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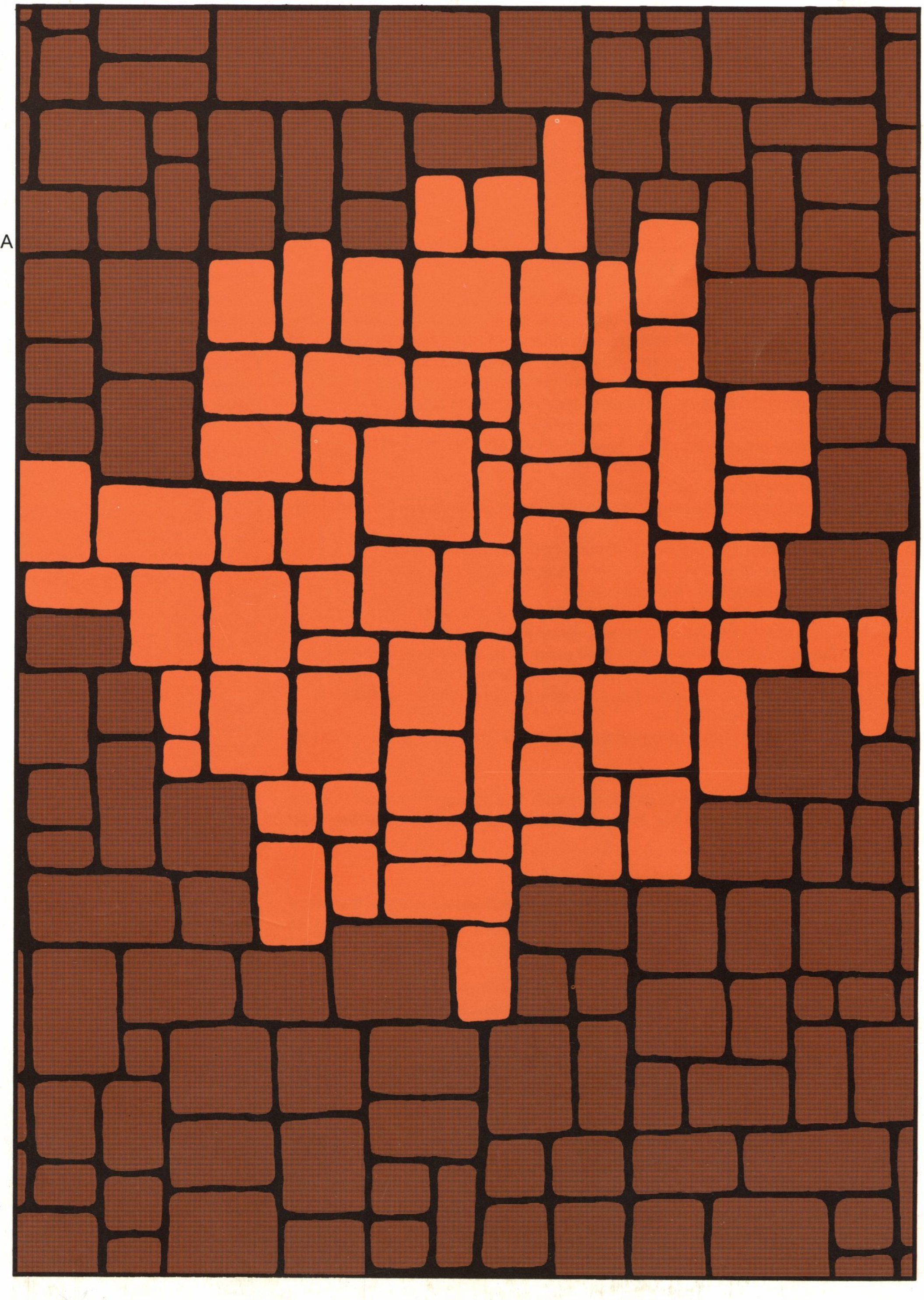

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Optimal fenestration retrofits by use of MILP programming technique

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Abstract

When buildings are subject for refurbishment, it is very important to add the optimal strategy at that very moment. If other solutions are chosen and implemented, it will no longer be possible to change the building at a later occasion with the same profitability. A suitable criterion for optimality is the point where the life-cycle cost (LCC) has its minimum value. This point can be calculated by using so-called mixed integer linear programming (MILP). This paper shows how building and possible fenestration retrofits are described in such a MILP program. Changing existing double-glazed windows to triple ditto will of course make the *U*-values lower, but at the same time less solar radiation is transferred through the glass panes. This must be properly addressed in the MILP model. Of vital importance are also the heating system and the energy tariff connected to it. Nowadays, time-of-use rates are common practice both for district heating and electricity. These facts make it unsuitable to write, optimise and solve the MILP model "by hand", and instead a computer program has been designed for writing the model in the form of a standard MPS data file. This file can in turn be scanned and optimised by MILP-solving programs available at the market today. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Mixed integer linear programming; Life-cycle cost; Fenestration retrofits; Windows; District heating

