. \\ b_i &= \text{Demand of product } i. \end{aligned}$$

The mathematical model can now be stated as

[PP]

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} y_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} y_j \geq b_i \qquad i = 1, \dots, m \\ & y_j \in \{0, 1\} \qquad j = 1, \dots, n. \end{aligned}$$

To apply this model for our case study we assume that we have boards of length 5,500mm. In Table 2 we give some potential cutting patterns to use. We note that this is a small subset of the total number possible.

For our example, model [**PP**] becomes

Cutting pattern	Scrap (mm)	Number of products *A*	*B*	*C*	*D*	*E*	*F*
1	100	0	0	0	0	0	6
2	400	0	2	0	0	0	1
3	0	1	1	0	0	0	1
4	200	0	0	1	0	2	1
5	500	2	0	0	0	0	0

Table 2: Five possible cutting patterns.

$$[\mathbf{P}] \quad \min \quad y_1 + y_2 + y_3 + y_4 + y_5 \ldots$$

$$\textbf{s.t.} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 6 \end{pmatrix} y_1 + \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} y_2 + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} y_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} y_4 + \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} y_5 \ldots \geq \begin{pmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \end{pmatrix}.$$

Problem [**PP**] is an integer programming problem. To solve it there is a need to generate all possible cutting patterns (or columns in the model) in advance or to use a column generation approach where the cutting patterns are generated dynamically based on dual information. The subproblem in a column generation scheme can be formulated as follows.

$$[\mathbf{Sub}] \qquad \min \quad \sum_{i=1}^{m} (c_i - \sum_{k=1}^{n} \alpha_k) x_i$$

$$\textbf{s.t.} \qquad \sum_{i=1}^{m} l_i x_i \leq 5,500$$

$$x_i \geq 0, \; integer \quad i = 1, \ldots, m.$$

Here, x_i is the number of products i used in the cutting pattern and l_i is the product length. The koefficient α_k is the value of the dual variable associated with constraint k in problem [**PP**]. Once the LP-relaxation of [**PP**] is solved, a subproblem [**Sub**] is solved to generate a new cutting pattern. This process is repeated until no cutting pattern with a negative reduced cost, i.e. $(c_i - \sum_{k=1}^{n} \alpha_k)$, is found. To find an optimal integer solution it is necessary to include a Branch& Bound algorithm. To describe this is outside of the scope of this paper. For our example, (after column generation together with Branch& Bound) the optimal solution is given in Table 3. The total number of boards needed is 1867.

Number of patterns	Pattern used, Number of products A	B	C	D	E	F
67	0	0	3	0	0	0
600	1	1	0	0	0	1
300	1	0	0	2	0	0
400	0	1	1	1	0	0
400	0	0	1	0	2	1
100	1	0	0	0	2	0

Table 3: Optimal solution for our example given defect free boards of length 5500mm.

3.3 Control strategies

There are three control strategies that we will study. The first one, *Fixed*, which is the commonly used one, is based on removing a product from the cutting bill once its demand is reached. The product value is fixed during the production.

[*Fixed*]
$$\bar{c}_i = \begin{cases} c_i & \text{if } p(i) < b_i \\ 0 & \text{otherwise} \end{cases}$$

Here, $\bar{c}_i$ is the adjusted value used in the cross cut optimization, c_i is the initially chosen product value and $p(i)$ is the current production of product i. The second strategy, *Scaled*, is based on a relative value based on the proportion of the current production level for each product. This relative value will be decreasing monotonically until demand is reached.

[*Scaled*]
$$\bar{c}_i = \begin{cases} c_i * p(i)/b_i & \text{if } p(i) < b_i \\ 0 & \text{otherwise} \end{cases}$$

The third, *Dynamic*, is based on a dynamically changed value. Here we use c_i^{old} to represent the value used for the previous board and $aver$ to represent the total current average production i.e. $aver = 1/m(\sum_{i=1}^{m} p(i)/b_i)$. The value $const$ is a positive constant (unless otherwise stated we have used the value 10). The product values will fluctuate depending on the current production levels of the different products. The idea is to increase the relative value if the production is falling behing the average production and vice versa if the production is ahead.

[*Dynamic*]
$$\bar{c}_i = \begin{cases} c_i^{old} + const * (p(i)/b_i - aver) & \text{if } p(i) < b_i \\ 0 & \text{otherwise} \end{cases}$$

We also need to select an initial value for each product. We will also study various rules but in the first tests we limit ourselves to two simple alternatives. The first initial value is to choose the same for all products (we have used 100) and the second is to choose a value which reflects the product length.

4 Production control with defect free boards

The numerical results presented in the remaining sections are obtained through a simulation where we have a cross cutting process. We have generated a large number of boards that represent the real situation at a production line. The simulation follow the description given in Figure 1. The implementation for the optimization routine is done in Visual Basic and the routines which implement the three strategies is implemented in Visual Basic linked with EXCEL.

In this section we will start with a simplified study and use defect free boards all with a length of 5,500mm. One reason for this is to be able to compare the solution obtained from the planning problem described earlier. Table 4 gives the results from these initial tests. We note that the optimal solution from the planing problem gives a lower bound of 1867 boards. It is clear from these results that updating may play an important role.

Strategy	Initial values	Number of boards used	Percentage (%)
Fixed	100	2168	100.0%
Fixed	length	2002	92.3%
Scaled	100	1901	87.7%
Scaled	length	1868	86.2%
Dynamic	100	1913	88.2%
Dynamic	length	1869	86.2%

Table 4: Results using the three strategies for defect free boards.

