Furniture Design by use of the Finite Element Method

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Abstract

The design of structural members of furniture is almost never the subject for mathematical considerations. Instead, the designer rests on empirical experience and constructs for example chairs with dimensions of structural members based on tradition and aesthetic reasons. By the much more common use of computers it is nowadays possible to use modern finite element programs in various stages of the design process. In this paper we show how already simple calculations lead to a totally different design of a chair. We also emphasise the need for more research on wood in "furniture size" and not only as part of building structures. There is no need for "triple- security" values when a chair is to be designed but instead it is possible to balance on the edge of the mechanical strength in the wooden members. Further, it is possible to use only wood details where no knots, or other errors are present. This will lead to substantially thinner members in the wooden chair.

Abstract

Möbel-Design mit Hilfe der FEM
Das Design von Möbelelementen ist nur selten Gegenstand mathematischer Überlegungen. Stattdessen stützen sich die Designer auf Ehrfahrungswerte und die Konstruktion von Einzelstücken aufgrund traditioneller oder ästhetischer Betrachtungen. Mit Hilfe von Computern ist es jedoch leicht möglich moderne FEM-Programme für die verschiedenen Entwicklungsschritte eines Designs zu nutzen. Wir betonen hier auch die Notwendigkeit weiterer Grundlagenforschung für Abmessungen im Möbelbereich nicht nur bei Baukonstruktionen. Für die Stuhlkonstruktion ist auch keine Dreifach-Sicherheit nötig, sondern man kann die mechanische Festigkeit der Konstruktionselemente voll ausnutzen. Zudem ist es möglich, durch Vorsortieren typische Holzfehler ganz unberücksicht zu lassen. Das wiederum ermöglicht schlankere Einzelteile für das Stuhldesign.

1 Introduction

In Sweden we have no tradition in using solid mechanics for furniture design. Instead, the design has been founded on empirical knowledge of how chairs and other furniture usually looks. Further, the scientific society has not published very much in this field so it seems that there is a vast field for new types of furniture based on modern methods in solid mechanics. However, some authors have shown some interest in this field. As far as we have found out it seems that the author C. A. Eckelman was the first one to consider strength design of furniture, (Eckelman 1978), see Reference [1]. From the authors reference list it is obvious that his work had started about 1966 with a paper in Forest Products Journal. In 1968 he followed with a dissertation, which unfortunately, is not published. The author considered the fact that the wooden members in the frame were not to have higher stresses than were allowable but he used values that are one third of the actual breaking strength. Certain parts of the frame were therefore thicker than they had to be. He says that the wooden members were "stressed at reasonably efficient levels" but also mentioned that the designer for aesthetic reasons wanted to use larger parts in the chair. He also deals in detail with determinate frames while calculations on indeterminate frames only got a few pages in mentioned reference. There is, of course, a reason for this because determinate frames are much easier to analyze.

We have also found an institution in Poland who have published papers about solid mechanics and furniture, see e.g. (Smardzewski, Dziegielewski 1993), i.e. Reference [2]. They present a likewise impressive publication list but unfortunately it seems that the most interesting paper about chairs and frames is only available in Polish.

At the Northeast Forestry University in Harbin, China, research about corner joints for case furniture is going on, (Cai, Wang 1993), see Reference[3]. In the referred paper they have studied the stiffness for joints on cabinets made of particle board. They also made their analysis by use of the finite element method.

2 Case study

Consider the chair in Figure 1.