1. Introduction

A building has a very long life-span, sometimes more than 100 years. During such a long period, a lot of repairs must be done or else the building will become dilapidated. Further, the building has to be heated at least in a country like Sweden, where winters are cold. A lot of money must therefore be spent over the years. Many times, the owner of the building only looks at the direct building cost and tries to build as cheap as possible, even if this will result in high operating and maintenance costs in the future. If all costs were added for the total life of the building, they would probably be constructed differently. One problem is that all costs do not emerge at the same time. A future cost is not of the same value as a present one, even if they add up to the same amount, e.g. in Swedish crowns, SEK. This can be dealt with by the so-called present value calculations, where a discount rate is used for transferring all future costs to present time. The sum of all these costs is called the life-cycle cost (LCC). Even if the concept has been known for a long time (see e.g. [1]), it has not been in widespread use until now. This is so because of all tedious calculations which must be performed before the LCC is found. Future costs are also not known in advance and the discount rate changes over the year in a way that is not easily predicted. Many LCCs must therefore be calculated with different data in order to widen the basis of decision. Such a procedure is called a sensitivity analysis. Heavy calculations are suitable for computers, and hence LCC calculations can nowadays be fulfilled in minutes or even seconds, if a model is constructed in the form of a computer program.

The objective is, however, not only to calculate the LCC, but also to find the lowest possible such cost. The strategy of renovation to choose is therefore the one where the LCC is minimised. "Classic" calculus provides a means for minimisation. The LCC of the building must then be expressed in a continuous function, which in turn is derived and set equal to 0. This method has been used in [2,3], where papers describe the so-called OPERA model. Sometimes, it is not easy or even preferable to find such continuous functions, for example when optimising windows. There are certain elements that are discrete by nature, e.g. number of glass panes, low radiation-emitting films, heavy gas inclusions and so forth. OPERA deals with such cases by testing a number of alternatives, and calculates the LCC for all these alternative solutions. A totally different concept is to find the lowest LCC by use of linear programming (LP); see [4] for details. As will be shown below, a LP model starts with an expression which shows the total LCC, i.e. the so-called objective function. This function contains a number of variables whose values must be set so that the function finds its lowest possible value. One way to minimise the situation

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is therefore to set all these variables to 0, but unfortunately this will lead to a building where no space or domestic hot water heating will be present, because the boiler thermal size is likewise set to 0. That solution is simple and the LCC is 0, but not preferred by humans. By the introduction of certain constraints, the LP ascertains that a suitable indoor temperature must prevail. This in turn might lead to extra insulation, e.g. by using oil for heating and perhaps triple-glazed windows. The name LP implies that the mathematical model must be totally linear. In real world, this behaviour is not common practice and the cost for changing an old boiler many times starts with a "step". For example, such a cost might occur for the demolition of the old boiler. Step functions like that are dealt with by introducing integers, especially binary variables that only can assume the values 0 or 1. If the value is 1, the boiler is chosen and a step cost is introduced, or else the value is 0 and no step exist and neither the boiler is present. This paper describes in detail how such a program is designed for a building with its accompanying LCC. First, however, a brief recapitulation of the thermal behaviour of windows might apply.

2. Heat transfer through windows

Heat transfer through windows has been of interest for the scientific society for more than 50 years. In, e.g. [5], the authors go back as early as 1946 in the search for suitable references in this field. However, they also contributed significantly themselves, and by using so-called finite difference technique they succeeded in calculating heat transfer over air gaps, i.e. double-pane windows, which in turn agreed with experiments. Those experiments, where the heat transfer was actually monitored, were published in [6]. Consider firstly, the case when the air gap between the panes is very small or not existing at all. As is stated in [5], Fourier's law now tells us that the heat transfer takes place by conduction, and if the air gap gets larger the transfer should be smaller. If the air gap is very large, the conduction heat transfer must be negligible but instead convection increases. The heat transfer by radiation was thought to be the same, no matter which size of air gap was considered. Heat transferred by convection is difficult to calculate and in, e.g. [7,8] some 100 pages deal with this problem. The calculations will always be complicated, because convection heat transfer is closely related to laminar and turbulent air flow. By the introduction of, e.g. Prandtl, Nusselt, Grashof and Rayleigh numbers, it is nonetheless possible to calculate the heat transfer, but the result is always an approximation and many times it differs a lot from monitored values. One important factor also is the so-called aspect ratio, A, which shows the rate between the height H of the window and the air gap thickness L. For long and narrow air gaps, where A is greater than 20, conduction was found to be dominant (see [5]). For A smaller than 10, convection was dominant. The air inside the gap was cooled by the outdoor window pane, and because of higher density sank to the bottom of the gap where in turn it was heated by the indoor pane. A so-called circulation cell was developed. For A about 17, however, a multi-cellular pattern occurred, and the authors found that the lowest possible heat transfer occurred when such a pattern was on the rim to emerge. Other papers dealing with heat transfer in such air gaps are, e.g. [9] where multi-cellular behaviour was calculated for an aspect ratio of 30, and [10] where the optimum gap is calculated for four cities in Turkey. This optimisation, however, seems to be based on the criterion of minimum heat flux for a specific dimensional outdoor winter temperature.

Normally, the gap between the window panes is filled with air. By changing this air to other gases, with lower thermal conductivity, the thermal performance of the window will improve. In [11], Table A-19, it is shown that, e.g. carbon dioxide and argon has lower thermal conductivity values than air, 0.0166, 0.0177 and 0.0261 W/m K for a temperature of 300 K, respectively. Even better gases exist, e.g. krypton and xenon but, according to [12], these are expensive to extract from the atmosphere.