To illustrate the behaviour of the strategies we have included a number of figures where we provide profiles of the product values and demand during the process. Figure 5 gives the demand (or production) profile for strategy *Fixed* where the initial values are 100 (left) and product length (right). Using initial values of 100 first favours a pattern with 6 product F, then a pattern with 2 product E and one product C, and so on. Having initial values scaled to the length of the products favours a cutting pattern with one product A, one B and one F, respectively. Initial values of 100 will favour patterns with as many pieces as possible whereas the second selection will favour patterns with as little waste as possible. We note that change in patterns appear as the production of certain products reaches its demand. This is a typical example of a non-smooth production.

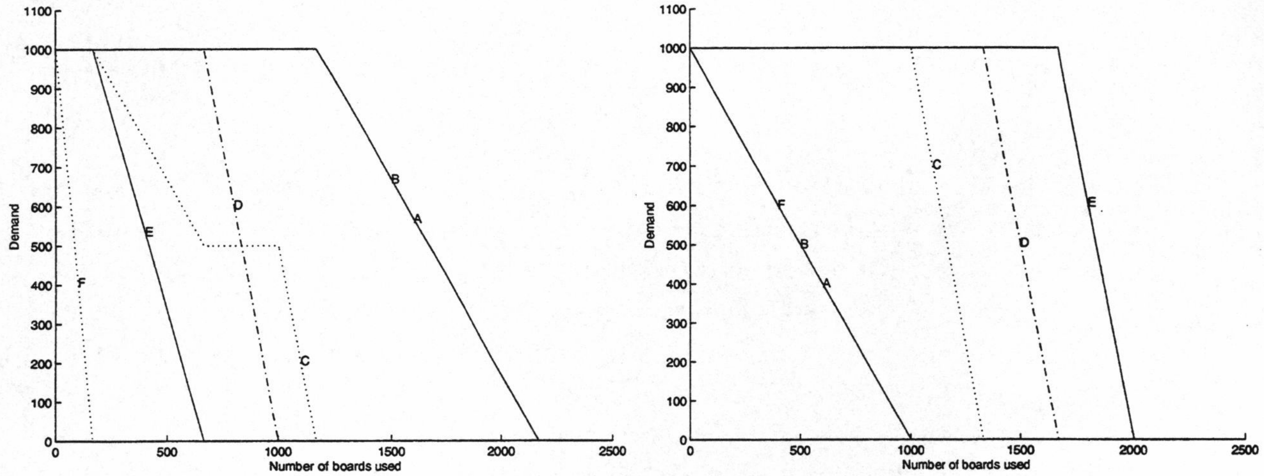

Figure 5: Demand profiles using the strategy *Fixed* with initial values of 100 (left) and product lengths (right)and defect free boards.

In Figure 6 we have used the *Scaled* strategy with initial values of 100. The left part gives the value profile and the right part the demand profile. It is obvious that this strategy provides a better production as all products meet their demand essentially at the same time. Furthermore, it requires much less boards as compared to the *Fixed* strategy. This is also true for the case when we have initial values corresponding to the length, see Figure 7. The number of boards used here is even less and the production profile is smoother.

Corresponding results with strategy *Dynamic* is given in Figures 8 and 9. We can clearly see that by using initial values corresponding to the length we obtain a more stable production. In Figure 8 there is a need for the products to establish some stable level for the values before the production becomes stable. For the case with initial values of length in Figure 9 we get a smooth production.

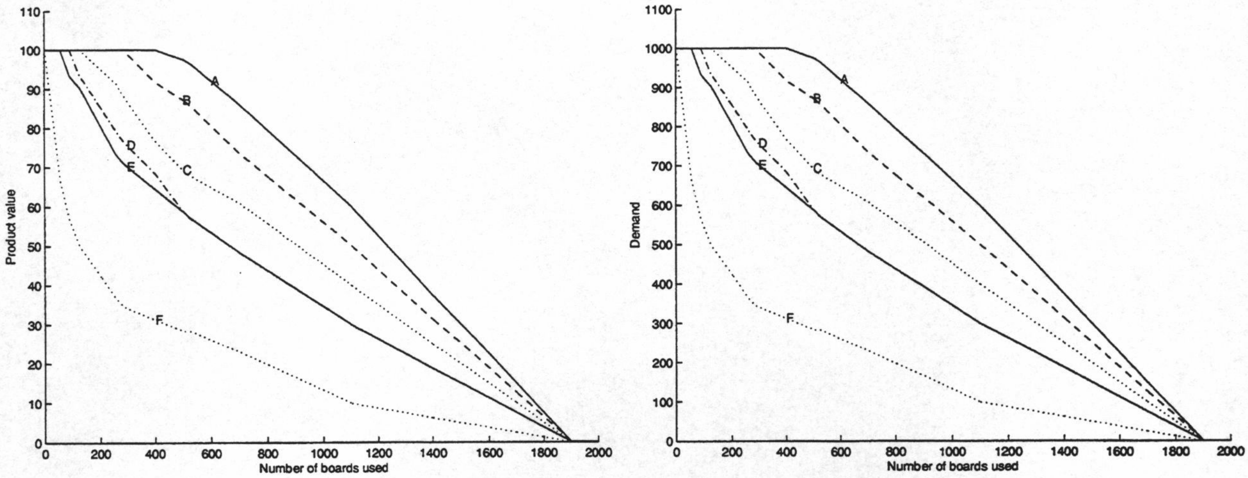

Figure 6: Value (left) and demand (right) profiles using the strategy *Scaled* with initial values of 100 and defect free boards.