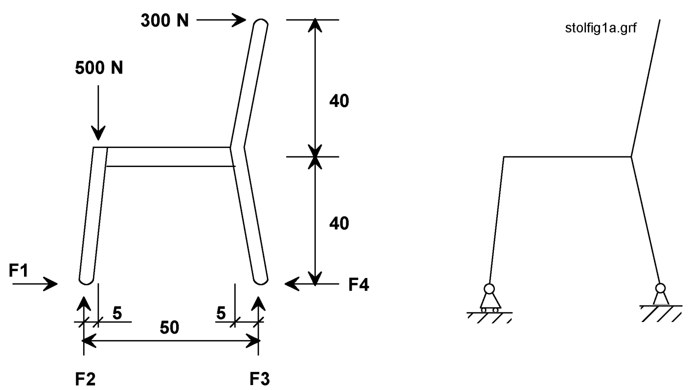

Figure 1: Drawing of a chair as a determinate frame. F_1, F_2, F_3 and F_4 are different forces acting on the chair. Schema eines Stuhls als Berechnungsgrundlage mit festen Größen. F_1 bis F_4 sind Kräfte, die auf den Stuhle einwirken.

The first thing to find out is the loads that are acting on the chair. In our example we use 300 N acting horizontally at the top of the back rail while we assume that 600 N act at the front of the chair in a vertical direction, values we have found in (Eckelman 1991), see Reference [4]. The following three equations might be drawn:

$$F_2 + F_3 - 600\ N = 0 \tag{1}$$

$$F_1 - F_4 + 300\ N = 0 \tag{2}$$

$$600\ N \times 0.05\ m + 300\ N \times 0.8 - F_3 \times 0.5\ m = 0 \tag{3}$$

The Equation, (1), considers the vertical forces, Eq. (2) the horizontal forces and the third one the moment, or bending force, around the bottom part of the left chair leg. We start with analysing a determinate frame assumed that we have no force F_1. The force F_4 must subsequently be 300 N. From the third equation F_3 must equal 540 N and finally F_2 will become 60 N. The moment, or bending force, at the middle of the back rail, but above the seat, is 120 Nm while the shear force must be 298 N, note that the force is not exactly perpendicular to the back rail. Immediately under the seat the moment must be:

$$300\ N \times 0.4\ m - 540\ N \times 0.05\ m = 93\ Nm \tag{4}$$

while the shear force equals 230 N. The floor reaction will also introduce an axial stress of 573 N. This in turn leads to a momentum of 213 Nm in the chair seat rail where the rail is concave on the upper side of the seat. At the same time a shear force of 540 N acts on the seat rail. Now it is time to calculate the internal stress in the frame. For a start, assume that the seat rail, where the maximum forces occur, has a thickness, w, of 0.02 m and a depth, d, of 0.05 m. Basic knowledge of solid mechanics shows that the stress σ is calculated as:

$$\sigma = \frac{N}{A} + \frac{M \times z}{I} \qquad \text{where} \qquad I = \frac{w \times d^3}{12} \tag{5}$$

and N is the axial force, A the cross sectional area of the rail and z the distance from the centre of gravity which is assumed to be located in the middle of the rail. In the seat rail the axial force is negligible so σ will become:

$$\sigma = 213\ Nm \times \frac{0.025\ m}{0.208 \times 10^{-6}} = 25.5 \times 10^{6}\ Pa = 25.5\ MPa \tag{6}$$

The question is now if this stress is permissible or not. Assume we choose red beech wood, *Fagus silvatica.* In (Tsuomis 1991), page 164, see Reference [5] it is shown that the tensile strength of beech is about 130 MPa and we are subsequently far from the limit here. On the other side of the rail, under the seat, the rail is subject to compression. A new look in the reference reveals that the compressive limit stress is only 46 MPa, which is also lower than the stress in the chair. In our own testing equipment we have found that values up to 72 MPa might be applicable before the cell structure is crushed. This is published in (Antic 1994), see Reference [6] but unfortunately the report is only available in Swedish. Due to the big difference between tensile and compressive breaking strength it is also common to examine the modulus of rupture for bending which in (Tsuomis 1991), Ref. [5], equals 104 MPa.

Further, it is necessary to consider the shear stress. From knowledge in solid mechanics this stress, note that we assume a rectangular cross section, is calculated as:

$$\tau = 1.5 \times \frac{F}{A} \tag{7}$$

The force in our case was 540 N which implies that (will equal 0.8 MPa. In (Kollmann and Côté 1984, page 402), see Reference [7] the applicable shear stress across the fibers is about 37 MPa while (Tsoumis 1991), Ref. [5], gives a value of 12 MPa. The two values are quite different but both are more than ten times larger than the actual stress.

