Stability problems in optimised chairs?

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Abstract

Chairs and other furniture is seldom designed by help of structural mechanics and modern computers. Even if the designer uses a sophisticated CAD program, he, or she, will not use for example, finite element programs, FEM, in order to optimise the construction. Most furniture is made of wood or wood composites. Usually, structural mechanics is used for designing wood members in roof constructions and so forth. Because of the consequences of a breakdown, the allowable design stresses must be very low, about one third of the measured breaking strength. Smaller wood details could be chosen with more care and for chairs the result of a break would not necessarily lead to a disaster. However, a lot of the knowledge about how to design small wood structures emanates from the pre-war aeroplane industry. The difference between tensile and compression strength properties in wood also makes ordinary FEM programs hazardous to use because the background theory assumes that these properties are equal in magnitude. In this paper we show how to calculate the internal stresses of an undetermined chair frame and also shows some material test results for Swedish alder, *Alnus glutinosa*.

INTRODUCTION

Strength design of furniture seems to interest only a few researchers in the world at least if one looks at the number of papers published during recent years. The authors to Reference [1] have examined the strength of furniture joints between structural members of laminated veneer lumber. This is a very important field to investigate because the joints seems to be the weakest point in various types of furniture. In Reference [2], which seems to be one of the first attempts to interest a wider population of researchers for furniture and strength design, the author has calculated the moments at the joints in an indeterminate frame of a chair. However, he did not show at all how he fulfilled the calculations. Corner joints in cabinets have interested the authors to Reference [3] and they have used FEM calculations to calculate the stiffness for such joints between members of particle board. Papers about cabinets and other furniture have also been written in Poland, see Reference [4]. The, for us, most interesting paper about FEM calculations and chairs, however, seems only to be available in Polish. They have also written several other papers, in German, about joints and adhesives. In Reference [5] we have shown how to calculate the internal stresses in an indeterminate chair frame by use of the so called displacement method. Further, experiments with this method by use of a computer are presented and the result showed that the stretcher was to be placed diagonally between the lowest part of the back rail to the highest part of the front member, see Figure 1.

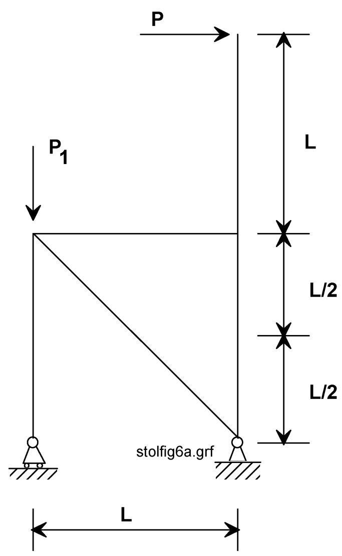

Figure 1: Chair with a diagonally placed stretcher for minimising the moment where the back rail meets the seat, see Reference [5]

The calculations for the frame in Figure 1 were made by a computer program, P-FRAME from Chalmers University of Technology, and they showed that the moments in the upper and lower part of the diagonally stretcher were very small, only about 20 Nm. However, they also showed that axial forces were introduced which were only of minor interest in Reference [5] but must be considered if the stretcher were made thinner. The axial forces would eventually make the stretcher collapse due to one of the Euler cases. This paper is therefore dedicated to such an investigation.

CASE STUDY

Consider the chair in Figure 1. For a start, assume that all the wooden members have the same cross sectional areas and are made of the same material, i. e. they have the same Young's Modulii, E, and moment of inertia, I. In order to calculate the stresses in the members we use the displacement method which can be studied in detail in Reference [6]. First we must elaborate the stiffness matrix. Here, we will only show the method for the first line in this matrix but the other lines are accomplished in the same way. In Figure 2, to the left, we have simplified the frame as much as possible while we in the right part have introduced a rotation in the upper left corner of the frame.