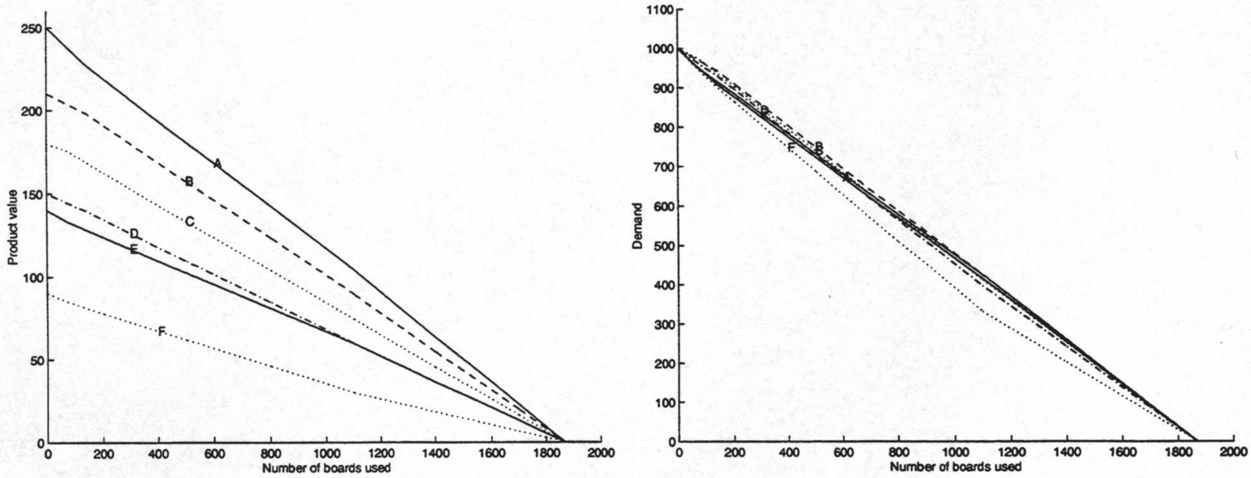

Figure 7: Value (left) and demand (right) profiles using the strategy *Scaled* with initial values corresponding to product lengths and defect free boards.

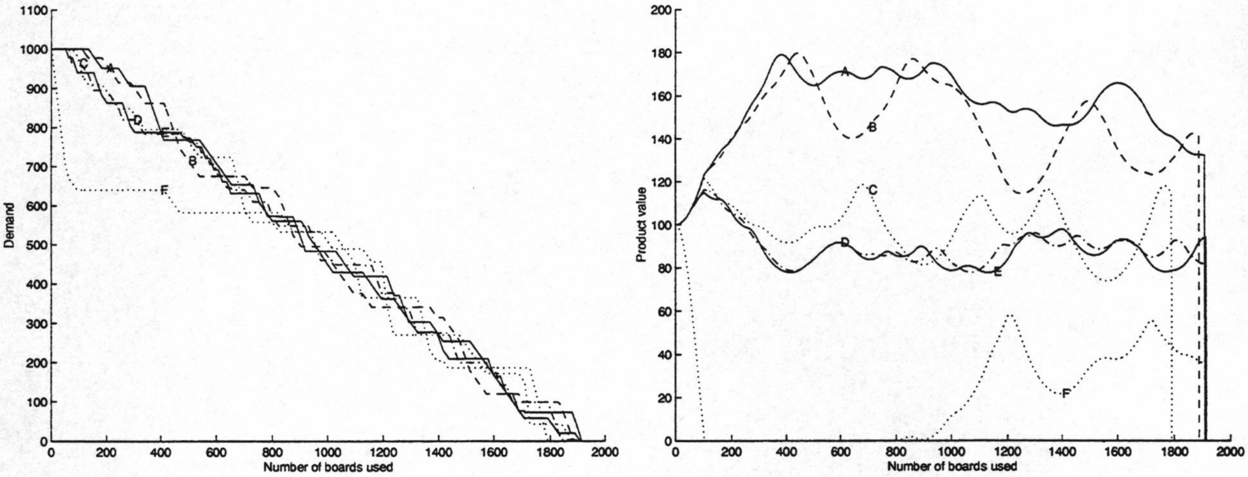

Figure 8: Value (left) and demand (right) profiles using the strategy *Dynamic* with initial values of 100 and defect free boards.

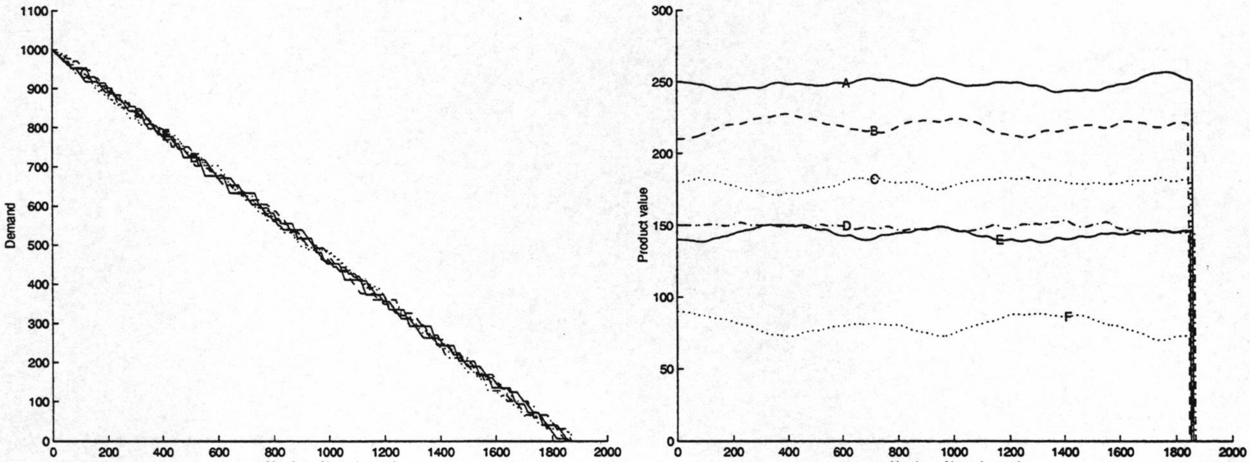

Figure 9: Value (left) and demand (right) profiles using the strategy *Dynamic* with initial values corresponding to product lengths and defect free boards.