From the above discussion it is shown that smaller rails could be chosen and the fact is that a rail of about 0.02 times 0.03 m might be applicable before the bending strength is reached. The other rails are subject to lower forces so in our case those members could be even smaller in size.

3 Indeterminate frames

Almost all chairs contain a stretcher between the front and back leg of the chair, see Figure 2.

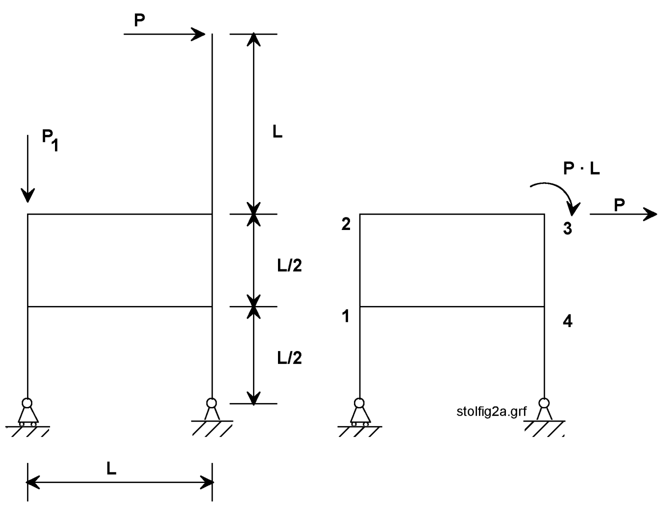

Figure 2: Indeterminate frame in a chair with a stretcher. Unbestimmts Berechnungsschema eines Stuhls mit Querversteifungen.

When the stretcher is introduced the calculation process gets severely harder even if the frame in Figure 2 has been simplified. The internal forces cannot be calculated only by use of the equations of static equilibrium. Instead we must assume the cross sections first and then calculate the rotations and movements in each joint. After this is done it is possible to calculate the internal forces. This work is nowadays made by computers but we will first show the method as it is used for hand calculations. However, it is only possible to show a small part of the calculation process here and hence the interested reader must find all details in e.g. (Asplund 1966), i.e. Reference [8]. The method we use is called the displacement method, see page 210 in the reference, where the stiffness matrix is elaborated by use of so called elementary cases. We use the matrix method here because of easier presentation. First consider the frame in Figure 2. The back rail has been eliminated and instead a moment and a force has replaced the rail. The first element in the stiffness matrix is elaborated by applying a rotation in one of the joints, e.g. joint number 1, while all the other joints are kept fixed. The forces required to obtain such a deformation will be equal to one column of the stiffness matrix. We apply the rotation clock wise but this is only for convenience, see Figure 3.

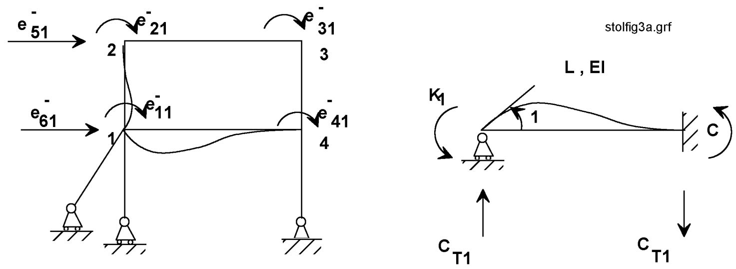

Figure 3: How to elaborate the first elements in the stiffness matrix, e^{-}_{ij} represent the element numbers. Bestimmung der Anfanges-Elemente e^{-}_{ij} zwichen den einzelnen Punkten.