One further means for improved thermal resistance is to introduce more air gaps, i.e. triple-glazed windows. See, however [13], where several different alternatives have been examined. The author writes that a lower U-value is achieved if an IR-transparent middle pane, i.e. not glass, is used. Therefore, double-glazed windows with a polyethylene or polypropylene middle sheet would be better than a window with three glass panes. Even better solutions exist, but those seem to be mostly of academic interest. If air is used in the gaps of an ordinary triple-glazed window, they become rather bulky, and this is also a reason for using, e.g. argon instead. The optimum air gap will by this become smaller, and for air, argon, krypton and xenon, it is supposed to be 20, 16, 12 and 8 mm, respectively [12]. Experiments and numerical calculations on a krypton-filled triple-glazed window is published in [14], and the gaps between the panes were 12 mm which supports the statement. Since some years, there is a European standard, EN673, where expressions can be found for calculating window U-values. This standard results in slightly larger optimal gaps and further there seem to be some discrepancies for large gaps compared to the findings in [5,6] (a new standard is under preparation, ISO15099).

In Sweden and other cold countries, it is important to take advantage of solar radiation transferred through the windows in the building. Such radiation resides in a wavelength region of about 0.3–2.0 μm ([7], p. 480), while the radiation from the interior of the building has a wavelength of about 3.5–30 μm ([7], p. 485). This behaviour is used in an ordinary greenhouse, and it is possible to design a window which transfers the shorter waves, but at the same time reflects the longer ones even better than ordinary glass. This is achieved by use of the so-called low-emittance coatings. Also for the radiative heat transfer, the problems with calculations are large. The solar beams are reflected in

Table 1
Average mean monthly outdoor temperatures in °C for Linköping, Sweden

January	February	March	April	May	June	July	August	September	October	November	December
−2.9	−3.0	−0.1	5.3	11.0	15.4	17.7	16.4	12.2	7.1	2.7	0.0

the first glass pane, and only a part is actually transferred, especially at oblique angles; see, e.g. [15], where such issues are addressed. Some of the transferred energy is absorbed in the glass pane, while the rest is lead into the air gap. The second glass pane also reflects some of the incoming radiation, which in turn is to a part reflected in the back side of the first pane and so forth. However, the reference also describes how to calculate the angle dependence of solar transmittance even if no knowledge of transmittance, reflectance or optical constants is present. In [16], the problems are dealt with in more detail.

Because of the difficulties shown above in accurately calculating the heat transfer, it is even more hazardous to actually minimise the LCC of the window construction. Probably, such efforts will not be worthwhile. In this study, we have therefore chosen to examine a number of alternative constructions.

3. Case study

For a number of years, we have used a basic case when designing our models. This case originates from a real building sited in Malmö, located in the very south of Sweden, see [17] where the first case study was published. Now, however, it is assumed that this building is located in Linköping, about 200 km south of Stockholm. It contains 14 apartments and is heated by means of a district heating system which is owned by the municipal utility. The thermal characteristics of the building have been calculated to 2056 W/K. (We will in the following, use the precise calculated values in order to make it possible to follow the calculations in detail.) In Linköping, the dimensioning outdoor temperature is set to −18°C according to the building code. Assuming that the inhabitants use an indoor temperature of +20°C implies that the heating system must provide about 78 kW in order to satisfy the need for space heating. It is further assumed that no extra heat capacity is needed for domestic hot water heating. The windows in the building are faced to the east, 27 windows with an area of 2.8 m^2 each, and west, 29 windows with an area of 2.4 m^2. There are no windows facing to the south and north, because other buildings adjoin the studied object. Each existing double-glazed window was supposed to have a U-value of 3.5 W/m^2 K, which in turn leads to a heat transfer of 508.2 W/m^2 K. About 25% of the heat demand is therefore used by the windows.

The outdoor temperatures for Linköping, calculated as monthly average mean values for a 30-year period are found in Table 1.

For January, with 744 h, this leads to a demand of $2056 \times (20 - (-2.9)) \times 744 = 35.0$ MWh. Values for the other months are present in Table 2. Energy is also needed for domestic hot water heating, and the annual consumption is assumed to be 42,000 or 3500 kWh each month. Some of the heating demand can be covered by free energy from appliances and so forth, and 50,000 kWh each year is supposed to be available, i.e. 4167 kWh each month. The windows are not only responsible for heat transfer from the inside to the outside, but solar radiation is also transferred in the other direction. Mentioned above are the difficulties when such values are to be obtained. In our case, a computer program called SORAD has been used which calculates the solar position each hour for full 1 year. By use of average

Table 2
Degree hours, energy demand and supply in kWh for the test building[a]

Month	Degree hours	Space heating	Hot water	Appliances	Solar	Boiler
1	17037.6	35035.3	3500.0	4167.0	591.0	33778.4
2	15456.0	31783.0	3500.0	4167.0	1635.0	29483.1
3	14954.4	30751.5	3500.0	4167.0	4303.7	25783.8
4	10584.0	21764.4	3500.0	4167.0	6377.2	14724.3
5	6696.0	13769.3	3500.0	4167.0	9149.1	3958.3
6	3312.0	6810.6	3500.0	4167.0	9374.1	3500.0
7	1711.2	3518.8	3500.0	4167.0	9372.7	3500.0
8	2678.4	5507.7	3500.0	4167.0	7681.1	3500.0
9	5616.0	11548.5	3500.0	4167.0	5196.7	5693.8
10	9597.6	19736.1	3500.0	4167.0	2578.8	16500.3
11	12456.0	25613.9	3500.0	4167.0	750.7	24207.3
12	14880.0	30598.5	3500.0	4167.0	313.6	29629.9

[a] Total annual demand from boiler = 194259.2.