5 Production control with defects

The boards used to test a practical situation with defects are randomly generated with a length uniformly distributed in the interval 4,000-7,000 mm (The average length is 5,500mm). The number of defects is limited to between 0 and 4 for each board, and the average number of defects for a 5,500mm board is 2.0. The results from applying the three strategies is given in Table 5. The potential decrease in the number of boards required is even larger in the case with defects. A potential reduction of 18% obviously correspond to a large savings in raw material.

Strategy	Initial values	Number of boards used	Percentage (%)
Fixed	100	3085	100.0%
Fixed	length	2364	76.6%
Scaled	100	2332	75.6%
Scaled	length	2236	72.5%
Dynamic	100	2279	73.9%
Dynamic	length	2220	72.0%

Table 5: Results using the different strategies for boards with defects.

To illustrate the behaviour of the strategies we have again included a number of figures. Figure 10 gives demand profiles with the *Fixed* strategy. With a common initial value of 100 (left) the production becomes non-smooth. With initial values relating to lengths the situation improves but there is still not a smooth production.

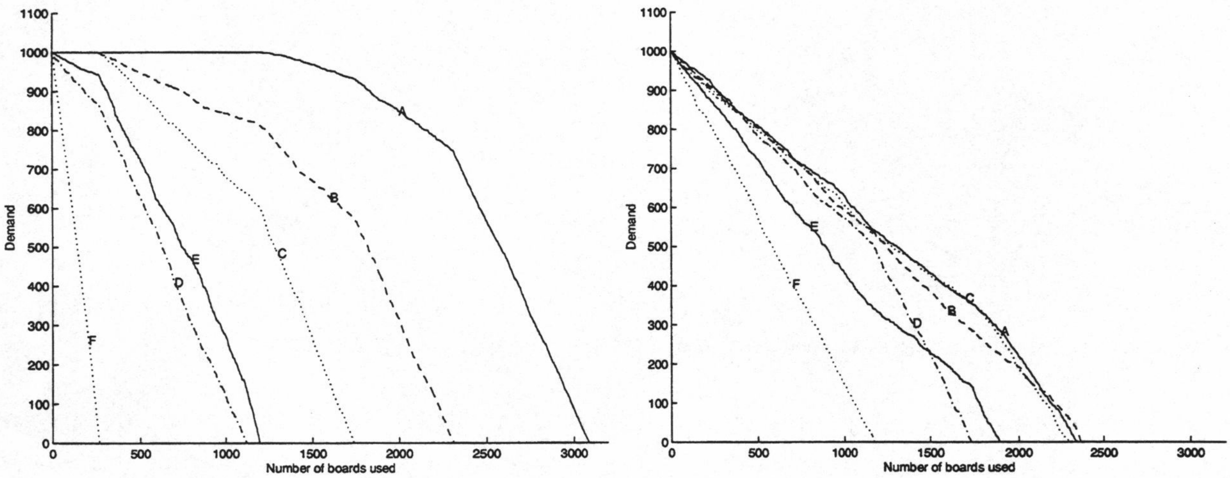

Figure 10: Demand profiles using the strategy *Fixed* with initial values of 100 (left) and product lengths (right)and boards with defects.

Using the *Scaled* strategy as in Figure 11 provide a production that brings the production ends of all products except product F together. The reason is that product F has a too large relative value as compared to the other. In Figure 12 where the length determines the initial values we have a much improved production profile.

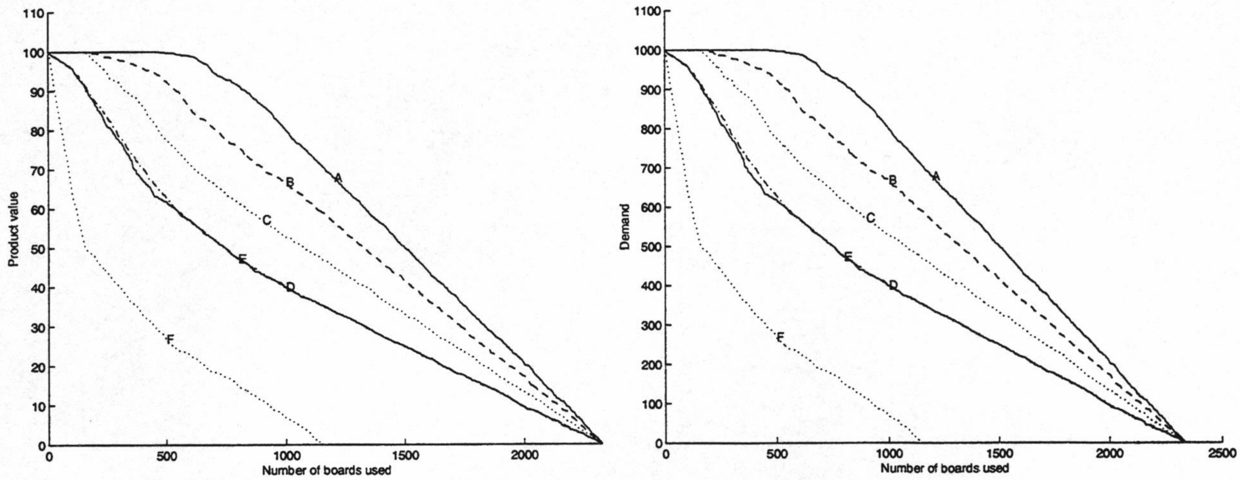

Figure 11: Value (left) and demand (right) profiles using the strategy *Scaled* with initial values of 100 and boards with defects.