Because of the diagonal stretcher the frame could not have a transverse

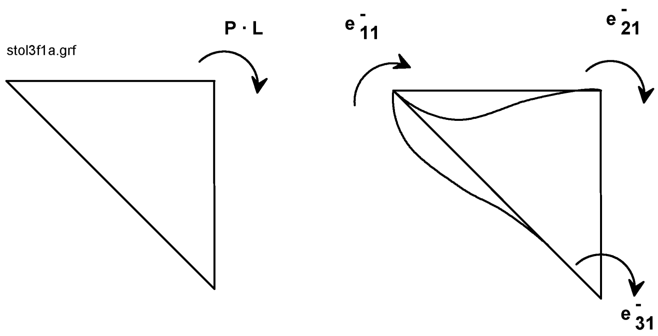

Figure 2: Simplified frame, to the left, and rotated corner for stiffness matrix elaboration, to the right

displacement, i. e. in first order theory. The joints could only rotate. Reference [6] now tells us that the stiffness matrix elements, made up of elementary cases, equal:

$$e_{\bar{11}} = \frac{4EI}{L} + \frac{4EI}{L \times 2^{0.5}} = \frac{6.83EI}{L}$$

$$e_{\bar{21}} = \frac{2EI}{L}$$

$$e_{\bar{31}} = \frac{2EI}{L \times 2^{0.5}} = \frac{1.41EI}{L}$$

The total stiffness matrix and the equation system to be solved is therefore:

$$\begin{bmatrix} 6.83 & 2 & 1.41 \\ 2 & 8 & 2 \\ 1.41 & 2 & 6.83 \end{bmatrix} \times \frac{EI}{L} \times \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ P \times L \\ 0 \end{bmatrix}$$

Solving this results in

$$\begin{aligned} q_1 &= -0.0345\tfrac{PL^2}{EI} \\ q_2 &= +0.1423\tfrac{PL^2}{EI} \qquad \text{and} \\ q_3 &= -0.0345\tfrac{PL^2}{EI} \end{aligned}$$

The elements of the frame must after this be separated in beams. The convention for positive signs of moments and rotations is shown in Figure 3.

The moments must therefore be calculated as, note that we have neglected translations, t, because of first order theory:

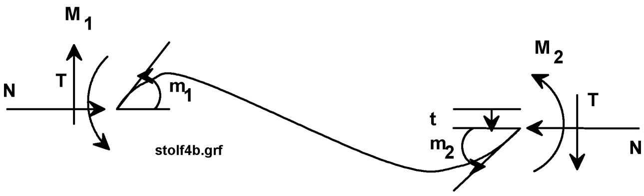

Figure 3: Positive signs for moments, translations and rotations, see Reference [5]

$$
\begin{aligned}
M_{12} &= 4 \times 0.0345 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 2 \times 0.1423 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= -0.1466PL \\
M_{21} &= 2 \times 0.0345 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 4 \times 0.1423 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= -0.5002PL \\
M_{23} &= 4 \times 0.1423 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 2 \times 0.0345 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= +0.5002PL \\
M_{32} &= 2 \times 0.1423 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 4 \times 0.0345 \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= +0.1466PL \\
M_{31} &= -4 \times 0.0345 \times 2^{-0.5} \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 2 \times 0.0345 \times 2^{-0.5} \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= -0.1466PL \\
M_{13} &= -2 \times 0.0345 \times 2^{-0.5} \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} - 4 \times 0.0345 \times 2^{-0.5} \times \tfrac{PL^2}{EI} \times \tfrac{EI}{L} &&= -0.1466PL
\end{aligned}
$$

Assuming that P equals 300 N and L equals 0.4 m, as in Reference [5], implies that:

$$
\begin{array}{lclll}
M_{12} & = & 17.59 & \mathrm{N}m & \text{(tension below)} \\
M_{21} & = & 60.0 & \mathrm{N}m & \text{(tension above)} \\
M_{23} & = & 60.0 & \mathrm{N}m & \text{(tension inside)} \\
M_{32} & = & 17.59 & \mathrm{N}m & \text{(tension outside)} \\
M_{31} & = & 17.59 & \mathrm{N}m & \text{(tension outside)} \\
M_{13} & = & 17.59 & \mathrm{N}m & \text{(tension inside)}
\end{array}
$$