Table 3
Calculated solar radiation transfer in kWh/m^2 through double-paned windows faced to east/west for Linköping, Sweden

January	February	March	April	May	June	July	August	September	October	November	December
4.07	11.26	29.64	43.92	63.01	64.56	64.55	52.90	35.79	17.76	5.17	2.16

mean values for clear and overcast days, insolation values, etc. we have in spite of the shortcomings calculated the amount of solar radiation which is transferred each month through a double-paned window. These values are shown in Table 3. (It should be noted here that it is also common practise to use monitored horizontal solar and climate data on an hourly basis, but such values were not easy available.) The total window area is 145.2 m^2, and therefore 590 kWh might be available in January (see Table 2).

In January, we need 35.0 MWh for space heating and 3.5 for domestic hot water. At the same time, there is free energy from appliances, 4.2 and 0.5 from solar radiation. The boiler must supply 33.8 MWh. During June–August, there is no need for space heating, and therefore only energy for hot water shows up in Table 2. It is assumed that it is not possible to transfer heat from appliances or solar radiation to domestic hot water.

4. The unavoidable cost

The first cost to calculate when dealing with LCC is the unavoidable, or inevitable, cost. This cost shows how much the owner must pay if nothing is done to the building at all. When, e.g. the old windows must be replaced with new ones, the same thermal standard is chosen as is present today. Old double-glazed windows are therefore assumed to be replaced by new double-paned ditto. An oil-fired boiler is replaced by the same type of boiler, etc. It is not possible to show all calculations in detail, but for the windows the following apply. The cost for changing the old to new ones are supposed to be reflected by Table 4. (The values found in Table 4 are not scientifically examined. We have only browsed some recent brochures and price lists from the Swedish manufacturer “Elitfönster”, but the values will hopefully be significant enough for this study; 1 € equals about 8.5 SEK).

Suppose for a moment that the existing windows must be replaced within 10 years. New windows are supposed to have a life-span of 30 years. With an interest rate of 5% and a total calculation time-span of 50 years, the present value for the existing windows will become

$$145.2 \times 2000 \times (1.05^{-10} + 1.05^{-30} - \tfrac{2}{3} \times 1.05^{-50}) = 202,647\,\text{SEK}$$

This cost must therefore be included in the so-called unavoidable cost. If the windows must be changed immediately, the following applies:

$$145.2 \times 2000 \times (1.05^{-10} + 1.05^{-30} - \tfrac{1}{3} \times 1.05^{-50}) = 349,150\,\text{SEK}$$

The cost for changing windows earlier then necessary is therefore about 150 kSEK. If the new windows are thermally better, it will result in energy cost savings, which must be higher than not only the actual window cost but also the cost for installing these new windows in advance. There are, of course, other unavoidable costs, and Table 5 shows those related to building measures. In this study, it is assumed that the windows must be replaced immediately, and therefore the sum of the inevitable costs for the windows is 350 kSEK. At the inside of the building much cheaper retrofits apply which must be considered, because it is possible to add extra insulation indoors. However, such retrofits are not often part of an optimal solution because of rent reduction due to smaller residential area. In the denser part of Linköping, district heating is one option for the owner of the building. The society supports that heating source and at least earlier the owner had to prove that an alternative was better if the owner applied for subsidies when the building was aimed for refurbishment. If this could not be proven, no subsidies were paid if not district heating was chosen. Because of this argument, almost all refurbished multi-family buildings are heated by use of district heating. The building in our case study is heated by firing oil in a boiler.

This boiler must be replaced within 5 years. If district heating is chosen, a salvage value for the oil-fired boiler therefore applies. Such boilers have an assumed cost of

Table 4
Relations between window type, window cost and *U*-value

Window type	Cost (SEK/m^2)	*U*-value (W/m^2 K)
Double-glazed windows	2000	3.0
Triple-glazed windows, TGW	2500	2.5
TGW, low emissivity coating	3000	2.0
TGW, low emissivity coating, argon-filled gap	3500	1.5

Table 5
Unavoidable costs in SEK for building measures in the studied building

External wall at the outside	184800
External wall at the inside	30800
Windows oriented to the east	181789
Windows oriented to the west	167361
Total	564750

$55{,}000 + 60 \times P_{\mathrm{oil}}$, where P_{oil} is the installed thermal power of the oil-fired boiler. Above it was shown that 78 kW had to be available and, assuming that the oil-fired boiler has an efficiency of 0.75, 104 kW must be used for P_{oil}. The oil-fired boilers must be replaced every 15 years, and hence a present value for all boilers during 50 years will become

$$(55,000 + 60 \times 104) \times (1.05^{-5} + 1.05^{-20} + 1.05^{-35}) = 85,233\,\mathrm{SEK}$$

In Table 2, it is shown that 194 MWh is needed each year. The so-called present value factor for annual recurring costs with an interest rate of 5% and an interval of 50 years is 18.26. The oil price is about 0.47 SEK/kWh, VAT excluded, when this paper is written and hence the unavoidable cost for oil is

$$\frac{194,259}{0.75} \times 0.47 \times 18.26 = 2,222,892\,\mathrm{SEK}$$

Adding all these unavoidable costs results in a total existing LCC of 2.873 MSEK. The question is now if this cost will be lower if new windows are installed, and further which type of window yields the lowest such LCC.