The right hand side of Figure 3 shows the elementary case where:

$$K_1 = \frac{4EI}{L} \qquad C = \frac{2EI}{L} \qquad \text{and} \qquad CT = \frac{6EI}{L^2} \tag{8}$$

Element number e^{-}_{11} will therefore consist of three parts, one for each structural element of concern. Consider the rail between point 1 and 4. The modulus of elasticity is E and the length is L, compare with the right part of Figure 3. We see that e^{-}_{11} has the same rotation as K_1, turn the elementary case upside down. Our first part of the element is therefore $\frac{4EI}{L}$. The second part, from point 1 to the floor, must be equal to null because of the wheels, while the third part, between point 1 and 2 also will correspond to K_1 above. Note that the length is now only $\frac{L}{2}$. Adding these cases results in the first element:

$$e^{-}_{11} = \frac{4EI}{L} + \frac{4EI}{\frac{L}{2}} = \frac{12EI}{L} \tag{9}$$

The element e^{-}_{21} corresponds to C in the elementary case, but with the length $\frac{L}{2}$, which results in $\frac{4EI}{L}$. Element e_{31}^{-} must be null because the structural elements at point 3 does not take part in the reaction against the rotation in point 1. Element e_{41}^{-} corresponds to C and will equal $\frac{2EI}{L}$. The "force" e_{51}^{-} is found in CT and will therefore become $-\frac{6EI}{(\frac{L}{2})^2}$ which equals $-\frac{24EI}{L^2}$, note the minus sign, and the same is valid for element number e_{61}^{-}, but with the opposite direction. All the joints in the frame must after this be dealt with and sometimes other elementary cases are applicable but they will not be shown here. However, the equation system to be solved is shown by the following stiffness matrix:

$$\begin{pmatrix} 12 & 4 & 0 & 2 & -\frac{24}{L} & \frac{24}{L} \\ 4 & 12 & 2 & 0 & -\frac{24}{L} & \frac{24}{L} \\ 0 & 2 & 12 & 4 & -\frac{24}{L} & \frac{24}{L} \\ 2 & 0 & 4 & 18 & -\frac{24}{L} & \frac{24}{L} \\ -\frac{24}{L} & -\frac{24}{L} & -\frac{24}{L} & -\frac{24}{L} & \frac{192}{L^2} & -\frac{192}{L^2} \\ \frac{24}{L} & \frac{24}{L} & \frac{24}{L} & \frac{24}{L} & -\frac{192}{L^2} & \frac{216}{L^2} \end{pmatrix} \times \frac{EI}{L} \times \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ P_2 \times L \\ 0 \\ P_2 \\ 0 \end{pmatrix} \tag{10}$$

Here the unknown rotations is denoted q while the linear deformations are called p. The load P_2 correspond to P in Figure 2. The unknown deformations can now be calculated and their values are:

$$\begin{aligned} q_1 &= 0.065 \ \tfrac{PL^2}{EI} \\ q_2 &= 0.048 \ \tfrac{PL^2}{EI} \\ q_3 &= 0.144 \ \tfrac{PL^2}{EI} \\ q_4 &= 0.077 \ \tfrac{PL^2}{EI} \\ p_1 &= 0.127 \ \tfrac{PL^3}{EI} \\ p_2 &= 0.080 \ \tfrac{PL^3}{EI} \end{aligned}$$

After this it is necessary to implement the displacements in a new set of equations which are elaborated from still another elementary case, see Figure 4.

The bending and shear forces in the beam between joint number 1 and 2 may now be calculated as, see (Asplund, 1966), i.e. Reference [8]:

$$M_1 = 4 \times m_1 \times \frac{EI}{L} + 2 \times m_2 \times \frac{EI}{L} + 6 \times t \times \frac{EI}{L^2} \tag{11}$$

$$M_2 = 2 \times m_1 \times \frac{EI}{L} + 4 \times m_2 \times \frac{EI}{L} + 6 \times t \times \frac{EI}{L^2} \tag{12}$$

$$T_1 = 6 \times m_1 \times \frac{EI}{L^2} + 6 \times m_2 \times EIL^2 + 12 \times t \times \frac{EI}{L^3} \tag{13}$$