It is obvious that the stretcher could be much thinner than the seat and back rails because the moments in the stretcher is only one third of the moments at the other frame joints. As mentioned above, we have also introduced axial forces in the frame and this could lead to stability problems. Up to now we have neglected these forces but this does not mean they are zero. Using equations of static equilibrium for each separated beam, see Figure 3, and joint, makes it possible to calculate the shear forces T and hence the axial forces, N. The axial force in the stretcher is by use of these facts calculated to 449 N, compressed, the force in the horizontal beam is 230 N, tensed, while the vertical beam is compressed by a force of 194 N. The question is now if the stretcher is in the vicinity to collapse because of the moments and the axial force. To start with we could compare the axial forces to the critical ones from an Euler IV case which are calculated as:

$$P_{crit} = 4 \times \pi^2 \times \frac{EI}{L^2}$$

We see that it is necessary to find values for E and I if we are to calculate the critical force. I is possible to calculate but E depends on the material. In e.g. Reference [7], page 164, values for E could be found. The author shows the value for the modulus of elasticity, MOE, which is calculated from bending tests. Tensile or compression moduluii cannot be found. Therefore, we have made some small tests on alder ourselves. In Figure 4 a tensile test is shown.

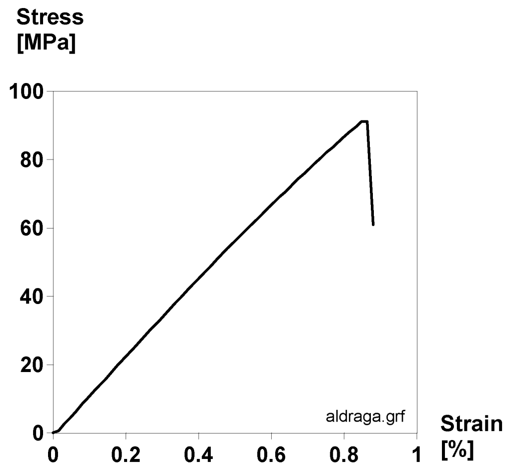

Figure 4: Tensile test for Swedish alder, *Alnus glutinosa*

From Figure 4 it is shown that the relationship between stress and strain follows an almost perfect straight line. The stress obtained at break was 91.1 MPa and the strain was 0.86 %. The E modulus is therefore calculated to 10,593 MPa. The breaking strength for European alder is 92 MPa according to Reference [7] which corresponds almost perfectly with the value in Figure 4. In Reference [8] the same type of diagram is published for Swedish birch, Betula. Birch wood is much stronger than alder, the stress obtained before breakage was higher than 160 MPa. Alder also seems to have another behaviour when tensed than birch because we could not observe that some fibres broke and other took over the load as was found for birch. Instead the alder wood in Figure 4 seemed to endure the load to a specific point and after this a total collapse occurred. Some other tests did not show this as clear as above but the main difference in behaviour between alder and birch seems to be valid. We have also made compression tests, see Figure 5.

The breaking strength for alder under compression parallel to the grain was about 58 MPa, see Figure 5, or about half the value found for tension. The value corresponds well to the one found in Reference [7] which is 54 MPa. The E-modulus calculated from the middle part of the curve in Figure 5 becomes approximately 2,900 MPa or about one third of the one found for tension. Inserting $L = 2^{0.5} \times 0.4 = 0.56$ m, $E =$ 2,900 MPa, $I = 0.01 \times \frac{0.03^3}{12} = 2.25 \times 10^{-8}$

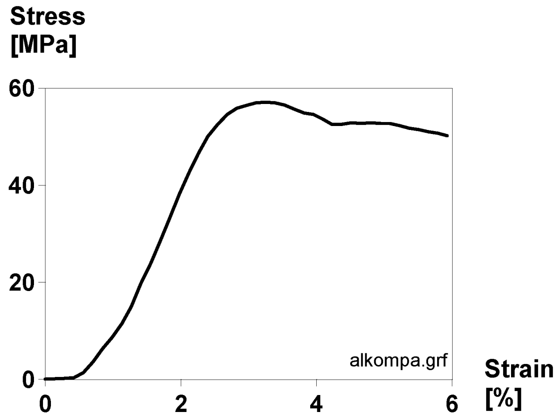

Figure 5: Compression test for Swedish alder, *Alnus glutinosa*

m^4 yields that P_{crit} equals 8,050 N which is far from the axial load calculated above. There is therefore no danger for a collapse in the same plane as the frame. The rod is weaker in the direction perpendicular to the frame which implies that an $I = 0.03 \times \frac{0.01^3}{12} = 2.5 \times 10^{-9}$ should be used instead. P_{crit} will then decrease to 894 N. The actual axial load is still about two times lower and therefore no risk seems to be apparent. Therefore, let us make the stretcher much thinner, for example 0.005 times 0.02 m. We must start the calculations all over again and elaborate a new stiffness matrix. The inertia moment, I, will now become 3.33×10^{-9} m^4 in the plane of the frame, which is about 6.75 times lower than before. The first three elements of the stiffness matrix will therefore change to:

$$\bar{e_{11}} = \frac{4EI}{L} + \frac{4EI}{6.75 \times 2^{0.5} \times L} = \frac{4.419EI}{L},$$

$$\bar{e_{21}} = \frac{2EI}{L},$$

$$\bar{e_{31}} = \frac{2EI}{6.75 \times 2^{0.5} \times L} = \frac{0.210EI}{L}$$

It is not possible to show the total calculation process but the resulting rotations are $q_1 = q_3 = \frac{0.0689PL^2}{EI}$, $q_2 = \frac{0.1594PL^2}{EI}$. Note that the first and last rotation is about twice as large as before, while q_2 did not change very much. The moments becomes:

$$M_{12} = 5.19\text{Nm (tension below)}$$

$$M_{21} = 60.0\text{Nm (tension above)}$$

$$M_{23} = 60.0\text{Nm (tension inside)}$$
$$M_{32} = 5.19\text{Nm (tension outside)}$$
$$M_{31} = 5.19\text{Nm (tension outside)}$$
$$M_{13} = 5.19\text{Nm (tension inside)}$$

Using equations for static equilibrium for each joint shows that the axial force in the stretcher will not change at all, the force is still 449 N. The critical Euler IV force will now become 1,200 N which is about three times higher than the actual load. Stability problems, however, arise when bending perpendicular to the frame plane is considered. The critical force will now become only 75 N, which is lower than calculated above. Therefore, the stretcher will probably collapse due to the axial force and it will bend perpendicular to the plane of the frame.

Using the Euler IV case as shown above is not quite accurate according to the theory of structural mechanics. The stretcher is influenced by the moment in each end. This implies that the stiffness matrix must be elaborated by use of so called Berry functions, see Reference [6] page 267. These functions change the stiffness elements according to a calculated value $\alpha = \frac{P}{P_{EUII}}$, i. e. the actual load divided by the Euler II critical load, which in our case equals,:

$$\alpha = 449 \times \frac{(0.4 \times 2^{0.5})^2}{\pi^2 \times 2,900 \times 10^6 \times 1.33 \times 10^{-8}} = 0.38$$

The stiffness $\frac{4EI}{L}$ must now be changed to $\frac{3.473EI}{L}$ while the value $\frac{2EI}{L}$ becomes $\frac{2.141EI}{L}$ for the beam under consideration. The total stiffness matrix will therefore change to:

$$\begin{pmatrix} 6.45 & 2 & 1.52 \\ 2 & 8 & 2 \\ 1.52 & 2 & 6.45 \end{pmatrix} \times \frac{EI}{L}$$

A stability problem now occurs only if the determinant of the matrix is negative or equals zero. In our case this is not the fact because this value is calculated to 275 which is well on the safe side. When the thinner stretcher is introduced the stiffness matrix will change to:

$$\begin{pmatrix} 5.43 & 2 & 2.04 \\ 2 & 8 & 2 \\ 2.04 & 2 & 5.43 \end{pmatrix} \times \frac{EI}{L}$$

which has a determinant of 175, which is lower than before but also on the safe side. Using Berry functions therefore confirms that no stability problem occurs for the stretcher under consideration, at least as long as only bending into the plane is dealt with. Even if the risk for collapse perpendicular to the frame is evident, because of the axial forces, it is interesting to study the stress at different points of the frame. The maximum moment occurs at the back rail above the seat, i. e. 120 Nm. By use of classic theory the stress could be approximated with:

$$\sigma = \pm\frac{M \times z}{I} \quad \text{or in our case} \quad \pm\frac{120 \times 0.015}{2.25 \times 10^{-8}} \quad \text{equalling} \quad 80 \quad \text{MPa.}$$

The stretcher is exposed to an axial force of 449 N, which results in a stress of 4.5 MPa and a moment of 5.19 Nm resulting in 15.6 MPa or a total stress of about 20 MPa if the thin stretcher is chosen. In Reference [7] page 164 the modulus of rupture in bending for European alder is set to 83 MPa, i. e. slightly stronger than the calculated stress. The compression E-modulus is lower than the calculated stress. It therefore seems necessary to reduce the moment in the back rail and at the same time change the cross sectional area of the stretcher if the wooden material should be utilised to maximum values. If a stretcher is introduced from the top of the back rail to the front of the seat the dangerous moment should be reduced, see Figure 6.