5. The MILP model

The first thing to start with when dealing with LP and MILP models is the objective function. This function, or expression, shows the total cost for the building, and it is this expression which must find its lowest value. In order to describe the climate conditions, we have split the year in 12 months (see Table 2). Starting with the oil-fired boiler we, hence, need 12 variables, one for each month, $P0_{\mathrm{oil}}$–$P11_{\mathrm{oil}}$. Note that we do not know in advance how big an oil-boiler we need, because window retrofits will influence the size of the boiler. For January with 744 h and February with 672 h, the objective function, because of the energy demand, will become (note that the expression is purely linear)

$$(P0_{\mathrm{oil}} \times 744 + P1_{\mathrm{oil}} \times 672 + \cdots) \times 18.26 \times \frac{0.47}{0.75} \tag{1}$$

If all the variables $P0_{\mathrm{oil}}$–$P11_{\mathrm{oil}}$ equal 0, the objective function will also have the value 0, but in that case no heat is produced in the boiler. Some constraints must therefore be introduced. For January, one such constraint must be

$$P0_{\mathrm{oil}} \times 744 \geq 33,778 \tag{2}$$

The value 33,778 is found in Table 2. $P0_{\mathrm{oil}}$ must therefore be greater than 45.4 kW, and a cost is also generated by the objective. $P0_{\mathrm{oil}}$ will also become as small as possible because of the minimisation of the objective. Due to the time segments, 12 such constraints must be used, one for each month. The installation cost of a new oil-fired boiler might depend on the largest of these $P0_{\mathrm{oil}}$–$P11_{\mathrm{oil}}$ values, and hence 12 new constraints are introduced just to find which of the values it is.

$$0.75 \times P_{\mathrm{oil}} - P0_{\mathrm{oil}} \geq 0 \tag{3}$$

P_{oil} can be very large and still constraint (3) is valid. By adding the cost for P_{oil} to the objective, the size of the oil-fired boiler will be as small as possible. This cost is supposed to be reflected by $55{,}000 + 60 \times P_{\mathrm{oil}}$, vide supra, which has a present value of $76{,}548 + 83.5 \times P_{\mathrm{oil}}$ SEK. One problem now emerges. This first cost must only be present in the total cost if the oil-fired boiler is chosen in the optimal solution. If not, the cost must be 0. This is achieved by the introduction of a binary variable, $A1$, which can only assume the value 0 or 1. The objective, (1), is therefore appended by

$$76,548 \times A1 + 83.5 \times P_{\mathrm{oil}} \tag{4}$$

If P_{oil} is greater than 0, $A1$ must be 1, otherwise it must be 0. One further constraint achieves this,

$$A1 \times M - P_{\mathrm{oil}} \geq 0 \tag{5}$$

Here M is a large number, i.e. larger than the largest value P_{oil} is supposed to ever take, say 150 in our case. If P_{oil} is 0, $A1$ can assume both 0 or 1, but because of the minimisation only 0 applies. If P_{oil} is greater than 0, $A1$ must be 1. M is not a variable and the problem is therefore still linear but changed from LP to MILP. The heating system must be able to supply a sufficient amount of heat even for bad winter conditions. This is shown by the dimensioning outdoor temperature, i.e. $-18^{\circ}\mathrm{C}$ in our case, which resulted in a thermal boiler size of 104 kW.

$$P_{\mathrm{oil}} \geq 104.0 \tag{6}$$

The model now consists of one binary and 14 ordinary variables and 26 constraints. Before adding more costs to the objective and even more constraints, a test is performed by optimising the model as it is. This is accomplished by use of special software in computers. A number of such programs exist, and here a program called ZOOM was used (see [18]). Other programs are, e.g. LAMPS and CPLEX. These programs can be used if the LP or MILP problems are presented in the form of standard MPS files. The files are in ordinary text format, but are very tedious to write when the models grow large. Therefore, a program written in C has been designed which writes the MPS file. If input data are changed, a new MPS file can be generated very fast. We have used an ordinary PC computer equipped with LINUX. ZOOM is in the form of a FORTRAN code with about 14,000 lines, which was compiled with the ordinary g77 compiler. The C program was designed in the form of a so-called GNOME application, i.e. one of the "windows" systems for LINUX, compiled by the ordinary gcc compiler.

Optimisation showed that the objective function found the lowest value at 2.305 MSEK which differs only by 0.003 MSEK from the calculations above. Note that the unavoidable cost in Table 5 has not yet been added to the model.