If the displacements, external forces where P equals 300 N, and the length L which equals 0.4 m, are implemented in the equations the following result is achieved:

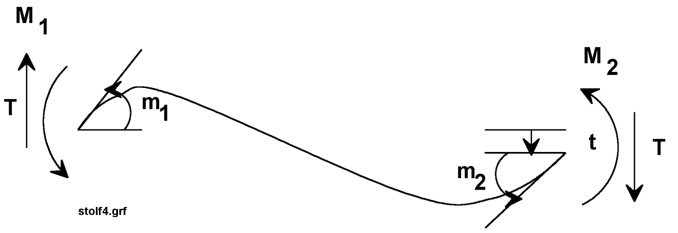

Figure 4: General representation of a beam. $M_{1,2}$ denotes bending forces and T shear forces. Vereinfachts Schema einer Quersprosse. $M_{1,2}$ = Biegemoment und T = Scherkräfte.

M_{14} = - 0.414 PL = 49.7 Nm (t. below)	M_{41} = - 0.438 PL = 52.6 Nm (t. above)
M_{12} = 0.416 PL = 49.9 Nm (t. outside)	M_{21} = 0.484 PL = 58.1 Nm (t. inside)
M_{23} = - 0.480 PL = 57.6 Nm (t. below)	M_{32} = - 0.672 PL = 80.6 Nm (t. above)
M_{34} = - 0.332 PL = 39.8 Nm (t. inside)	M_{43} = - 0.064 PL = 7.7 Nm (t. outside)
M_{45} = 0.498 PL = 59.8 Nm (t. outside)	T_{14} = -0.85 P = 255 N
T_{12} = 1.8 P = 540 N	T_{23} = - 1.15 P = 345 N
T_{34} = - 0.792 P = 237 N	

Note that the letter "t" in the list above stands for tension.

The moment M_{14} is the bending force in point number one in the direction of point number four, while M_{41} is the moment in point number 4 in the direction to number one. Interesting is now to study what happened to the moment of 213 Nm which was present in the determinate frame. The moment in the same point with a stretcher introduced is 80.6 Nm i.e. a reduction by over 50 %. It is therefore possible to design the rails in the chair as very thin details. However, it is also obvious that some parts of the chair are not stressed as much as others. It would thus be very interesting to see what happens if some of the rails, and the stretcher, are made of other dimensions than e.g. the back rail. The members would also be stressed differently if the stretcher is moved up or down and it is not obvious that the lowest stress is present when the stretcher is horizontal.

4 Computer Calculations

In order to calculate this we have used the computer program "P - frame" which is developed at the department of structural mechanics at Chalmers University of Technology. Thank you Lars Bernspång who helped us with the calculations. The result is shown in Table 1. Each column represent one case of the location of the stretcher where the values correspond to the vertical measure for the two joints. The zero point is in the low left corner in Figure 2. The first column therefore shows the case calculated by hand above. In the second column the stretcher is moved horizontally upwards 0.04 m.

Moving the stretcher upwards results in surprisingly small changes in the

Mom	.20,.20	.24,.24	.16,.16	.20,.24	.20,.16	.24,.20	.24,.16	.16,.20	.16,.24
M_{14}	48.7	49.7	47.6	52.7	44.2	44.2	38.7	51.0	53.7
M_{41}	53.4	57.2	49.4	65.7	42.2	44.9	34.2	60.6	72.3
M_{12}	48.7	49.7	47.6	52.7	44.2	44.2	38.7	51.0	53.7
M_{21}	56.6	55.9	57.2	58.4	54.8	53.2	50.8	58.3	59.1
M_{23}	56.6	55.9	57.2	58.4	54.8	53.2	50.8	58.3	59.1
M_{32}	81.3	77.1	85.6	85.5	79.1	73.3	71.6	88.4	92.5
M_{34}	38.7	42.8	34.8	34.5	40.9	46.7	48.4	31.6	27.5
M_{43}	6.6	14.8	1.3	6.3	5.8	15.1	13.8	0.6	0.3
M_{45}	60.0	72.0	48.0	72.0	48.0	60.0	48.0	60.0	72.0