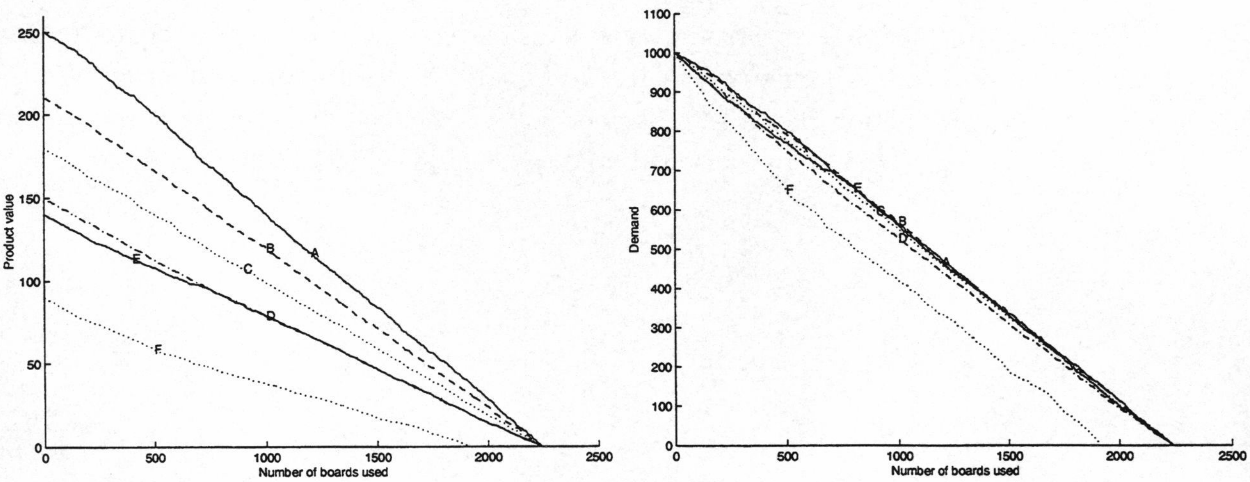

Figure 12: Value (left) and demand (right) profiles using the strategy *Scaled* with initial values corresponding to product lengths and boards with defects.

When we use the *Dynamic* strategy, see Figure 13, with board lengths as initial values we have a smooth production and all products will reach their demand in about the same time. A problem occur for the case when we use the same initial values, see Figure 14. Here, the value profiles get a tendency to increase as long as product F is in demand. The reason behind this is that there is nothing which limits the total product value which is down at 0. When product F obtain a value of 0 and the production is ahead of the other products then all products (except F) will have a production profile which is below the average. All these products will now increase their values in order to compensate for this. However, product F will continue to produce as there will be short pieces where only product F fits. Hence, the total value will increase as long as F is below average production and has not reached its demand. To overcome this we make a modification which enforces the total value to be constant for the active products (i.e. products that have not reached their demand). The way we implement this is to use the same updating formula combined with a rescaling. This modification is given below.

[*Dynamic*]

$$\bar{c}_i = \hat{c}_i * sum_1/sum_2$$

where

$$\hat{c}_i = \begin{cases} c_i^{old} + const * (p(i)/b_i - aver) & \text{if } p(i) < b_i \\ 0 & \text{otherwise} \end{cases}$$

$$sum_1 = \sum_{i \in I} \hat{c}_i$$

$$sum_2 = \sum_{i \in I} c_i^{old}$$

The index set I is defined through $I = \{i : \hat{c}_i > 0\}$ which denote all products that have not reached their demand. Using this modification (which we keep for all tests using the *Dynamic* strategy) we get profiles as given in Figure 15.

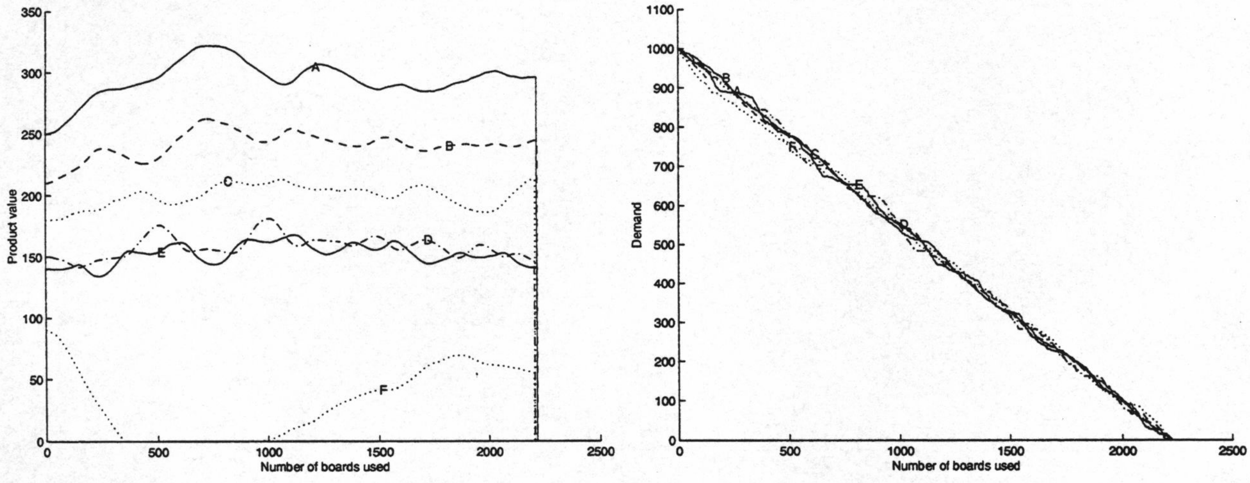

Figure 13: Value (left) and demand (right) profiles using the strategy *Dynamic* with initial values corresponding to product lengths and boards with defects.