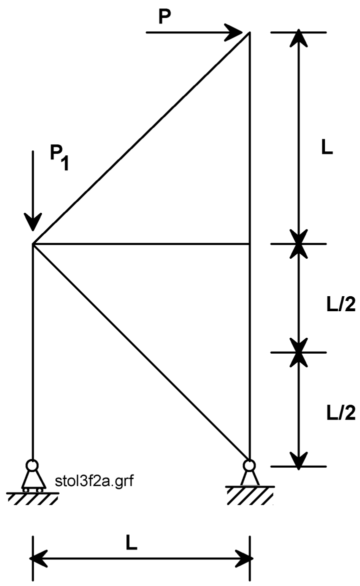

Figure 6: Chair with two stretchers for minimising the moment in the back rail.

The chair in Figure 6 is probably not very practical but is used here for academic reasons. In real life the top stretcher could be made of cord instead of massive wood because it will be exposed almost only for tension axial forces. Another way is to build a new frame above the seat which could be used as a place to rest your arms. Unfortunately, the structure in Figure 6 cannot be analysed without considering the axial deformation of the stretcher between point 1 and 4. The stretcher is however a very thin element and therefore it cannot endure large moments. It is therefore satisfactory to assume that the moment in point 4 equals zero and hence it is possible to calculate the axial force in the stretcher to 424 N which in turn implies a prolongation of less than 0.001 m. Further, assuming that the top of the back rail deflects this amount results in a moment in the back rail at point 2 of only about 1 Nm. Computer calculations show that the moment is even closer to zero. In real life, the burden a chair must endure does not correspond with the loads assumed in this paper. Real truss chairs therefore do not seem to be recommendable but if the structure could get closer to a truss, much smaller members could be chosen.

CONCLUSIONS

If the structure of a chair could be changed to a truss or a bar structure only axial forces are of interest. Some of the bars will become compressed and therefore stability problems will occur when the wooden material shall be utilised to the brink of collapse. Some of the structural members will only be tensed which means that very thin dimensions could be used because of the high strength of tensed wood parallel to the fibres. When the cross sectional areas of the wooden members get smaller more interest must be paid to the different loads actually implemented on the structure. One example is to study how the load is distributed on the back and seat of the chair. The beam under the seat must of course carry a distributed load which will imply rotations in the ends of the member. Because of firm joining between the parts in the chair moments will be introduced in other parts of the structure as well. Hence, the truss model cannot be used without great care when real chairs are to be designed.

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References

[1] Wang S. Y. and Juang H. B. Structural Behaviour of Various Joints in Furniture Components Made of Softwood Laminated Veneer Lumber. *Mokuzai Gakkaishi*, 40(9):911–921, 1994.

[2] Eckelman C. A. A Look at the Strength Design of Furniture. *Forest Products Journal*, 16(3):21–24, 1966.

[3] Cai L. and Wang F. Influence of the Stiffness of Corner Joint on Case Furniture Deflection. *Holz als Roh- und Werkstoff*, 51:406–408, 1993.

[4] Smardzewski J. and Dziegielewski S. Stability of Cabinet Furniture Backing Boards. *Wood Science Technology*, 28:35–44, 1993.

[5] Gustafsson S. I. Furniture Design by use of the Finite Element Method. *Holz als Roh- und Werkstoff*, 53(?):257–260, 1995.

[6] Asplund S. O. *Structural Mechanics: Classical and Matrix Methods.* Prentice-Hall / Englewood Cliffs, N.J., 1966.

[7] Tsuomis G. *Science and Technology of Wood.* Van Nostrand Reinhold, 1991. ISBN 0-442-23985-8.

[8] Gustafsson S.-I. Finite Element Modelling Versus Reality for Birch Chairs. *Holz als Roh- und Werkstoff*, 54(?):355–359, 1996.