Table 1: Bending force, or moment, at the end of the beams in the chair

bending forces. For instance, the force is almost identical in point M_{14}, i.e. 49.7 compared to 48.7 Nm in the left side of the stretcher, see column one and two. The right side moment in the same item is increased by 3 Nm while the force in the horizontal part in the top right corner is decreased by 3 Nm. The major differences occur in point 4 where M_{43} is doubled to 14.8 Nm and M_{45} increases to 72 Nm.

A move downwards by 0.04 m, see column three, results in similar small changes, e. g. M_{23} equals 57 Nm or is increased by only one Nm. The moment in the horizontal beam at the top right corner is now 85.9 Nm or an increase by 5 Nm compared with the first case. At the same time the vertical beam in the same corner will loose about 5 Nm. M_{45} will decrease to 48 Nm.

In order to make the moments more equal it therefore seems as if we must raise the stretcher in its right part. This, because the bending force in the top right horizontal part decreased when the stretcher was moved upwards. However, further investigations showed that if the stretcher is lowered by 0.04 m in the right side and raised by the same amount in the left side, column number seven, the moment in the right corner of the top horizontal beam will equal only 71.6 Nm. If the opposite strategy is chosen, i. e. lowering the left side while at the same time raising the right side, the critical moment is increased to 92 Nm, see column number nine. This result encourages us to move the stretcher to the lowest point in the right side of the frame and at the same time at the top left joint. The moment in the top left corner will now become 17.9 Nm where the horizontal beam is drawn underneath. The same moment occurs in the top left corner of the diagonal element which is tensed on the upper side. In point three the horizontal beam has a moment of 61.5 Nm while the vertical beam has a moment of 58.5 Nm. In the bottom right corner 16.6 Nm occurs. The frame in the chair should from the point of solid mechanics be designed as shown in Figure 5.

5 Discussion and Conclusions

From Figure 5 it is clear that ordinary design of chairs does not take solid mechanics into account as to minimise the stresses in the wooden frame. However, much more knowledge is needed in this field of research. Perhaps it would be better to change the cross sectional area for the details instead of changing their location. If, for instance, the diagonal part in the frame is designed only for taking up the bending force a very thin stretcher could be used. This in turn emphasises the need for taking stability problems into account. The maximum

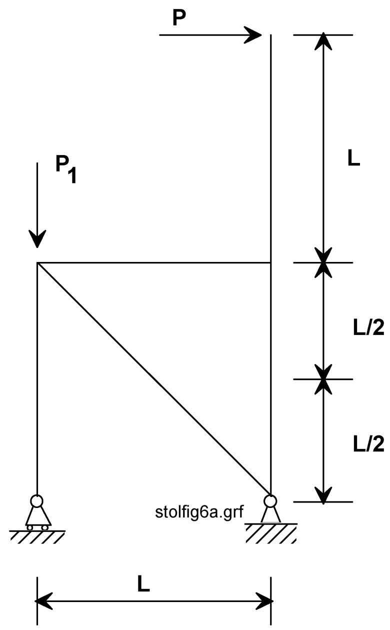

Figure 5: Frame which makes the critical moment in the right side of the horizontal beam as small as possible. Stuhlschema zur Minimierung des kritischen Moments der Quersprossen.

axial force is 628 N which occurs at the same time as the joints are bent. Another problem which is not dealt with here is the fact that the limit strength for wood differs very much between tension and compression. In (Antic, 1994), i.e. Reference [6] it is shown that a total compression load of 2700 N could be implemented in a rod of 0.035 times 0.01 m without stability problems. The rod was made of beech and had a free length of 0.72 m. The case tested was a so called Euler IV where the ends of the rod were tightly fixed. There was no bending force applied at all but the example shows that much weaker constructions could be designed if the wooden material should be utilised to maximum stress.

6 Acknowledgement

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