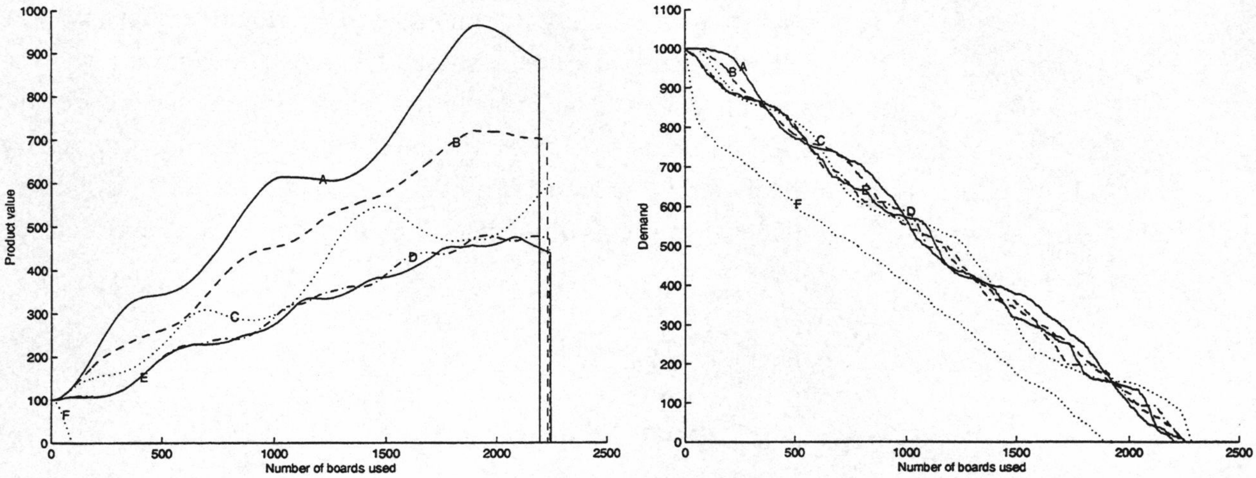

Figure 14: Value (left) and demand (right) profiles using the strategy *Dynamic* with initial values of 100 and boards with defects.

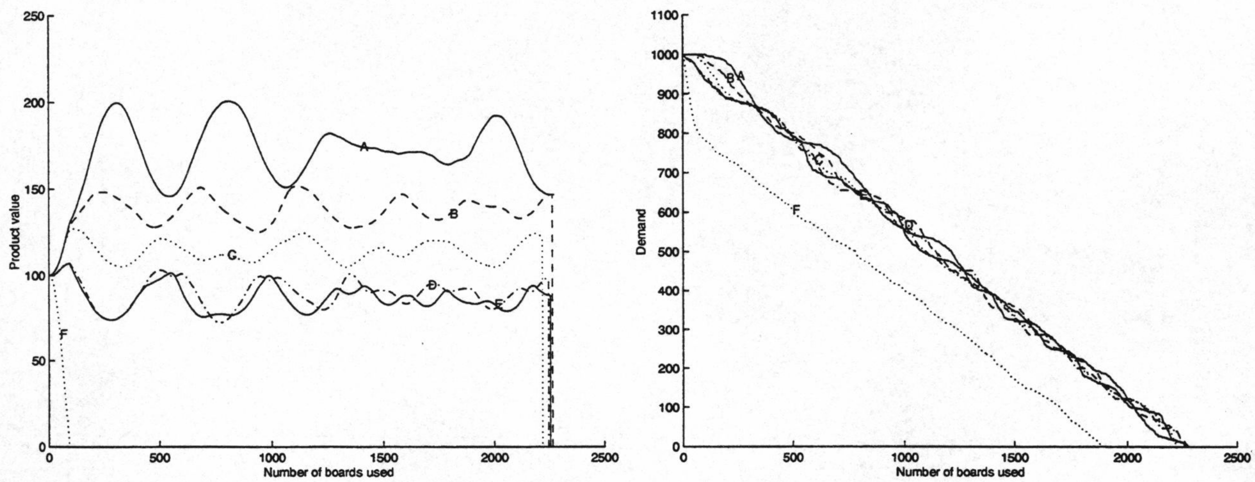

Figure 15: Value (left) and demand (right) profiles using the modified strategy *Dynamic* with initial values corresponding to product lengths and boards with defects.

6 Initial value selection

It is obvious that properly selected initial values has a positive impact of the overall performance. This is more critical for the *Fixed* and *Scaled* strategies. To investigate the impact of the internal weighting between products we study the case when only two products are in demand. In Figure 16 we have demand profiles using A and B (left) and products A and F (right). In both these situations we have used the board lengths as initial values. It is obvious that it is very difficult to get a smooth production with this strategy. Using the comparison with products A and B for the *Dynamic* strategy, as in Figure 17, we get a more controlled production. An interesting aspect is that the value of B occasionally become larger than the value for A. Hence, to get a smooth production it is important to allow the values to dynamically change and pass each other. When we compare products A and F in Figure 18 we note that it is more difficult to control the production. The reason is that product F does not compete with product A for a large proportion of the boards. Even if the value of A would increase infinitely it is not possible to decrease the production of F below a certain level. When the production profiles becomes tangled, the value for F flips between 0 and a very low value in order to keep up with the production of A. A different result appear when we compare products D and F. Then we get a profile as given in Figure 19 where there indeed is a more intense competition between the products.