Until now no alternatives are included in the model and, therefore some possible window retrofits must be added. Consider for a start constraint (2) above. If thermally better windows are installed, the right hand side (RHS) of the constraint will be lower. Each alternative window construction will contribute in a different way. There is, hence, a need for a new binary variable called $W1$. If the window type is selected, it will equal 1 and if not, 0. Above it was shown that the existing windows transferred 508.2 W/K. This value is reduced with 72.6 by choosing new double-paned windows according to Table 4. (The U-value decreases from 3.5 to 3.0 W/m^2 K.) The new double-paned window is supposed to transfer solar radiation in the same way as the existing type, so the RHS must be reduced with $72.6 \times 17{,}037 \times 10^{-3} = 1237$ kWh. Constraint (2) must therefore be changed to

$$P0_{\text{oil}} \times 744 + 1237 \times W1 \geq 33{,}778 \tag{7}$$

Note that no energy is saved outside of the heating season (see Table 2). The corresponding constraints for June–August must therefore not be reduced at all. For May, the reduction first appears to be 486 kWh, but a closer look at Table 2 shows that only 458 kWh applies. This might seem to be of only academic interest, but when even better windows are considered this fact might change the optimal solution. When also other retrofits are considered, e.g. if extra insulation should be added on the attic floor, it is hard to predict the precise reduction value for all possible retrofit combinations. This is a problem that still has to be solved when dealing with practical MILP for buildings.

Also constraint (6) must be changed and the difference is $(76.2 \times (20 - (-18)) \times 10^{-3})/0.75 = 3.7$ kW.

$$P_{\text{oil}} + 3.7 \times W1 \geq 104.0 \tag{8}$$

The objective function must normally also be appended with the cost for the new windows. As is shown above, this might lead to a change also of the unavoidable cost, i.e. if the original windows have some years left before they are worn out. This is not the case in this study, and because of the necessary change from the old to new double-paned windows no extra cost applies. The optimal solution for this new model shows that the LCC is reduced from 2.305 to 2.215 MSEK and that a change to new double-paned windows is a profitable retrofit. This is natural because no extra cost was added to the objective.

Dealing with triple-glazed or coated windows is a bit more complicated. Firstly, we must add the extra cost for the thermally better windows to the objective function (see Table 4). The difference between double- and triple-glazed windows is therefore 500 SEK/m^2 and present value calculations reveal that about 87 kSEK should be added to the objective if such windows are optimal. New binary variables must also be introduced, $W2$, $W3$ and $W4$ for the alternative windows in Table 4. The objective (1) must therefore be appended with

$$W2 \times 87{,}287 + W3 \times 174{,}575 + W4 \times 261{,}863 \tag{9}$$

It is very important that only one of the alternatives is chosen by the optimisation, and hence the following constraint applies:

$$W1 + W2 + W3 + W4 \leq 1 \tag{10}$$

The transmission of solar radiation decreases when thermally better windows are installed. This fact must be present in the model, which therefore must be added to constraint (7). Suppose that solar transmission is reduced by 10% for each window type in Table 4. For January, the following should be added to the left hand side of (7):

$$-W1 \times 59.1 - W2 \times 118.2 - W3 \times 177.3 - W4 \times 236.4 \tag{11}$$

One problem arise for the summer months, where not all solar radiation can be utilised. The model is therefore equipped with a routine, similar to the one used for calculating Table 2, which examines how much of the available solar radiation is actually used, and if no solar radiation can be used the "summer part" of constraint (11) is set to 0. Constraints (7) and (8) also include values of how much the maximum demand in kW and monthly energy need in kWh is reduced, if $W1$, $W2$, $W3$ or $W4$ is chosen.

The model now contains 18 ordinary, five 0/1 variables and 27 constraints, and the lowest value of the objective was calculated to 2.100 MSEK, i.e. lower than before. The solution includes windows of the best type, and $W4$ is accordingly set to 1, while the other W-variables are set to 0. The heat demand is now, of course, also lower and is decreased from 104 to 79 kW.

Oil is a very expensive means for space heating. In denser parts of Linköping, Sweden, it is possible to connect the buildings to a district heating network owned by the municipality. The tariff for buying district heat is shown in Table 6.

The tariff is divided in intervals, where the high limit for the values in Table 6 is 100 kW. Considering that the oil-fired boiler, with an inferior efficiency, has a maximum demand of 104 kW makes it plausible that the demand is lower than 100 kW and subsequently the prices above apply. The value of P_{dh1} is calculated for an outdoor temperature of −20°C. The efficiency of the district heating heat exchanger is set to 0.9, and therefore P_{dh1} will equal 91.3 kW, and the connection cost will become 77,783 SEK if district heating is the only alternative. P_{dh2} is a calculated value and is based on the average of the annual use for the latest 2 years divided by a so-called category number which is set to 2200 for residences. In order to add these facts in the model, a new

Table 6
Applicable parts of the district heating tariff for Linköping, 2000 (VAT excluded)

Connection fee (only once)	$6600 + 779 \times P_{\text{dh1}}$ SEK
Subscription fee (annual)	$880 + 227 \times P_{\text{dh2}}$ SEK
Energy cost for residences	0.285 SEK/kWh

binary number, $A2$, must be introduced which is 1 if district heating applies and 0 if not. It might be the cheapest solution to keep the oil-fired boiler, so both $A1$ and $A2$ can assume the value 1 at the same time, if optimal. The constraints ascertain that not both are 0! If both an oil-fired boiler and a district heating system are optimal, we do not know how large the value of P_{dh1} should be, and hence the objective must be added with (see Table 6)

$$A2 \times 6600 + 779 \times P_{\mathrm{dh1}} \quad (12)$$

$$(A2 \times 880 + 227 \times P_{\mathrm{dh2}}) \times 18.26 \quad (13)$$

$$(P0_{\mathrm{dh}} \times 744 + P1_{\mathrm{dh}} \times 672 + \cdots + P11_{\mathrm{dh}} \times 744) \times 18.26 \times \frac{0.285}{0.9} \quad (14)$$

where 18.26 is the present value factor (see above). P_{dh1} is found because of a constraint

$$P_{\mathrm{dh1}} - P_{\mathrm{dh}} \times \frac{-20}{-18} \geq 0.0 \quad (15)$$

where −20 and −18 correspond to the outdoor temperatures. P_{dh2} is found by

$$P_{\mathrm{dh2}} - \frac{P0_{\mathrm{dh}} \times 744 + P1_{\mathrm{dh}} \times 672 + \cdots + P11_{\mathrm{dh}} \times 744}{2200} \geq 0.0 \quad (16)$$

The district heating equipment is supposed to cost $40{,}000 + 60 \times P_{\mathrm{dh3}}$, and these are dealt with in the same ways as in conditions (4) and (5) but using $A2$ instead.