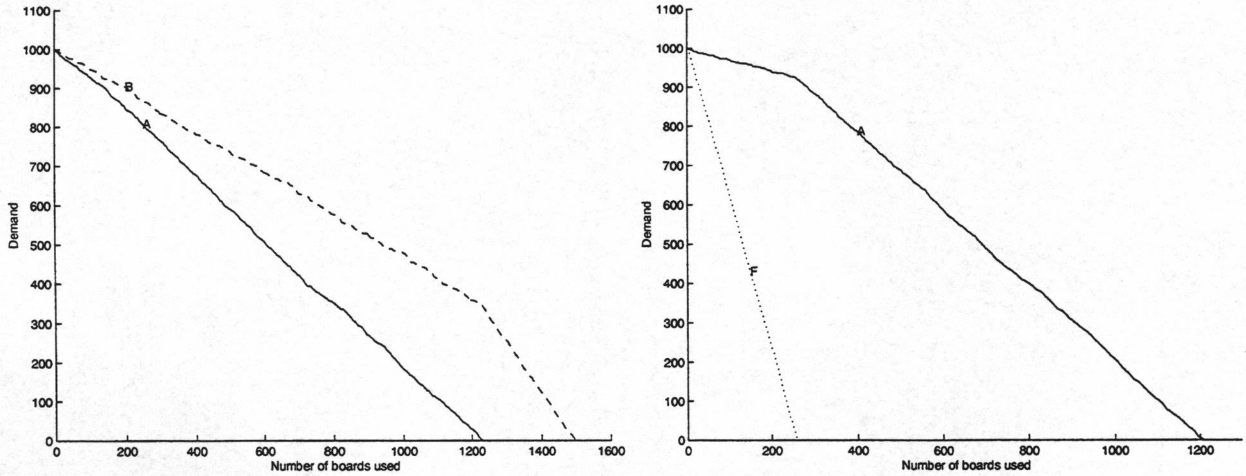

Figure 16: Demand profiles with strategy *Fixed* when there is a demand of only products A and B (left) and products A and F (right).

If we compare two products that are very similar in length e.g. products D and E we find that the *Fixed* strategy gives a production that favours D as it has a slightly higher value. This is illustrated in the left part of Figure 20. The right part of the

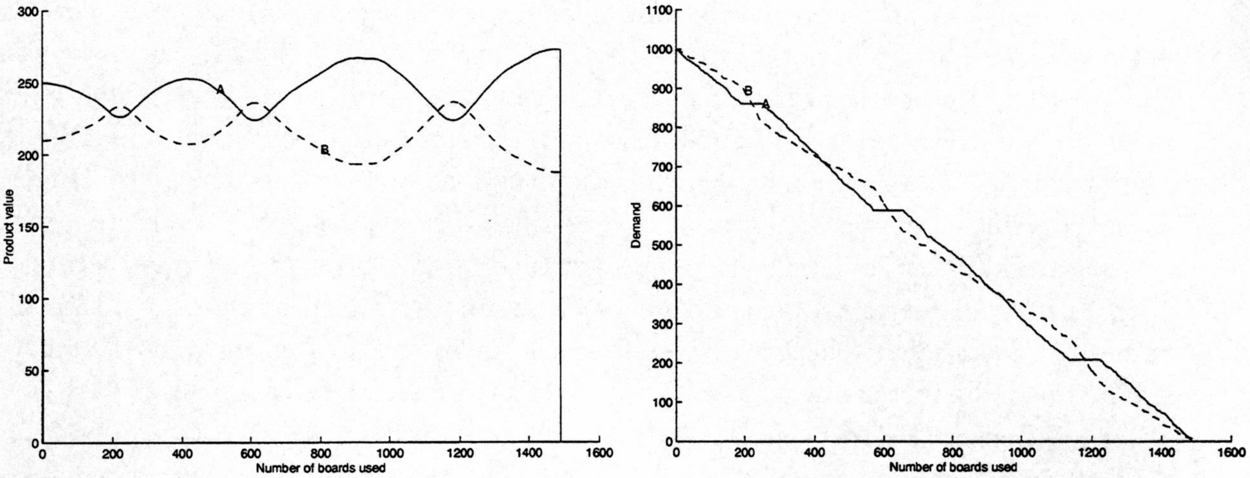

Figure 17: Value (left) and demand (right) profiles with strategy *Dynamic* when there is a demand of only products A and B.