The model now consists of 42 constraints, 34 ordinary variables and six binary ditto, and optimisation reveals that the minimum objective function value is 1.328 MSEK, i.e. a significant decrease with about 36%. This is so because the oil-fired boiler is abandoned when it comes to the basic heat load, only the peak is covered. District heating is used instead. Because of the lower operating cost for energy, the thermally better windows are likewise abandoned and the double-glazed type is optimal (see Fig. 1). Note that the graph shows the amount of heat used in the building. If the demand of fuel is considered, these values must be divided by the efficiency, e.g. 0.75 for the oil-fired boiler. From the view of the model, no oil energy at all is used which, of course, cannot be true in reality. Such a calamity must be solved by some short time segments during winter conditions.

The main result, however, will still be the same. Thermally better windows cannot compete if the district heating tariff in Linköping is applicable.

6. Sensitivity analysis

When dealing with LCC calculations, predictions must be made about future energy prices, interest rates, etc. By a so-called *ceteris paribus* analysis, i.e. changing one variable and letting all others be the same, it is possible to examine how much the optimal solution changes for a certain change in one variable. Assume now that the energy price in the district heating tariff is increased from 0.25 to 0.6 SEK/kWh (see Table 7).

Table 7
Minimum LCC in MSEK and solution

Energy price	LCC	Solution
0.25	1.192	$A1 = 1, A2 = 1, W1 = 1$
0.30	1.386	$A1 = 1, A2 = 1, W1 = 1$
0.35	1.560	$A1 = 1, A2 = 1, W4 = 1$
0.40	1.716	$A1 = 1, A2 = 1, W4 = 1$
0.45	1.872	$A1 = 1, A2 = 1, W4 = 1$
0.50	2.027	$A1 = 1, A2 = 1, W4 = 1, P0_{\mathrm{ob}} = 1.08$ kW
0.55	2.100	$A1 = 1, A2 = 0, W4 = 1$
0.60	2.100	$A1 = 1, A2 = 0, W4 = 1$

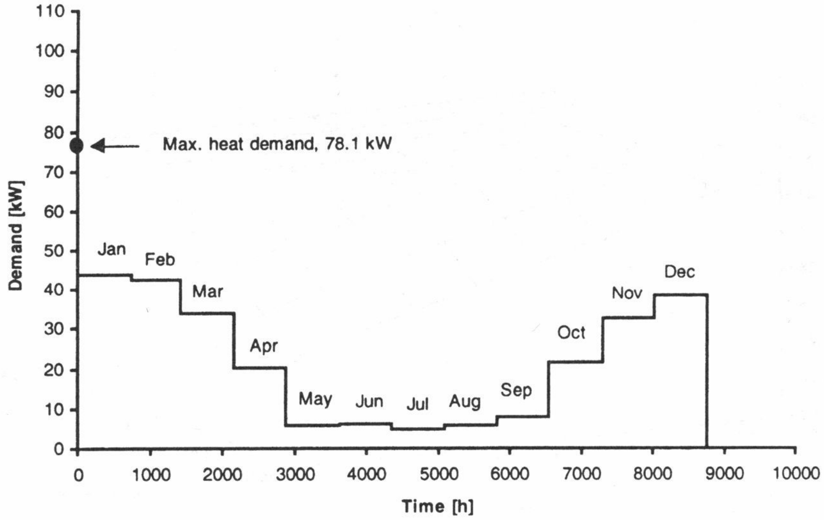

Fig. 1. Heat demand for the studied building.

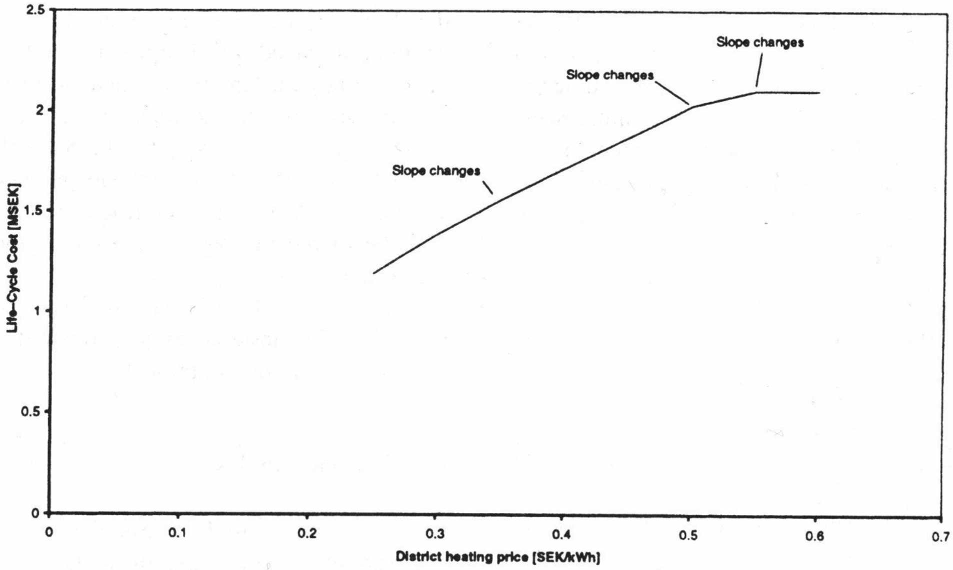

Fig. 2. Life-cycle cost vs. district heating price.