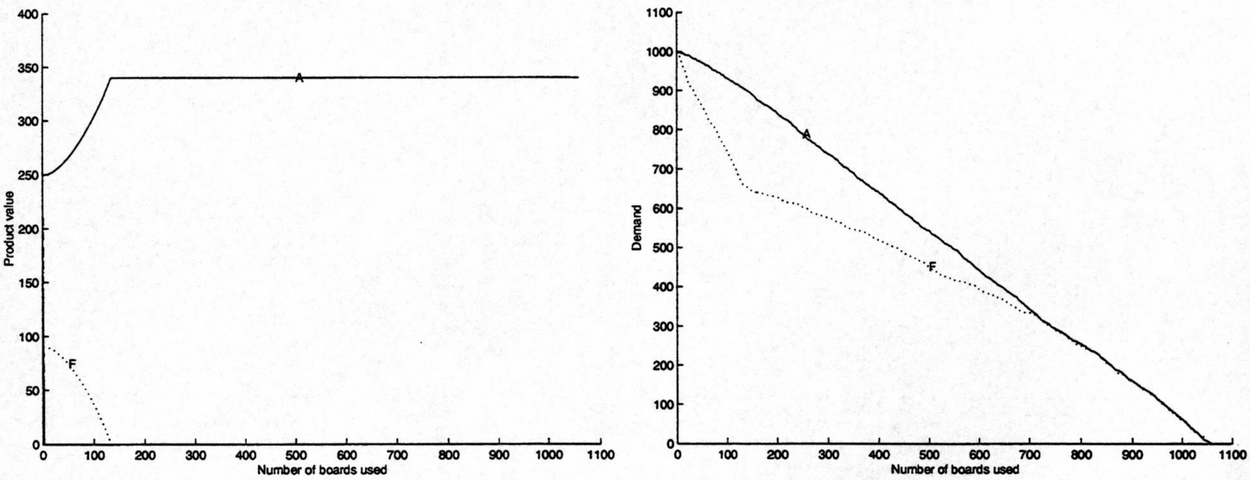

Figure 18: Value (left) and demand (right) profiles with strategy *Dynamic* when there is a demand of only products A and F.

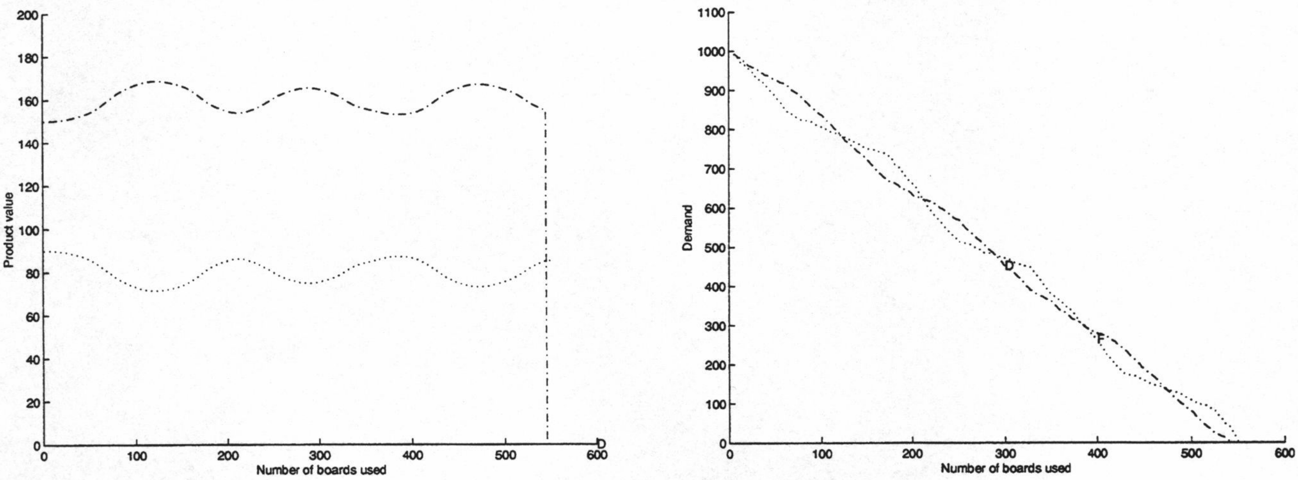

Figure 19: Value (left) and demand (right) profiles with strategy *Dynamic* when there is a demand of only products D and F.

Figure shows the demand profile for products A and F. Using strategies *Scaled* and *Dynamic* gives much better performance, in Figure 21 we give the value and demand profiles using the *Dynamic* strategy for products D and E.

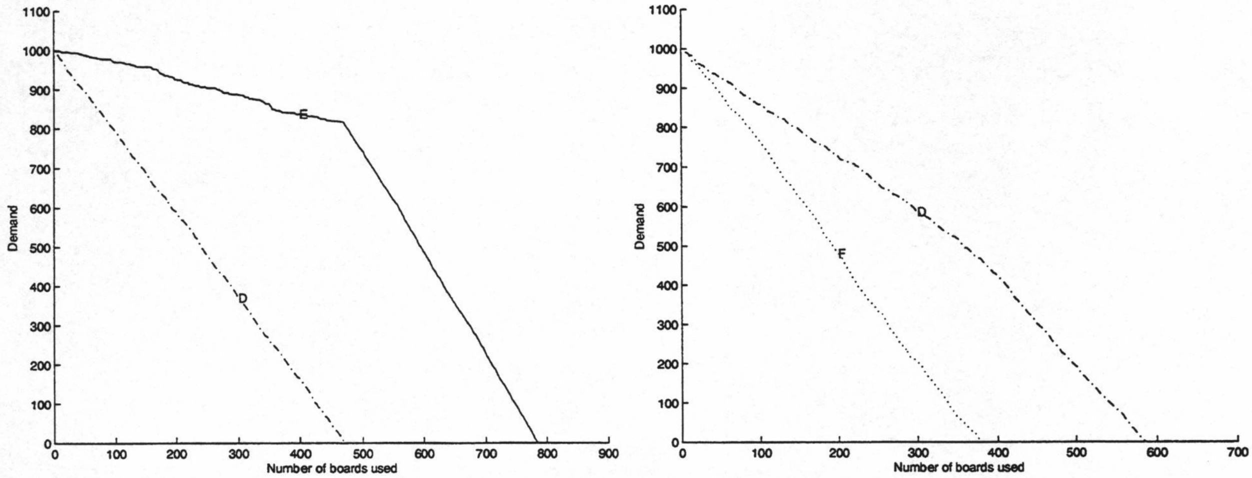

Figure 20: Demand profiles with strategy *Fixed* when there is a demand of only products D and E (left) and products D and F (right).

6.1 Pairwise comparison

One approach to find weightings in other applications is an idea by Saaty [14]. This is based on finding a relative weight between all pair of products. In Table 6 we give