For a low district heating price, only double-glazed windows apply, i.e. $W1 = 1$. The oil-fired boiler is used for the thermal peak, $A1 = 1$, while district heating is used for the base, $A2 = 1$. When the price goes up to 0.35 SEK/kWh, better windows are optimal, and hence $W4 = 1$. Starting from the price 0.5 SEK/kWh, it is also optimal to use the oil-fired boiler for the base load, and for 1 month, i.e. January, about 1 kW is used. The "flip-over" to the oil-fired boiler alone was found for a price of 0.55 SEK/kWh, where district heating was abandoned. The LCC is now no longer affected by a still increasing district heating price (see Fig. 2). The reason for the model to "choose" both the oil-fired boiler and district heating comes from the high "initiating costs" for the district heating system, where costs are coupled to the $A2$ binary variable (see expressions (12) and (13)). It must be noted here that it is not possible to act in this way in real life. If shorter time segments were introduced for the winter months and, e.g. 20 h (peak hours) were dealt with in separate segments, the difference between the peak coming from the dimensioning outdoor temperature and the largest average demand based on 1 h would not be as large as in the present model.

Of some interest might also be to study at which window installation cost and U-values, thermally better windows are optimal when the building is coupled to the district heating system. Triple-paned windows were supposed to cost 2500 SEK/m^2 and have a U-value of 2.5 W/m^2 °C (see Table 4). In Fig. 3, such a sensitivity analysis is

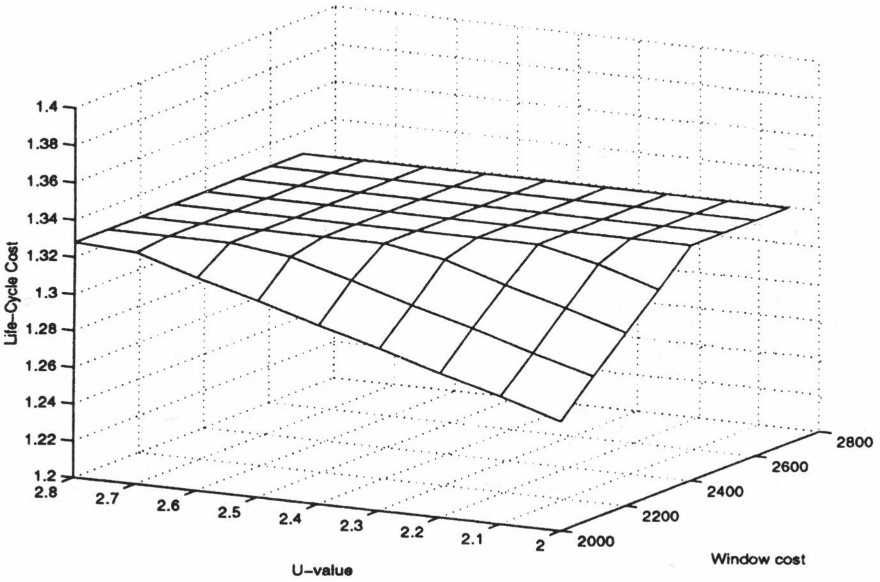

Fig. 3. Sensitivity analysis for life-cycle cost, depending on U-value and window cost.

presented. For high U-values and high costs, the LCC is constant, 1.33 MSEK. If the cost for a triple-glazed window increase nothing will happen just if the cost is over a certain limit. The same applies for the U-value. If the value is higher than, e.g. 2.8 W/m^2 °C, the LCC will not change. Below these limits, the LCC is decreased for decreasing U-values and window cost. The reduction is of course linear because of the linear model.

7. Conclusions

It has been shown that thermal calculations for windows are connected with several difficulties, and it is hazardous to predict the exact thermal behaviour when the construction is changed. For example, the air gap between the window panes influences the thermal conduction in one way but the convection in the opposite direction. More air gaps aggravate the scientific situation. Adding one more pane or coating will lead to a lower U-value but also to a reduced transfer of solar radiation. The solution to overcome all these calamities is to deal with a number of different alternatives. When it comes to optimisation, i.e. to find the best solution, we have chosen the alternative which yields the lowest LCC for the building.

Because there are a number of alternatives to be examined, classic calculus cannot be used because there are no continuous functions. Instead, the method called MILP has been used, and it is shown how to build such a mathematical model which is able to deal with, not only different window constructions, but also energy tariffs for district heating in common use, at least in Sweden. With the prices valid in Sweden today, December 2000, heating systems based on private oil-fired boilers will lead to solutions with high-tech low U-value windows, while the use of district heating makes such windows unprofitable.

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